The Automorphism Groups of Reed-Solomon Codes *

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A class of maximum distance separable codes is introduced which
we call "Cauchy codes" because of the special form of their
generator matrices. They include Reed-Solomon codes [1],
extended Reed-Solomon Codes [1,2], and other cyclic or pseudo-
cyclic maximum distance separable codes studied recently by Rocha
[3]. Our notion of Cauchy codes slightly extends the notion of
generalized Reed-Solomon codes [1] while preserving the algebraic
character of Reed-Solomon codes. We show that Cauchy codes of
length n and dimension k, $3 \leq k \leq n-3$, can be characterized by
certain algebraic equations for the entries of their generator
matrices, and hence form a closed submanifold of dimension $2n-4$
in the $k \times (n-k)$-dimensional algebraic manifold of all maximum
distance separable codes of lenth n and dimension k .

For every Cauchy code we determine the automorphism group and
its underlying permutation group. For example, the permutation
group of the doubly-extended Reed-Solomon codes over GF(q) is
the semilinear fractional group $P\Gamma L(2,q)$. It is interesting
to compare this result with the situation of other codes which
admit highly transitive permutation groups [4,5,6,7,8,9,10,11] .

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