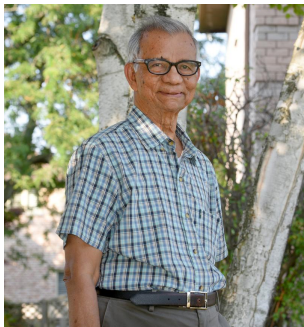


## PREFACE TO THE SPECIAL ISSUE FOR THE 2021 LATTICE PATH CONFERENCE

CYRIL BANDERIER, CHRISTIAN KRATTENTHALER, AND MICHAEL WALLNER

### 1. THE LATTICE PATH CONFERENCE

**1.1. History of the conference.** Following an interdisciplinary interest for lattice paths after the publication in 1979 of the two books *Lattice Path Combinatorics with Statistical Applications* by Tadepalli Venkata Narayana and *Lattice Path Counting and Applications* by Sri Gopal Mohanty, Mohanty initiated a series of conferences on this topic.



Sri Gopal Mohanty, father of the Lattice Path Conference.

The first two events were held at McMaster University in 1984 and 1990, and then at the University of Delhi in 1994, University of Vienna in 1998, University of Athens in 2002, East Tennessee State University in 2007, University of Siena in 2010, Cal Poly Pomona University in 2015, Centre International de Rencontres Mathématiques in 2021. A new initiative started by dedicating the fourth conference in Wien to the memory of Germain Kreweras (1918–1998) and Tadepalli Venkata Narayana (1930–1987), both of whom made a significant contribution to the field. In the same spirit, the 2002 conference was dedicated to the memory of István Vincze (1912–1999). The 2015 conference was dedicated to Shreeram Shankar Abhyankar (1930–2013), Philippe Flajolet (1948–2011), and Lajos Takács (1924–2015).

For each conference there is the tradition to have a journal special issue where anyone could submit work related to lattice paths.<sup>1</sup> We shall come back to the current special issue in Section 2. Before, let us say a few words about the event itself.

**1.2. The 2021 Lattice Path Conference.** The 9th Lattice Path Conference was held in hybrid form (given the circumstances of the global Covid pandemic) during 21–25 June 2021 at the Centre International de Rencontres Mathématiques (CIRM) in Luminy, France. This event attracted 227 registered participants with affiliations from the “four corners of the world” from 34 different countries of which 24 attended in person. Moreover, about 40 more people attended casually online without being registered (we made the event fully open), and some colleagues even organized some watch parties in their university, so the real number of participants in the event will remain unknown.

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<sup>1</sup>See the website of the [Lattice Path Conference](#) for links to these special issues and further information.



As visible via some details in the above photos,  
the 2021 Lattice Path Conference was a hybrid conference.

The conference consisted of 27 invited talks and 22 accepted posters, leaving also time for scientific discussions. The presentations covered a wide range of topics, including

- algebraic combinatorics (Young tableaux, representation theory, symmetric polynomials, representation theory, ...),
- analytic combinatorics (kernel method, symbolic method, asymptotics, ...),
- enumerative combinatorics (walks in convex and non-convex cones, alternating sign matrices, generating functions, bijections, ...),
- probability theory (Brownian motion, Markov chains, random trees, ...),
- computer algebra (orthogonal polynomials, holonomy theory, ...),
- theoretical computer science (random generations, context-free grammars, ...),
- number theory (integer partitions, continued fractions, ...),
- theoretical physics (Liouville quantum gravity, square ice, ...).

The conference was a great success, with many stimulating talks and discussions. We would like to thank all of the participants, including the invited speakers and the attendees, for their contributions to the conference. We would also like to extend our gratitude to the staff at CIRM, who, like always, was warmly welcoming, even in this difficult period: CIRM reopened for our meeting after many months of closure due to the Covid containment and the interdiction of meetings. Finally, we would like to thank Kilian Raschel who was the main sponsor of this event via his ERC starting grant COMBINEPIC, which allowed us to help CIRM who financially suffered a lot of so many months of closure.

This event successfully brought together leading experts in the field and contributed to the development of new research directions. Several conjectures were even solved during the event! We hope that our special issue dedicated to this conference will serve as a valuable resource for researchers in combinatorics, probability theory, and statistical physics, and that they will inspire further research on lattice paths.

Let us now give the lists of organizers, speakers, and participants of this meeting.

