

CARTOUCHE WRITING OF GALOIS AND ADJOINT PAIRS  
AND THEIR APPLICATIONS FOR GEOMETRICALLY  
DEPICTING CONSEQUENCE RELATIONS IN LOGICS  
AND IN RELATIONAL DATA BASES

BY

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Four fundamental remarks are giving the motivation and are explaining the developments of this paper :

1) In some real dynamical systems one can find action pairs whose properties can be described as Galois or adjoint pairs between ordered sets or, more generally, as  $j\check{j}$  pairs between sets (Galois or adjoint pairs being  $j\check{j}$  pairs with special properties, corresponding to some basic order relations.)

2) As it is well known, Galois and adjoint pairs between ordered sets are but a special case of henceforth classical notions of Galois and adjoint pairs between categories.

3) The *cartouche writing* (as we call it) of  $j\check{j}$  pairs supplies a suggestive geometrical way of depicting them and also of depicting adjoint pairs between categories.

4) The algebraic modelisation of a real system is not a category, but a more general structure that we call an *actegory*.

As a consequence of these remarks, we are led

to build foundations for an actegory-theory and to show Galois and adjoint pairs in ordered sets or in categories can be formulated and studied in the framework of actegory-theory.

to show the expressive power of the *cartouche writing* by examples coming from algebra, logic and theory of relational data bases.

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