PROGRAM OF THE WPI WORKSHOP -
MATHEMATICS OF MOIST ATMOSPHERIC DYNAMICS:
MODELING, ANALYSIS AND COMPUTATIONS

Monday, June 12

14:00-14:05: *opening*

14:05-15:30: Olivier Pauluis: *Tutorial 1: Thermodynamic properties of cloudy air*

15:45-16:30: Didier Bresch: *Tutorial 1: Mathematical analysis of relevant compressible geophysical models*

17:00-18:00: Rupert Klein: *How Mathematics helps structuring climate discussions* (Public Lecture)

Tuesday, June 13

9:00-10:30: Olivier Pauluis: *Tutorial 2: Thermodynamic cycles and heat engines*

11:00-11:45: Didier Bresch: *Tutorial 2: Mathematical analysis of relevant compressible geophysical models*

12:00-14:00: lunch break

14:00-15:30: Didier Bresch: *Tutorial 3: Mathematical analysis of relevant compressible geophysical models*

Wednesday, June 14

10:00: *general discussion*

12:00-15:00: lunch break

15:00-15:45: Sam Stechmann: *Precipitating Quasi-Geostrophic Equations and Minimal Cloud Microphysics*

16:00-16:45: Olivier Pauluis: *Thermodynamic analysis of atmospheric motions*

17:00-17:45: Boualem Khouider: *A zonally symmetric model for the monsoon-Hadley circulation with stochastic convective forcing*

19:00 *social event*

Thursday, June 15

9:00-9:45: Tom Dörffel: *Intensification of atmospheric vortices through asymmetric diabatic heating*

10:00-10:45: Rupert Klein: *The role of multiscale convection in hurricane intensification*

11:00-11:45: Piotr Smolarkiewicz: *Finite-volume integrators for cloud-resolving simulations of global atmospheric flows*

12:00-14:00: lunch break

14:00-15:15: Wojtec Grabowski: *Modeling condensation in cloud-scale models*

15:30-16:15: Annette Müller: *The DSI as an indicator for diabatic processes across the scales*

Friday, June 16

9:00-9:45: Matthias Hieber: *Thermodynamical Consistent Modeling and Analysis of Heat-Conducting Fluids*

10:00-10:45: Manuel Baumgartner: *Diffusional Growth in Clouds*

11:00-11:45: Jinkai Li: *Some mathematical analyses on two dynamical models for atmosphere with moisture*

12:00: *closing*

Abstracts for the tutorials:

Didier Bresch: *Mathematical analysis of relevant compressible geophysical models*

In this talk, we talk about mathematical results related to compressible fluid systems with applications to geophysical flows. We focus on pressure laws, viscosity effects, bi-fluid flows description. Some singular limits are also discussed.

Olivier Pauluis:

Tutorial 1: Thermodynamic properties of cloudy air

In this tutorial, I will review the thermodynamic properties cloudy air and how they are typically treated in numerical models. This will include the concepts of saturation, equation of state for moist air, moist entropy and potential temperature of many kinds. We will then discuss the implications for buoyancy and convective processes.

Tutorial 2: Thermodynamic cycles and heat engines

The atmosphere can be describe as a heat engine that continuously generates kinetic energy by transporting energy from a warm source, i.e. the Earth surface, to a cold sink, i.e the colder troposphere. However, the ability of the atmosphere to generate kinetic energy is strongly reduced by the hydrological cycle. We will analyze how the impacts of moist processes can be a quantified in terms of a Gibbs penalty associated with the evaporation of water in unsaturated air and its removal as liquid water.

Abstract for the public lecture:

Rupert Klein: *How Mathematics helps structuring climate discussions*

Mathematics in climate research is often thought to be mainly a provider of techniques for solving the continuum mechanical equations for the flows of the atmosphere and oceans, for the motion and evolution of Earth's ice masses, and the like. Three examples will elucidate that there is a much wider range of opportunities.

Climate modellers often employ reduced forms of "the continuum mechanical equations" to efficiently address their research questions of interest. The first example discusses how mathematical analysis can provide systematic guidelines for the regime of applicability of such reduced model equations.