### 1.3. Scientific and organizing committee.

- Cyril Banderier (CNRS, Université Sorbonne Paris Nord)
- Jehanne Dousse (CNRS, Université Lyon 1)
- Enrica Duchi (Université Paris Diderot)
- Christian Krattenthaler (Universität Wien)
- Greta Panova (University of Southern California)
- Kilian Raschel (CNRS, Université de Tours)
- Michael Wallner (TU Wien)

#### 1.4. Invited talks.



George Andrews (Pennsylvania State University):  
*Schmidt type partitions and partition analysis*



Andrei Asinowski (Alpen-Adria-Universität Klagenfurt):  
*Vectorial kernel method and lattice paths with patterns*



Philippe Biane (CNRS, Université Paris-Est):  
*Mating of discrete trees and walks in the quarter-plane*



Alin Bostan (INRIA, Saclay):  
*How to prove or disprove the algebraicity of a generating function using a computer*



Mireille Bousquet-Mélou (CNRS, Université de Bordeaux):  
*Invariants for walks avoiding a quadrant*



Timothy Budd (Radboud University):  
*Winding of simple walks on the square lattice*



Philippe Di Francesco (University of Illinois at Urbana-Champaign and IPhT Saclay):  
*Triangular ice: combinatorics and limit shapes*



Sergi Elizalde (Dartmouth College):  
*Counting lattice paths by the number of crossings and major index*



Ilse Fischer (University of Vienna):  
*The alternating sign matrices/descending plane partitions relation:  
 $n+3$  pairs of equivalent statistics*



Ira Gessel (Brandeis University):  
*Redundant generating functions in lattice path enumeration*



Vadim Gorin (MIT):  
*Addition of matrices at high temperature*



Tony Guttmann (Melbourne University):  
*Extracting asymptotics from series coefficients*



Nina Holden (ETH Zürich):  
*Random triangulations and bijective paths to Liouville quantum gravity*



Mourad E.H. Ismail (University of Central Florida):  
*Orthogonal polynomials, moments, and continued fractions*



Satya Majumdar (CNRS, Université Paris Sud):  
*Nonintersecting Brownian bridges in the flat-to-flat geometry*



Olya Mandelshtam (University of Waterloo):  
*A Markov chain on tableaux that projects to a multispecies totally asymmetric zero range process*



Irène Marcovici (Université de Lorraine):  
*Bijections between walks inside a triangular domain and Motzkin paths of bounded amplitude*



Stephen Melczer (University of Pennsylvania):  
*Lattice walks and analytic combinatorics in several variables*



Robin Pemantle (University of Pennsylvania):  
*Generating function technologies: applications to lattice paths*



Bruno Salvy (INRIA / ENS Lyon):  
*Computation of tight enclosures for Laplacian eigenvalues*



Michael Singer (North Carolina State University):  
*Differentially algebraic generating series for walks in the quarter plane*



Perla Sousi (University of Cambridge):  
*The uniform spanning tree in 4 dimensions*



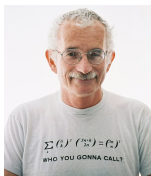
Andrea Sportiello (CNRS, Université Paris Nord):  
*Boltzmann sampling in linear time: irreducible context-free structures*



Xavier Viennot (CNRS, Université de Bordeaux):  
*Heaps and lattice paths*



Karen Yeats (University of Waterloo):  
*Łukasiewicz walks and generalized tandem walks*



Doron Zeilberger (Rutgers University):  
*Using symbolic dynamical programming in lattice paths combinatorics*



Paul Zinn-Justin (Melbourne University):  
*Generalized pipe dreams and lower-upper scheme*

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A recording of all the talks can be found at <https://lipn.fr/~banderier/LPC/2021/>.

