follows the stability of the beam for $-\infty < v^2 < v_*^2$ in the metric (4). In [4], and also independently by another method, the author has proved the instability of the beam for $v \geqslant v_*$.

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UNITARY REPRESENTATIONS WITH HIGHEST WEIGHT OF THE GROUP OF DIFFEOMORPHISMS OF A CIRCLE

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Let Diff_0 be the group of diffeomorphisms of the circle S^1, preserving orientation. It is natural to consider its Lie algebra as the algebra Vect of vector fields on S^1. The algebra Vect has a one-dimensional central extension **Vect**, constructed by Gel'fand and Fuks [3]. A basis in $\mathbf{Vect}_\mathbf{C}$ ($\mathbf{Vect}_\mathbf{C}$ is the complexification of **Vect**) is made up of elements e_k $(k \in \mathbf{Z})$ and z, for which one has the commutation relations

$$[e_k, e_n] = (n-k)\, e_{n+k}, \qquad \text{if} \quad n+k \neq 0,$$

$$[e_{-k}, e_k] = 2ke_0 + \frac{1}{12}(k^3 - k)\, z, \quad [z, e_k] = 0,$$

here the elements e_k in $\mathbf{Vect}_\mathbf{C}/\mathbf{C}z = \mathrm{Vect}_\mathbf{C}$ correspond to the vector fields $e^{ik\varphi}\partial/\partial(i\varphi)$.

Let M(h, c) be the Verm module over $\mathbf{Vect}_\mathbf{C}$, generated by a vector v such that $e_0 v = hv$, $zv = cv$ (h, c . L(h, c) is its irreducible quotient-module (see [2, 5-7]). The present note is devoted to the problem of unitarization of the modules M(h, c) and L(h, c) over **Vect** and their integration to unitary projective representations of the group Diff_0.

1. Let K be a complex Hilbert space with scalar product $\langle\cdot, \cdot\rangle$. Let $O_0(K)$ and $\mathrm{Sp}_0(K)$ be the groups of operators, preserving, respectively, $\mathrm{Re}\langle\cdot, \cdot\rangle$ and $\mathrm{Im}\langle\cdot, \cdot\rangle$, and representable in the form $U(1 + T)$, where U is a unitary operator and T is the Hilbert–Schmidt operator. We denote by Spin the spinor representation of O_0, and by W the Shale–Weyl representation of the group Sp_0 (these representations are projective; see the constructions in [1]). Let $\mathrm{Spin} = \mathrm{Spin}_+ \oplus \mathrm{Spin}_-$, $W = W_+ \oplus W_-$ be the decomposition of Spin and W into irreducible subrepresentations (in even and odd functions). We note that W can be extended canonically [1] to a projective unitary representation of the group $\mathrm{Sp}_0 \cdot K$, the semidirect product of Sp and the additive group of the space K.

2. We consider in the space K of real functions on S^1 with zero mean the scalar product

$$\langle f_1, f_2\rangle = \int_0^{2\pi}\int_0^{2\pi} \ln\left|\sin\frac{\varphi_1 - \varphi_2}{2}\right| f_1(\varphi_1)\, f_2(\varphi_2)\, d\varphi_1\, d\varphi_2.$$

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We introduce in K a complex structure with the help of the Hilbert transform

$$Hf(\varphi) = \frac{1}{2\pi} \text{ v. p.} \int_0^{2\pi} \operatorname{ctg}\left(\frac{\varphi - \psi}{2}\right) f(\psi)\, d\psi$$

(if = Hf). Let the group Diff_0 act on K by the formula $T(\psi) f(\varphi) = f(\psi(\varphi))\psi'(\varphi)$, where $\psi \in \text{Diff}_0$.

THEOREM 1. The operators $T(\psi)$ lie in $\text{Sp}_0(K)$.

Now we note that the formula

$$Z_{\alpha,\beta}(\psi) f(\varphi) = T(\psi) f(\varphi) + \alpha(\psi' - 1) \dot{+} \beta\psi''/\psi'$$

defines an additive action of Diff_0 on K. Thus, we have a series of imbeddings of Diff_0 in $\text{Sp}_0(K)\cdot K$, and, consequently, a series of unitary representations $N_{\alpha,\beta}$ of the group Diff_0.

THEOREM 2. The representation $N_{\alpha,\beta}$ is a unitary projective representation of Diff_0 with highest weight $(h, c) = (\frac{1}{2}(\alpha^2 + \beta^2),\ 1 + 12\beta^2)$. For $(h, c) \neq \left(\frac{n^2}{4}, 1\right)$, where $n \in \mathbf{Z}$, $N_{\alpha,\beta}$ is irreducible.

3. Let the two-sheeted covering of the group $\textbf{Diff}_0$ act on the space L^2 of odd complex functions (for s = 0, real functions) on S^1 by the formula

$$R_s(\psi) f(\varphi) = f(\psi(\varphi))\psi'(\varphi)^{\frac{1}{2}+is}.$$

We realify our space and in it we introduce a new complex structure with the help of the operator

$$H_s f(\varphi) = \frac{1}{2\pi\Gamma(2is)} \text{ v. p.} \int_0^{2\pi} \frac{\overline{f(\psi)}\operatorname{sgn}(\sin(\varphi - \psi))\, d\psi}{|\sin(\varphi - \psi)|^{1+2is}}.$$

THEOREM 3. The operators $R_s(\psi)$ lie in the group O_0.