Meteorologists define "climate", in a narrow sense, as "the statistical description in terms of the mean and variability of relevant quantities over a period of time" (World Meteorological Society, http://www.wmo.int; see the website for a broader sense definition). Now, climate researchers are most interested in changes of the climate over time, and yet there is no unique, well-defined notion of "time dependent statistics". In fact, there are restrictive conditions which data from time series need to satisfy for classical statistical methods to be applicable. The second example describes recent developments of analysis techniques for time series with non-trivial temporal trends.

Modern climate research has joined forces with economy and the social sciences to generate a scientific basis for informed political decisions in the face of global climate change. One major type of problems hampering progress of the related interdisciplinary research consists of often subtle language barriers. The third example describes how mathematical formalization of the notion of "vulnerability" has helped structuring related interdisciplinary research efforts.

Abstracts for the talks:

Manuel Baumgartner: *Diffusional Growth in Clouds* (with Peter Spichtinger)

Diffusional growth is the most important growth mechanism for newly formed cloud droplets and ice crystals. Non-linear diffusion equations control the transport of water vapor towards the cloud particles. Although the solution of these diffusion equations is circumvented in numerical cloud models, it remains computationally expensive to include the details of diffusional growth due to severe timestep restrictions. Moreover, as soon as ice crystals are present in a cloud consisting mostly of cloud droplets, the Wegener-Bergeron-Findeisen process becomes active and the ice crystals grow at the expense of the cloud droplets.

In the first part of the talk, we discuss the aspect of locality of the Wegener-Bergeron-Findeisen process, i.e. an ice crystal does only affect its immediate vicinity. Its presence decouples the diffusional growth behavior of nearby droplets from environmental conditions. We show some simulation results and a possible way to include locality in the context of bulk-microphysics.

The second part considers the case of a liquid cloud. In the context of numerical models, the microphysical details of the diffusional growth and the timestep restrictions are effectively avoided through the technique of saturation adjustment. We will show some of these techniques and analyze an air parcel model containing activation of new droplets using asymptotics.

Tom Dörffel: *Intensification of atmospheric vortices through asymmetric diabatic heating* (with Ariane Papke, Rupert Klein)

The dynamics of atmospheric vortices such as tropical storms, hurricanes and mid-latitude cyclones is driven by a variety of interacting scales. [1] developed an asymptotic theory for the dynamics of strongly tilted atmospheric vortices in the gradient-wind regime, embedded into a synoptic-scale geostrophic background field.

One central outcome of the theory is the evolution equation for the nearly axisymmetric primary circulation. It predicts that Fourier-mode 1 of asymmetric diabatic heating/cooling patterns can spin up or spin down a vortex depending on the relative arrangement of the heating dipole relative to the vortex tilt.

Based on this methodology further investigations led to the conclusion that this theory is generalizable to Rossby numbers of order 1 and higher, i.e. cyclostrophic balance. Accompaning the asymptotics numerical experiments are conducted to test the theory within an anelastic model [2]. In this talk we present the latest results showing consistency of numerical simulations and theoretical predictions.

[1] E. Päschke, P. Marschalik, A. Z. Owinoh and R. Klein, *Motion and structure of atmospheric mesoscale baroclinic vortices: dry air and weak environmental shear*, J. Fluid Mech. 701: 137–170, (2012)

[2] J. M. Prusa, P. K. Smolarkiewicz and A. A. Wyszogrodzki, *EULAG, a computational model for multiscale flows*, Comput. Fluids 37: 1193–1207 (2008)

Wojciech W. Grabowski: *Modeling condensation in cloud-scale models*

Condensation of water vapor to form and grow cloud droplets is the most fundamental process of cloud and precipitation formation. It drives cloud dynamics through the release of latent heat and determines the strength of convective updrafts. Cloud-scale models simulate condensation by applying two drastically different methods. The first one is the bulk condensation where condensation/evaporation is assumed to always maintain saturated conditions. The second approach involves prediction of the in-cloud super- or sub-saturation and can be used in models that predict not only condensate mass but also relevant features of the droplet size distribution (e.g., models with the 2-moment microphysics or with the bin microphysics). This presentation will address the question whether the difference between the two approaches has a noticeable impact on convective dynamics. Model simulations with the bin microphysics for shallow non-precipitating convection and with the double-moment bulk microphysics for deep convection will be discussed to document the differences in cloud field simulations applying the two methodologies. For the shallow convection, the differences in cloud field simulated with bulk and bin schemes come not from small differences in the condensation, but from more significant differences in the evaporation of cloud water near cloud edges as a result of entrainment and mixing. For the deep convection, results show a significant dynamical impact of finite supersaturations and a strong microphysical effect associated with upper-tropospheric anvils. Implications of these results for modeling convective dynamics will be discussed and a possible intermediate modeling methodology will be suggested.

