The Equivalence Problem for Finite Structures

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Beginning

Syracuse, early 90's

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The Computer Scientist showed that this problem traces back to equivalences of terms over commutative rings.

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Finite Automatons, Formal Languages, Montreal, Brno, Ekaterinburg

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- Syntactic Monoids

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It was Shown that this problem traces back to equivalences of terms over monoids.

Some Definitions and Examples

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Let $\mathbf{A}$ be an algebra and let t and s be two terms over $\mathbf{A}$.

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- No, $M_n(\mathbb{F})$ is not commutative

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- $[[x, y], [x, z]]^2 \overset{?}{\equiv} 1$ over S_4
- Yes, $[x, y] \in A_4 \implies [[x, y], [x, z]] \in A_4'$
  $A_4' \simeq Z_2 \times Z_2$

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- What is the complexity of the problem for certain class of structures?
- Goal: Prove dichotomy: $\mathrm{TERM\text{-}EQ}$ is either in P or coNP-complete

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For nilpotent groups $TERM\text{-}EQ$ is in P.

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$TERM\text{-}EQ$ is in P for metacyclic groups (semidirect product of cyclic groups).

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SAT can be formulated

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Multiplication:

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and

$$0 \cdot s = 0 = s \cdot 0 \qquad \forall s \in S_{\mathbf{M}}$$

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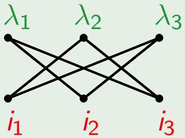

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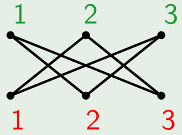

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- $(a_x, b_y) \in E_t \iff xy$ is subword of t

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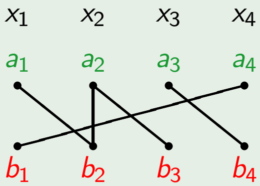
Example

$t = x_1 x_2^2 x_3 x_4 x_1 x_2$



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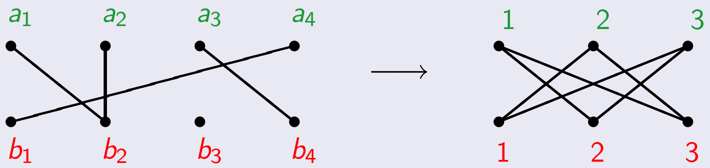

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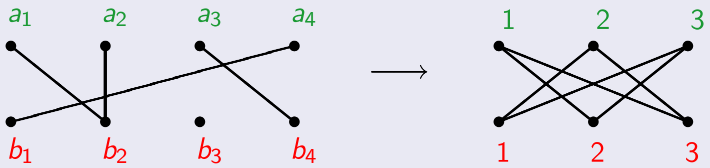

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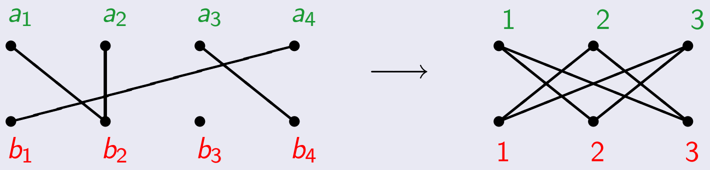

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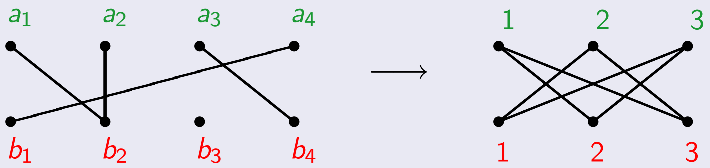

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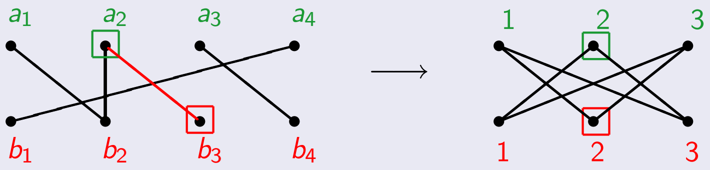

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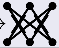
$G_t \to$  $\quad a_x \mapsto \lambda$, $b_x \mapsto i$ if $x = \langle i, \lambda \rangle$

