Exploring Ground States and Excited States of Spin-1 Bose-Einstein Condensates with/without external magnetic field

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Abstract

In this talk, I will first give a brief introduction to the spinor Bose-Einstein condensates. Then I will present two recent results, one is numerical, the other is analytical.

In 1925, Bose and Einstein predicted that massive bosons could occupy the same lowest-energy state at low temperature and formed the so-called Bose-Einstein condensates (BECs). It was realized on several alkali atomic gases in 1995 by laser cooling technique.

In the numerical study of spinor BEC, a pseudo-arclength continuation method (PACM) was proposed and employed to compute the ground state and excited state solutions of spin-1 BEC. Numerical results on the wave functions and their corresponding energies of spin-1 BEC with repulsive/attractive and ferromagnetic/antiferromagnetic interactions are presented. Furthermore, it is found that the component separation and population transfer between the different hyperfine states can only occur in excited states due to the spin-exchange interactions.

In the analytical study, the ground states of spin-1 BEC are characterized. First, we present the case when there is no external magnetic field. For ferromagnetic systems, we show the validity of the so-called single-mode approximation (SMA). For antiferromagnetic systems, there are two subcases. When the total magnetization $M \neq 0$, the corresponding ground states have vanishing zeroth ($m_F = 0$) components (so call 2C state), thus are reduced to two-component systems. When $M = 0$, the ground states are also reduced to the SMA, and there are one-parameter families of such ground states. Next, we study the case when an external magnetic field is applied. It is shown analytically that, for antiferromagnetic systems, the ground state has a bifurcation from 2C state to 3C state as the external magnetic field increases. The key idea in the proof is a redistribution of masses among different components, which reduces kinetic energy in all situations, and makes our proofs simple and unified.

The numerical part is a joint work with Jen-Hao Chen and Weichung Wang, whereas the analytical part is jointly with Liren Lin.