Matthias Hieber: *Thermodynamical Consistent Modeling and Analysis of Heat-Conducting Fluids*

In this talk, we derive and discuss thermodynamically consistent models for heat-conduction fluids. Our approach is based on the entropy principle.

Boualem Khouider: *A zonally symmetric model for the monsoon-Hadley circulation with stochastic convective forcing*

Idealized models of reduced complexity are important tools to understand key processes underlying a complex system. In climate science in particular, they are important for helping the community improve our ability to predict the effect of climate change on the earth system. Climate models are large computer codes based on the discretization of the fluid dynamics equations on grids of horizontal resolution in the order of 100 km, whereas unresolved processes are handled by subgrid models. For instance, simple models are routinely used to help understand the interactions between small-scale processes due to atmospheric moist convection and large-scale circulation patterns.

Here, a zonally symmetric model for the monsoon circulation is presented and solved numerically. The model is based on the Galerkin projection of the primitive equations of atmospheric synoptic dynamics onto the first modes of vertical structure to represent free tropospheric circulation and is coupled to a bulk atmospheric boundary layer (ABL) model. The model carries bulk equations for water vapor in both the free troposphere and the ABL, while the processes of convection and precipitation are represented through a stochastic model for clouds. The model equations are coupled through advective nonlinearities, and the resulting system is not conservative and not necessarily hyperbolic.

This makes the design of a numerical method for the solution of this system particularly difficult.

We develop a numerical scheme based on the operator time-splitting strategy, which decomposes the system into three pieces: a conservative part and two purely advective parts, each of which is solved iteratively using an appropriate method. The conservative system is solved via a central scheme, which does not require hyperbolicity since it avoids the Riemann problem by design. One of the advective parts is a hyperbolic diagonal matrix, which is easily handled by classical methods for hyperbolic equations, while the other advective part is a nilpotent matrix, which is solved via the method of lines. Validation tests using a synthetic exact solution are presented, and formal second-order convergence under grid refinement is demonstrated. Moreover, the model is tested under realistic monsoon conditions, and the ability of the model to simulate key features of the monsoon circulation is illustrated in two distinct parameter regimes.

This is joint work with Michále De La Chevrotiáre.

Rupert Klein: *The role of multiscale convection in hurricane intensification*

Paeschke et al (2012) showed analytically how non-axisymmetric external diabatic forcing of a tilted vortex in dry air can amplify or attenuated the flow depending on the relative orientation of vortex tilt and the "heating dipole". Here we include a bulk moist microphysics closure and describe how boundary layer processes and multiscale deep moist convection can interact to produce this effect self-consistently.

Jinkai Li: *Some mathematical analyses on two dynamical models for atmosphere with moisture* (with Sabine Hittmeir, Rupert Klein, Edriss S. Titi)

In this talk, we will present some recent mathematical results, mainly the global well-posedness and convergence of the relaxation limit, on two kinds of dynamical models for the atmosphere with moisture. In the first part of this talk, which is a joint work with Edriss S. Titi [1], we will consider a tropical atmosphere model introduced by Frierson, Majda, and Pauluis (Commum. Math. Sci. 2004); for this model, we will present the global well-posedness of strong solutions and the strong convergence of the relaxation limit, as the relaxation time ε tends to zero. It will be shown that, for both the finite-time and instantaneous-relaxation systems, the H^1 regularities on the initial data are sufficient for both the global existence and uniqueness of strong solutions, but slightly more regularities than H^1 are required for both the continuous dependence and strong convergence of the relaxation limit. In the second part of this talk, which is a joint work with Sabine Hittmeir, Rupert Klein, and Edriss S. Titi [2], we will consider a moisture model for warm clouds used by Klein and Majda (Theor. Comput. Fluid Dyn. 2006), where the phase changes are allowed, and we will present the global well-posedness of this system.