THEOREM 4. The representation $\text{Spin}(R_0(\psi))$ is a unitary projective representation of Diff_0 with highest weight. Here $\text{Spin}_+(R_0(\psi))$ corresponds to $L(0, 1/2)$, and $\text{Spin}_-(R_0(\psi))$ to $L(1/2, 1/2)$.

The construction given is the Fermion analog of the constructions of [4].

4. THEOREM 5. The module M(h, c) is unitarizable (i.e., admits a positive definite **Vect**-invariant Hermitian form) if and only if $(h, c) \in \{h > 0, c \geqslant 1\} \setminus \{c = 1, h = n^2/4, n \in \mathbf{Z}\}$.

COROLLARY. For $h \geqslant 0$, $c \geqslant 1$ the module L(h, c) is unitarizable.

However, Theorem 4 gives examples of strongly degenerate unitarizable modules L(h, c) with (h, c) lying outside the domain indicated.

Considering the tensor products of the representations constructed in Secs. 2 and 3, we get that the following L(h, c) are integrable to projective representations of $\textbf{Diff}_0$:

1. L(h, c), where $(h, c) \in \{h \geqslant 0, c \geqslant 1\} \setminus \bigcup_{k \geqslant 0} A_k$ are open triangles with vertices (0, 1 + k/2), (0, 1 + (k + 1)/2), (1/48, 1 + (k + 1)/2). In particular, this domain contains the set $\{h \geqslant 1/48, c \geqslant 1\}$.
2. L(0, 0), which is the identity representation.
3. L(0, 1/2), L(1/16, 1/2) (constructed by Ismagilov), L(1/2, 1/2).

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EXTENSION OF BOUNDED HOLOMORPHIC FUNCTIONS FROM AN ANALYTIC CURVE IN GENERAL POSITION TO A POLYDISK

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The present paper is devoted to the problem of constructing a continuous linear operator for extending bounded holomorphic functions from an analytic curve in general position in a polydisk to bounded holomorphic functions on the polydisk (see [2-6, 9]).

We formulate the basic result.

THEOREM. Let A be an analytic curve, defined in a neighborhood of the polydisk D^n, such that:

(i) the singular points of A are situated strictly inside D^n;

(ii) at each point $a \in A \cap \Gamma_i$ or $a \in \Gamma_{ij}$ the intersection of A with Γ_i or Γ_{ij} is transverse; here $\Gamma_i = D_1 \times \ldots \times D_{i-1} \times T_i \times D_{i+1} \times \ldots \times D_n$, $\Gamma_{ij} = \Gamma_i \cap \Gamma_j$.

Then there exists a continuous linear extension operator $L: H^\infty(A) \to H^\infty(D^n)$. If the function $g \in H^\infty(A)$ is continuous on $A \cap \bar{D}^n$, then L(g) is continuous on $\bar{D}^n$.

For the necessity of the conditions of (ii), see [2].

The proof of the theorem is based on the construction of local extensions, which are then made compatible. A local extension, with an estimate in a neighborhood of a point $a \in A$, lying strictly inside D^n or lying strictly inside $A \cap \Gamma_i$, can be constructed using the local description of the analytic curve and condition (ii) of the theorem [1].

The local extension with an estimate in the neighborhood of a point $a \in A \cap \Gamma_{ij}$ satisfies the following lemma:

LEMMA. Suppose there are defined in C^1 with coordinate $z = x + iy$ two domains $B_1(1) = \{z: |z| < 1, x > 0\}$, $B_2(1) = \{z: |z| < 1, y < 0\}$. Let f(z) be a bounded function on $B_{12}(1) = B_1(1) \cap B_2(1)$ which is holomorphic at interior points of $B_{12}(1)$. Then there exist two functions $f_1(z) = L_1 f(z)$ and $f_2(z) = L_2 f(z)$, bounded on $B_1(1/2)$ and $B_2(1/2)$, respectively, and holomorphic at interior points of these domains, and such that

$$(i)\ f = f_1|_{B_{12}(1)} + f_2|_{B_{12}(1)}, \qquad (ii)\ \|f_k\|_\infty \leqslant K \cdot \|f\|_\infty.$$

If f(z) is continuous on $B_{12}(1)$, then $f_1(z)$ and $f_2(z)$ are continuous on $B_1(1/2)$ and $B_2(1/2)$, respectively.

Sketch of Proof of the Lemma. The functions sought $f_1(z)$ and $f_2(z)$ are defined by

$$f_1(z) = \frac{1}{2\pi i}\int_{-1}^{0} \frac{f(iy)\, i\, dy}{iy - z} + \frac{1}{2\pi i}\int_{0}^{1} \frac{f(-iy)\, i\, dy}{iy - z} + \frac{1}{2}\varphi_{12}(z) + \frac{1}{2}\psi_{12}(z),$$

$$f_2(z) = \frac{1}{2\pi i}\int_{-1}^{0} \frac{f(-x)\, dx}{x - z} + \frac{1}{2\pi i}\int_{0}^{1} \frac{f(x)\, dx}{x - z} + \frac{1}{2}\varphi_{12}(z) + \frac{1}{2}\psi_{12}(z),$$

where

$$\varphi_{12}(z) = \frac{1}{2\pi i}\int_{\substack{|\zeta|=1 \\ \{\frac{3}{2}\pi \leqslant \arg\zeta \leqslant 2\pi\}}} \frac{f(\zeta)\, d\zeta}{\zeta - z}, \qquad \psi_{12}(z) = \frac{1}{2\pi i}\int_{\substack{|\zeta|=1 \\ \{\frac{\pi}{2} \leqslant \arg\zeta \leqslant \pi\}}} \frac{f(-\zeta)\, d\zeta}{\zeta - z}.$$

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