### 1.5. Accepted posters.

- Ault Shaun, Charles Kicey: *From lattice paths to standard Young tableaux*
- Cyril Banderier, Marie-Louise Lackner, Michael Wallner: *Latticepathology and symmetric functions*
- Nicholas Beaton: *Walks obeying two-step rules on the square lattice*
- Swee Hong Chan, Igor Pak, Greta Panova: *Log-concavity in posets and random walks*
- Sergey Dovgal, Mohamed Lamine Lamali, Philippe Duchon: *A phase transition in non-deterministic walks with two or more variables*
- Andrew Elvey Price: *Enumeration of walks with small steps by winding angle*
- Rigoberto Flórez, José L. Ramírez, Fabio A. Velandia, Diego Villamiza: *Restricted Dyck paths*
- Xi Chen, Bishal Deb, Alexander Dyachenko, Tomack Gilmore, Alan Sokal: *Coefficientwise total positivity of some matrices defined by linear recurrences*
- Hans Höngesberg: *Weight-preserving bijections between integer partitions and a class of alternating sign trapezoids*
- Heba Ayeda, David Beecher, Alan Krinik, Jeremy J. Lin, David Perez, Thuy Vu Dieu Lu, Weizhong Wong: *Lattice paths with alternating probabilities*
- Josef Küstner, Michael Schlosser, Meesue Yoo: *Lattice paths and negatively indexed weight-dependent binomial coefficients*
- Florian Lehner, Christian Lindorfer, Wolfgang Woess: *The language of self-avoiding walks*
- Satya Majumdar, Francesco Mori, Gregory Schehr: *Distribution of the time between maximum and minimum of random walks*
- Stéphane Ouvry, Alexios Polychronakos, Shuang Wu: *Algebraic area counting for lattice closed random walks*
- Alan Krinik, Gerardo Rubino: *The exponential-dual matrix method: applications to Markov chain analysis*
- Andrei Asinowski, Benjamin Hackl, Sarah Selkirk: *Down-step statistics in generalized Dyck paths*
- Myrto Kallipoliti, Robin Sulzgruber, Elini Tzanaki: *Patterns in Shi tableaux and Dyck paths*
- Malvina Vamvakari: *On  $q$ -order statistics*
- Florian Aigner, Gabriel Frieden:  *$qRSt$ : A probabilistic Robinson–Schensted correspondence for Macdonald polynomials*
- Quang-Nhat Le, Sinai Robins, Christophe Vignat, Tanay Wakhare: *A continuous analogue of lattice path enumeration*
- Jisun Huh, Sun-Young Nam, Meesue Yoo: *LLT polynomials in a nutshell: on Schur expansion of LLT polynomials*
- Benjamin De Bruyne, Satya Majumdar, Gregory Schehr: *Generating discrete-time constrained random walks.*

The poster session was organized on the online platform [Gather.town](https://gather.town), where all the posters can still be perused.

1.6. **The Hotel Latticepathologia escape game.** The image below shows a part of the “Hotel Latticepathologia” (a virtual location designed by Cyril Banderier, Jehanne Dousse, and Michael Wallner), where the poster session was taking place.



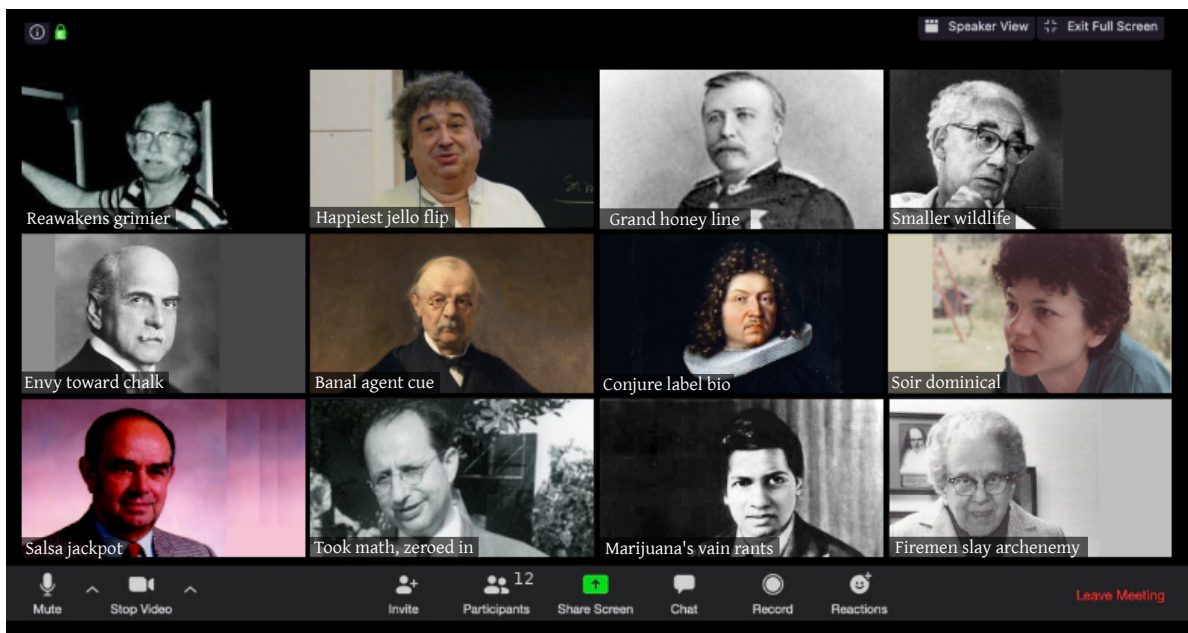
This virtual place gave the online participants the opportunity to socialize and to participate in an escape game! In its first part, you have to identify a mysterious mathematician in each room in the Hotel Latticepathologia (<https://tinyurl.com/s6njbsku>), using clues scattered throughout. Will you succeed?

