Some examples of parabolic SPDEs that can be handled by the existence result.

Example 1 $\quad V = H = V' \Rightarrow \mathbb{R}^d$

This is a system of ODEs. In principle, we have already solved it, but under the extra assumption of a Lipschitz continuous A.

The theorem ensures that the Lipschitz continuity assumption can be dropped. It suffices to take

$$A_1(\cdot,\omega) : \mathbb{R}^d \to \mathbb{R}^d \quad \text{continuous \& strongly monotone}$$

For instance $\quad A_1(u) = |u|^{p-2} u, \quad p \in (1,2]$

Example 2 $\quad V = H_0^1(D) \subset L^2(D) = H$

$$\langle A_2(u,\omega), v \rangle = \int_\Omega \mathbb{A}(x,\omega) \nabla u(x) \cdot \nabla v(x) \, dx$$

with $\mathbb{A} : D \times \Omega \to \mathbb{R}^{d \times d}$ bounded and measurable and $\mathbb{A}\xi \cdot \xi \geq \alpha |\xi|^2$ for all $\xi \in \mathbb{R}^d$ and some $\alpha > 0$.

This corresponds to linear SPDEs

$$du - \Delta u \, dt = B \, dW$$

<u>Example 3</u> $A_1 + A_2 : H^1_0(D) \longrightarrow H^{-1}$ gives the <u>semilinear</u> SPDE:

$$du - \Delta u \, dt + |u|^{p-2} u \, dt = B \, dW$$

<u>Example 4</u> We can consider the <u>quasilinear</u> SPDE:

$$du - \nabla \cdot (\beta(\nabla u)) \, dt = B \, dW$$

Where $\beta : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ (possibly also depending on ω) is smooth, linearly bounded, and monotone, i.e.

$$\beta(\xi) \cdot \xi \geq \alpha |\xi|^2 \quad \forall \xi \in \mathbb{R}^d$$

and some $\alpha > 0$.

This includes the cases:

p-Laplacian: $\beta(\xi) = |\xi|^{p-2} \xi$ $\qquad p \in (1, 2]$

Stefan problem: $\beta(\xi) = -\xi^- + (\xi - 1)^+$

the case $p > 2$ can be treated as well.

Example 5 higher order SPDEs:

$$du + \Delta^2 u\, dt - \Delta u\, du = B\, dW$$

with $V = \{ u \in H^2(D) : u = 0,\ \nabla u = 0 \text{ on } \partial D \}$

Example 6 Stochastic damped wave equation $(\alpha > 0)$ written as first order system:

$$\begin{cases} dv + \alpha A v\, dt + A w\, dt = B\, dW \\ dw - v\, dt + \alpha w\, dt = 0 \end{cases}$$

Assume that $A : V \longrightarrow V'$ is the Riesz isomorphism (one can always do so) and rewrite the system as

$$\begin{cases} dv + A w\, dt = B\, dW \\ A(dw) - A v\, dt + \alpha A w\, dt = 0 \end{cases}$$

$$\hookrightarrow \quad A w(t) = A w(0) + \int_0^t A v\, dt - \alpha \int_0^t A w$$

Define $u = (v, w)$ $V = (H^1_0(D))^2$, $H = (L^2(D))^2$

$$\left\langle \mathcal{A}\begin{pmatrix} v \\ w \end{pmatrix}, \begin{pmatrix} \bar{v} \\ \bar{w} \end{pmatrix} \right\rangle = \langle A w, \bar{v} \rangle - \langle A v, \bar{w} \rangle + \alpha \langle A v, v \rangle + \alpha \langle A w, w \rangle$$