[1] Jinkai Li; Edriss S. Titi: *A tropical atmosphere model with moisture: global well-posedness and relaxation limit*, Nonlinearity, **29** (2016), 2674–2714.

[2] Sabine Hittmeir; Rupert Klein; Jinkai Li; Edriss S. Titi: *Global well-posedness for*

passively transported nonlinear moisture dynamics with phase changes, arXiv:1610.00060

Annette Müller: *The DSI as an indicator for diabatic processes across the scales*

In atmospheric flows, the Dynamic State Index (DSI) indicates local deviations from a steady wind solution. This steady wind solution is based on the primitive equations under adiabatic and inviscid conditions. Hence, from theoretical point of view, atmospheric dynamics is regarded relative to a solution derived from fluid mechanic's first principles. Thus, this parameter provides a tool to capture diabatic processes. The DSI can be designed for different fluid mechanical models on distinguished scales, we will introduce a DSI_{QG} for the quasi-geostrophic flow, a DSI_{Ro} for the Rossby model and DSI_{mois} that is based on the equations of motions including moisture processes.

Olivier Pauluis: *Thermodynamic analysis of atmospheric motions*

In this talk, I will show how to extract thermodynamic cycles from high resolution simulations of atmospheric flows. On the one hand, thermodynamic processes are typically analyzed in terms of the behavior of individual parcel trajectories. On the other hand, most atmospheric flows are associated with infinitely many turbulent lagrangian trajectories. The Mean Air Flow As Lagrangian Dynamics Approximation (MAFALDA) has been recently developed to address this problem. It MAFALDA, the flow is first averaged in isentropic coordinates, typically pressure and equivalent potential temperature, and the mean flow is then treated as a set of thermodynamic cycles. This offer a systematic procedure to analyze the thermodynamic transformation in atmospheric flows, which is applied here to compare the thermodynamics behavior of convection and hurricanes.

Piotr Smolarkiewicz: *Finite-volume integrators for cloud-resolving simulations of global atmospheric flows*

This work extends to moist-precipitating dynamics a recently documented high-performance finite-volume integrators for simulating global all-scale atmospheric flows (doi:10.1016/j.jcp.2016.03.015). A key objective of the current development is a seamless coupling of the conservation laws for moist variables engendered by cloud physics with the semi-implicit, non-oscillatory forward-in-time integrators already proven for dry dynamics. The representation of the water substance and the associated processes in weather and climate models can vary widely in formulation details and complexity levels. The adopted representation assumes a canonical "warm-rain" bulk microphysics parametrisation, recognised for its minimal physical intricacy while accounting for the essential mathematical complexity of cloud-resolving models. A key feature of the presented numerical approach is global conservation of the water substance to machine precision — implied by the local conservativeness and positivity preservation of the numerics — for all water species including water vapour, cloud water, and precipitation. The moist formulation assumes the compressible Euler equations as default, but includes reduced anelastic equations as an option. The theoretical considerations are illustrated with a benchmark simulation of a tornadic thunderstorm on a reduced size planet, supported with a series of numerical experiments addressing the accuracy of the associated water budget.

Sam Stechmann: *Precipitating Quasi-Geostrophic Equations and Minimal Cloud Microphysics*

Two simplified models are presented for precipitating atmospheric dynamics. First, a minimal version of cloud microphysics is presented. The time scales of all microphysical processes are assumed to be fast, and the resulting microphysics has only one parameter, the terminal velocity of falling rain drops. It is shown that, despite its simplicity, this minimal microphysics scheme can reproduce distinct canonical modes of convective organization (scattered convection and a squall line) under appropriate environmental conditions. This suggests that the essential physical processes underlying moist convection are simply phase changes and falling rain drops. Second, a precipitating version of the quasi-geostrophic (QG) equations is presented. The precipitating QG (PQG) equations include phase changes between water vapor and liquid water, which arise as Heaviside nonlinearities in the new PQG PDEs. Finally, we present an initial application of the PQG equations, in a linearized setting that can be solved analytically, to understanding meridional moisture transport by baroclinic eddies.