In its second part, you have to solve a collection of puzzles designed in collaboration with Vivien Ripoll; see <https://lipn.fr/~cb/LPC/2021/Puzzles/>. It includes a musical concert by 9 conference participants. Don't miss it!

Below is a sample puzzle from the escape game. Can you solve it?

## Confusing Dream

*I had the strangest dream last night: a Zoom meeting with some of our [mathematical heroes](#). Luckily I was able to take a screenshot before waking up! They look utterly **confused** by the situation, though I note they all have one extraordinary feature.*



*They were discussing the discovery of a new lattice path. I managed to copy it there:* <https://00.00/00000000>

**1.7. Registered participants.** Most of the 227 registered participants were online. A few of them had the opportunity to enjoy CIRM's beautiful surroundings.



List of registered participants: David Adame-Carrillo, Mohammed Ageel, Florian Aigner, Marie Albenque, Seamus Albion, Ian Alevy, Irha Ali, George Andrews, Omer Angel, Margaret Archibald, David Ash, Andrei Asinowski, Shaun Ault, Jean-Christophe Aval, Heba Ayeda, Arvind Ayyer, Beáta Bényi, Cyril Banderier, Josaphat Baolahy, Elena Barcucci, Jean-Luc Baril, Erik Bates, Nicholas Beaton, David Beecher, Chiheb Ben Bechir, Sudip Bera, Olivier Bernardi, Antonio Bernini, David Bevan, Philippe Biane, Arthur Blanc-Renaudie, Aubrey Blecher, Alin Bostan, Mireille Bousquet-Mélou, Cédric Boutillier, Jérémie Bouttier, Timothy Budd, Théophile Buffière, Ariane Carrance, Giulio Cerbai, Swee Hong Chan, Linxiao Chen, Shaoshi Chen, Frédéric Chyzak, Lapo Cioni, Alice Contat, Michael Coopman, Sylvie Corteel, Logan Crew, Cesar Cuenca, Nicolas Curien, Stéphane Dartois, Benjamin De Bruyne, Bishal Deb, Nachum Dershowitz, Hiranya Kishore Dey, Philippe Di Francesco, Lucia Di Vizio, Ruiwen Dong, Robert Donley, Jehanne Dousse, Sergey Dovgal, Thomas Dreyfus, Michael Drmota, Enrica Duchi, Dennis Eichhorn, Sergi Elizalde, Andrew Elvey Price, Sen-Peng Eu, Wenjie Fang, Valentin Féray, Luca Ferrari, Ilse Fischer, Rigoberto Flórez, Luis Fredes, Gabriel Frieden, Éric Fusy, Ira Gessel, Sudhir Ghorpade, Juan Gil, Tomack Gilmore, Vadim Gorin, Adam Gregory, Tony Guttmann, Hans Höngesberg, Benjamin Hackl, Aliakbar Haghighi, Eva-Maria Hainzl, Charlotte Hardouin, Kilian Hermann, Clemens Heuberger, Pawel Hitczenko, Hung Hoang, Nina Holden, Sam Hopkins, Yueyun Hu, Justin Hua, Mourad E. H. Ismail, Svante Janson, Helen Jenne, Frédéric Jouhet, Josef Küstner, Wonwoo Kang, Manuel Kauers, Rinat Kedem, Ghizlane Kettani, Mikhail Khristoforov, Charles Kicey, Donghyun Kim, Sergey Kirgizov, Victor Kleptsyn, Arnold Knopfmacher, Isaac Konan, Irina Kourkova, Christian Krattenthaler, Alan Krinik, Nishu Kumari, Raunak Kundagrami, Florian Lehner, Helder Lima, Zhicong Lin, Christian Lindorfer, Martin Loeb, Baptiste Louf, Torsten Mütze, Satya Majumdar, Pritam Majumder, Olya Mandelshtam, Jean-Francois Marckert, Irène Marcovici, Barbara Margolius, Hana Melánová, Stephen Melczer, Laurent Menard, Sri Gopal Mohanty, Derrick Mohlala, Francesco Mori, Lukas Nabergall, Philippe Nadeau, Victor Nador, Mehdi Naima, Hiroshi Naruse, Andreas Nessmann, David Nguyen, Hadrien Notarantonio, Soichi Okada, Stéphane Ouvry, J. E. Paguyo, Nimisha Pahuja, Greta Panova, Jay Pantone, Eveliina Peltola, Robin Pemantle, Karol Penson, Leonid Petrov, Renzo Pinzani, Thomas Prellberg, Helmut Prodinger, Sanjay Ramassamy, José Ramírez, Kilian Raschel, Vivien Ripoll, Tom Roby, Martin Rubey, Gerardo Rubino, Tiadora Ruza, Nasser Saad, Bruce Sagan, Manjil Pratim Saikia, Bruno Salvy, Yoshio Sano, Gilles Schaeffer, Gregory Schehr, Michael Schlosser, Jeanne Scott, Blair Seidler, Sarah Selkirk, Timo Seppalainen, Michael F. Singer, Alexandros Singh, Erik Slivken, Rebecca Smith, Alan Sokal, U-Keun Song, Perla Sousi, Andrea Sportiello, Richard Stanley, Dennis Stanton, Benedikt Stuffer, Adrian Tanasa, Benjamin Terlat, Vasu Tewari, Paul Thévenin, Mikhail Tikhonov, Jordan Tirrell, Jessica Tomasko, Joonas Turunen, Eleni Tzanaki, Malvina Vamvakari, Roger Van Peski, Zoé Varin, Ekaterina Vassilieva, Fabio Velandia, Xavier Viennot, Christophe Vignat, Diego Villamizar, Michael Voit, Trung Vu, David Wahiche, Tanay Wakhare, Michael Wallner, Harriet Walsh, Guoliang Wang, Sebastian Wild, Mark Wilson, Peter Winkler, Wolfgang Woess, Elaine Wong, Zaidan Wu, Karen Yeats, Meesue Yoo, Sergey Yurkevich, Doron Zeilberger, Noam Zeilberger, Jiang Zeng, Yan Zhuang, Paul Zinn-Justin.



