Österreichische Gesellschaft für Wissenschaftsgeschichte

XI. Österreichisches Symposion zur Geschichte der Mathematik

Der Blick aufs Ganze.

Gibt es große Linien in der Entwicklung der Mathematik?

Tagung, 22. April bis 28. April 2012, Miesenbach (Niederösterreich)

Errata

Infolge eines bedauerlichen Versehens fehlen im Tagungsband
die beiden letzten Seiten des Beitrags von
Bernhelm Booss-Bavnbek und Phil Davis
(nach Seite 152)


BBB: Longing for unity

and role of mathematics and insisted on the distinction between meaningful and meaningless abstractions and constructions:

> We have a *perestroika* in our time. We have computers which can do everything. We are not obliged to be bound by two operations - addition and multiplication. We also have a lot of other tools. I am sure that in 10 to 15 years mathematics will be absolutely different from what it was before.[14]

And

> An important side of mathematics is that it is an adequate language for different areas: physics, engineering, biology. Here, the most important word is adequate language. We have adequate and nonadequate languages. I can give you examples of adequate and nonadequate languages. For example, to use quantum mechanics in biology is not an adequate language, but to use mathematics in studying gene sequences is an adequate language.

The emergence of a new type of unity, oriented differently, may be sensed in the outspoken ethical stand of M.F. ATIYAH, another exponent of the classical GALILEAN quote. Firstly, as president of the Royal Society and later as president of the Pugwash nuclear disarmament movement, he blamed the development and the consequent degradation of much mathematics on its applications to war and to juke boxes.

[Bernhelm] Promising offshoots and developments

There are clearly distinguishable mainstreams in mathematics. The active research mathematician has continuously to make a choice as to what are the prominent and promising fields to enter into or to rely on their own originality and inspiration. The difficult and often narrow problem of choice goes back to LAGRANGE who expressed very definitely his conviction that now all what could be solved in mathematics had been solved while at the same time opening wide fields of new mathematical research.[15]

In 1933, NORBERT WIENER characterized the *hierarchy* of mathematical objects:[16]

> In the hierarchy of branches of mathematics, certain points are recognizable where there is a definite transition from one level of abstraction to a higher level. The first level of mathematical abstraction leads us to the concept of the individual numbers, as indicated for example by the Arabic numerals, without as yet any undetermined symbol representing some unspecified number. This is the stage of elementary arithmetic; in algebra we use undetermined literal symbols, but consider only individual specified combinations of these symbols. The next stage is

[14] L.c., p.xiv.

[15] "There is but one universe, and it can happen to but one man in the world's history to be the interpreter of its laws." That is what Lagrange is quoted to have said about Newton, according to Th. Kuhn, The function of dogma in scientific research, in: A.C. Crombie (ed.), *Scientific Change*, Heinemann, New York, 1963, pp. 347-369, here p. 353. Kuhn's own comment: "In receiving a paradigm the scientific community commits itself, consciously or not, to the view that the fundamental problems there resolved have, in fact, been solved once and for all."

[16] Norbert Wiener, *The Fourier Integral and Certain of its Applications*, Cambridge University Press, Cambridge, 1933, p. 1.

> that of analysis, and its fundamental notion is that of the arbitrary dependence of one number on another or of several others -- the function. Still more sophisticated is that branch of mathematics in which the elementary concept is that of the transformation of one function into another, or, as it is also known, the operator.

Today, we might be inclined rather to make long lists of promising developments and major themes abandoned to illustrate the nature of contemporary productivity[17]. Since Lagrange's pronouncement of the victorious end of mathematics and its putative revitalization, there has been permanent and productive tension between what has been accomplished and what new theoretical insights might lead to new fields. Every time a question seemed to be settled and a new fact established, new concepts have arisen on a higher level of abstraction. BØRGE JESSEN once quoted to me a remark of HARALD BOHR that all developments require and receive consolidation: for example, invariants were consolidated in groups, equations in operator algebras, statistics in probability, optimization in functionals. Instead of the much feared atomization of mathematics, a world of cross connections has been discovered and elaborated. With hindsight it is incomprehensible why JOHN VON NEUMANN declined the invitation to the 1954 Amsterdam ICM to give a HILBERT style talk that would present a list of the most important and as yet unsolved mathematical problems. On the basis of his work for the US Atomic Energy Commission (ACE) he would have been the ideal witness for the ever and ever more manifest unity of mathematics. To me it seems that only regards to military security prevented him of demonstrating that it had become easier to oversee mathematics since HILBERT's 1900 and not more difficult and certainly not impossible, as VON NEUMANN claimed in his famous letter.

Underlying all specializations and generalizations, there is one dominant theme in the development of mathematics, namely, striving for meaning: for human meaning. Such meaning may be found in many directions, aesthetic, cognitive or utilitarian. To me, when all has been said, the search for, the discovery and the construction of meaning establish a kind of unity within mathematics.

[Phil] The search for *Unity within Diversity* as a never ending process

There is certainly unity within mathematics. The Brown University catalog lists 50 different courses under one heading: Mathematics. Mathematicians of the world gather together every four years. On the other hand, Applied Math at Brown split off from Pure Math, and Computer Science split off from Applied Math.

I think that the phenomenon we are dealing with goes under the name of *Unity within Diversity*. This is a vast topic that spans all intellectual disciplines (Google the italicised phrase!) and the search for such unity within diversity is a never ending process.

[17] See Philip J. Davis, The rise, fall, and possible transfiguration of triangle geometry: A mini-history, *The American Mathematical Monthly* **102**/3 (March 1995), 204-214.