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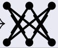
$G_t \to$  $\quad a_x \mapsto \lambda$, $b_x \mapsto i$ if $x = \langle i, \lambda \rangle$

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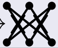
$G_t \to$  $\quad a_x \mapsto \lambda$, $b_x \mapsto i$ if $x = \langle i, \lambda \rangle$

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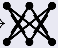
- $t(\vec{a}) \neq 0 \iff G_t \to$  is a homomorphism;

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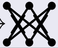
$G_t \to$  $\quad a_x \mapsto \lambda$, $b_x \mapsto i$ if $x = \langle i, \lambda \rangle$

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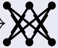
- $t(\vec{a}) \neq 0 \iff G_t \to$  is a homomorphism;
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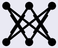
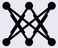
- $G_t \to$  is a homomorphism $\iff$ $G_s \to$  is a homomorphism;

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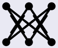
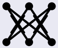
- $G_t \to$  is a homomorphism $\iff$ $G_s \to$  is a homomorphism;
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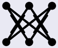
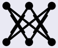
- $G_t \to$  is a homomorphism $\iff$ $G_s \to$  is a homomorphism;
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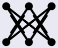
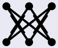
- $G_t \to$  is a homomorphism $\iff$ $G_s \to$  is a homomorphism;
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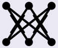
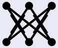
- $G_t \to$  is a homomorphism $\iff$ $G_s \to$  is a homomorphism;
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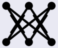
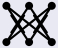
- $G_t \to$  is a homomorphism $\iff$ $G_s \to$  is a homomorphism;
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Multiplication:

$$\langle i, g, \lambda \rangle \langle j, h, \mu \rangle = \begin{cases} \langle i, g\mathbf{M}(\lambda, j)h, \mu \rangle, & \text{if } \mathbf{M}(\lambda, j) \in G \\ 0, & \text{if } \mathbf{M}(\lambda, j) = 0 \end{cases}$$

and

$$0 \cdot s = 0 = s \cdot 0 \qquad \forall s \in S_{\mathbf{M}}$$

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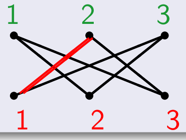
For the semigroup S_P we define a bipartite graph:

$$P = \begin{pmatrix} 0 & a & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$



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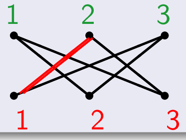

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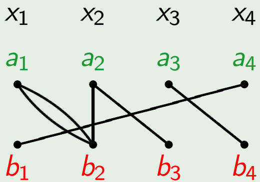

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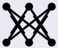
- 2HOM() $\overset{\text{poly}}{\Longleftrightarrow}$ $\mathrm{TERM\text{-}EQ}(S_P)$

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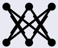
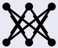
- 2HOM() $\overset{\text{poly}}{\Longleftrightarrow}$ $\mathrm{TERM\text{-}EQ}(S_P)$
- 2HOM() coNP-complete.

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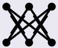
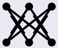
- 2HOM() $\overset{\text{poly}}{\iff}$ $\mathrm{TERM\text{-}EQ}(S_P)$
- 2HOM() coNP-complete.

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Hilbert Theorem 90's

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w is a word that proves coNP-completeness for $M_{n_1}(\mathbb{F}_1) \oplus \cdots \oplus M_{n_k}(\mathbb{F}_k)$ then
w^n proves the coNP-completeness for $\mathcal{R}$

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$\mathit{POL\text{-}EQ}(S) \equiv \mathit{CSP}(B)$

- unfortunately $\mathrm{TERM\text{-}EQ}$ is in P for these structures (Seif, Szabó)
- 0-simple semigroups
  - S_P $\surd$ (Goldberg, VV)
  - Hope: For any CSP(B) $\exists$ a 0-simple semigroup S such that: $\mathrm{POL\text{-}EQ}(S) \equiv \mathrm{CSP}(B)$?
- NP-complete for bands $\surd$ (Klima)
- NP-complete for the Brandt monoid $\surd$ (Klima, Seif)

Problems

Semigroups

- with constants (polynomials) $\equiv$ CSP

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- Combinatorial semigroups ?

Vége = The End

What about your favorite structure?