## 2. OUR SPECIAL ISSUE IN THE “SÉMINAIRE LOTHARINGIEN DE COMBINATOIRE”

The “Séminaire Lotharingien de Combinatoire” is an international biannual seminar, cofounded in 1980 by Dominique Foata (Strasbourg), Adalbert Kerber (Aachen and Bayreuth), and Volker Strehl (Erlangen). The name of the seminar comes from the fact that these cities were almost covered by Lotharingia, a part of the Carolingian Empire. In 1994, an eponymous journal was launched and quickly gathered a wider international audience, welcoming also articles independent of any participation to the actual seminar. On some occasions, the journal has had special issues dedicated to Festschriften or conference proceedings. All volumes are freely accessible at <https://www.mat.univie.ac.at/~slc/>.

We are pleased to add another volume to this collection of the journal of the Séminaire Lotharingien de Combinatoire. Our volume, dedicated to the themes of the 2021 Lattice Path Conference, contains the following contributions:

1. Helmut Prodinger: *A walk in my lattice path garden*
2. Anthony J. Guttmann and Václav Kotěšovec: *A numerical study of  $L$ -convex polyominoes and 201-avoiding ascent sequences*
3. David W. Ash: *Introducing DASEP: the doubly asymmetric simple exclusion process*
4. Stéphane Ouvry and Alexios P. Polychronakos: *Signed area enumeration for lattice paths*
5. Aubrey Blecher and Arnold Knopfmacher: *Left-to-right maxima in Dyck paths*
6. Rigoberto Flórez, Toufik Mansour, José L. Ramírez, Fabio A. Velandia, and Diego Villamizar: *Restricted Dyck paths on valleys sequence*
7. Sergi Elizalde: *Counting lattice paths by crossings and major index II: tracking descents via two-rowed arrays*
8. Jang Soo Kim and Dennis Stanton: *Three families of  $q$ -Lommel polynomials*
9. Malvina Vamvakari: *On  $q$ -order statistics*
10. Thomas Dreyfus: *Differential algebraic generating series of weighted walks in the quarter plane*
11. Rodolphe Garbit and Kilian Raschel: *The generating function of the survival probabilities in a cone is not rational*
12. Rafik Aguech, Asma Althagafi, and Cyril Banderier: *Height of walks with resets, the Moran model, and the discrete Gumbel distribution*

We would like to thank all the authors for their patience during the interactions with the editors, which (hopefully!) resulted in a pleasantly polished volume. . . Last but not least, we would like to also thank all the referees for their excellent work. We hope that the reader will enjoy this volume and will eventually be motivated to contribute to the next Lattice Path Conference!

### 3. A PANORAMA ON LATTICE PATHS

Let us end this preface with two beautiful photos taken during a random walk in the proximity of the conference location (warm thanks to Andreas Nessmann and Sergey Dovgal for sharing them with us).



A path... what else could end our preface?