SEMISTANDARD MONOMIALS AND INVARIANT THEORY

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For a long time, the problem of extending to arbitrary characteristic the Fundamental Theorems of classical invariant theory remained untouched. The technique of Young bitableaux, introduced by Doubilet, Rota and Stein, succeeded in providing a simple combinatorial proof of the First Fundamental Theorem valid over every infinite field.

Since then, it was generally thought that the use of bitableaux, or some equivalent device, as the cancellation lemma proved by Hodge after a suggestion of D.E.Littlewood, and that single tableaux would not suffice.

We provide a simple combinatorial proof of the First Fundamental Theorem of invariant theory, valid in any characteristic, which uses only single tableaux or "brackets", as they were named by Cayley.

We believe the main combinatorial tool in the proof is the notion of semistandard monomial; indeed, we prove that semistandard monomials are linearly independent in the bracket polynomial space, thus obtaining a generalization of the crucial property of standard tableaux.