Identification of Factors: the Andrews - Burge Determinant

In[2]:=
```
m[i_, j_] := Binomial[mu + i + j, 2 i - j];
```
Binomialkoeffizient

In[3]:=
```
Rows[n_, val_] := (
    Var = Sum[c[i] * Table[m[i, j] /. mu → val, {j, 0, n - 1}], {i, 0, n - 1}];
    Var = Solve[Var == Table[0, {n}], Table[c[i], {i, 0, n - 1}]];
    Table[c[i], {i, 0, n - 1}] /. Var);
Columns[n_, val_] := (
    Var = Sum[c[j] * Table[m[i, j] /. mu → val, {i, 0, n - 1}], {j, 0, n - 1}];
    Var = Solve[Var == Table[0, {n}], Table[c[j], {j, 0, n - 1}]];
    Table[c[j], {j, 0, n - 1}] /. Var);
```
summiere Tabelle löse Tabelle Tabelle Tabelle summiere Tabelle löse Tabelle Tabelle Tabelle

In[6]:=
```
Rows[5, -5]
```
Solve: Equations may not give solutions for all "solve" variables.

Out[6]=
```
{{c[0], c[1], -c[1], c[1], -c[0] - 2 c[1]}}
```

In[7]:=
```
<< zb.m
```

> Fast Zeilberger Package by Peter Paule and Markus Schorn (enhanced by Axel Riese) — © RISC Linz — V 3.54 (02/23/05)

In[8]:=
```
Zb[Binomial[n, k], {k, 0, n}, n, 1]
```
Binomialkoeffizient

```
If `n' is a natural number, then:
```

Out[8]=
```
{2 SUM[n] - SUM[1 + n] == 0}
```

In[9]:=
```
Zb[Binomial[n - 2, j - 1] Binomial[-n + i + j, 2 i - j], {j, 0, n - 1}, n, 2]
```
Binomialkoeffizient Binomialkoeffizient

```
If `-1 + n' is a natural number and
   none of {-2 + i - n, -2 + n} is a negative integer, then:
```

Out[9]=
```
{-3 (-1 + n) SUM[n] + (-1 - 3 i + 2 n) SUM[1 + n] + (-i + n) SUM[2 + n] == 0}
```

LU - Factorisation: the

Vandermonde Determinant (again)

In[10]:=
```
U[n_] :=
 Table[Switch[Sign[i - j], -1, g[i, j], 0, 1, 1, 0], {i, 0, n - 1}, {j, 0, n - 1}]
L[n_] := Table[If[i ≥ j, k[i, j], 0], {i, 0, n - 1}, {j, 0, n - 1}]

LU1[n_] :=
  Module[{Var}, Var = Solve[Map[(# == 0) &, Flatten[Transpose[W[n]].U[n] - L[n]]],
      Flatten[Join[Table[k[i, j], {i, 0, n - 1}, {j, 0, i}],
        Table[g[i, j], {i, 0, n - 1}, {j, i + 1, n - 1}]]]];
   UU = Factor[U[n] /. Var[[1]]]; LL = Factor[L[n] /. Var[[1]]];
   Print[""];
   Print["Your upper triangular matrix is"];
   Print[""];
   Print[UU];
   Print[""];
   Print["Your lower triangular matrix is"];
   Print[""];
   Print[LL];];
LU2[n_] := Module[{Var}, Var = Solve[Map[(# == 0) &, Flatten[W[n].U[n] - L[n]]],
     Flatten[Join[Table[k[i, j], {i, 0, n - 1}, {j, 0, i}],
       Table[g[i, j], {i, 0, n - 1}, {j, i + 1, n - 1}]]]];
  UU = Factor[U[n] /. Var[[1]]]; LL = Factor[L[n] /. Var[[1]]];
  Print[""]; Print["Your upper triangular matrix is"]; Print[""];
  Print[UU];
  Print[""]; Print["Your lower triangular matrix is"]; Print[""];
  Print[LL];]
```

In[14]:=
```
W[n_] := Table[x[i]^(j), {i, 0, n - 1}, {j, 0, n - 1}]
```

In[15]:=
```
LU1[5]
```

Your upper triangular matrix is

$$\left\{\left\{1, -1, \frac{x[1]-x[2]}{x[0]-x[1]}, -\frac{(x[1]-x[3])\,(x[2]-x[3])}{(x[0]-x[1])\,(x[0]-x[2])}, \frac{(x[1]-x[4])\,(x[2]-x[4])\,(x[3]-x[4])}{(x[0]-x[1])\,(x[0]-x[2])\,(x[0]-x[3])}\right\}, \left\{0, 1, -\frac{x[0]-x[2]}{x[0]-x[1]}, \frac{(x[0]-x[3])\,(x[2]-x[3])}{(x[0]-x[1])\,(x[1]-x[2])}, -\frac{(x[0]-x[4])\,(x[2]-x[4])\,(x[3]-x[4])}{(x[0]-x[1])\,(x[1]-x[2])\,(x[1]-x[3])}\right\}, \left\{0, 0, 1, \frac{(x[0]-x[3])\,(x[1]-x[3])}{(x[1]-x[2])\,(-x[0]+x[2])}, -\frac{(x[0]-x[4])\,(x[1]-x[4])\,(x[3]-x[4])}{(x[0]-x[2])\,(-x[1]+x[2])\,(x[2]-x[3])}\right\}, \left\{0, 0, 0, 1, -\frac{(x[0]-x[4])\,(x[1]-x[4])\,(x[2]-x[4])}{(x[2]-x[3])\,(-x[0]+x[3])\,(-x[1]+x[3])}\right\}, \{0, 0, 0, 0, 1\}\right\}$$

Your lower triangular matrix is

$$\{\{1, 0, 0, 0, 0\}, \{x[0], -x[0]+x[1], 0, 0, 0\}, \{x[0]^2, -(x[0]-x[1])\,(x[0]+x[1]), (x[0]-x[2])\,(x[1]-x[2]), 0, 0\}, \{x[0]^3, -(x[0]-x[1])\,(x[0]^2+x[0]\times x[1]+x[1]^2), -(x[1]-x[2])\,(-x[0]+x[2])\,(x[0]+x[1]+x[2]), -(x[0]-x[3])\,(x[1]-x[3])\,(x[2]-x[3]), 0\}, \{x[0]^4, -(x[0]-x[1])\,(x[0]+x[1])\,(x[0]^2+x[1]^2), -(x[1]-x[2])\,(-x[0]+x[2])\,(x[0]^2+x[0]\times x[1]+x[1]^2+x[0]\times x[2]+x[1]\times x[2]+x[2]^2), -(x[2]-x[3])\,(-x[0]+x[3])\,(-x[1]+x[3])\,(x[0]+x[1]+x[2]+x[3]), (x[0]-x[4])\,(x[1]-x[4])\,(x[2]-x[4])\,(x[3]-x[4])\}\}$$

Interlude: A Word about Guessing

In[16]:=
```
<< rate2.m
```

Out[16]= 

In[17]:=
```
Rate[1, 3, 6, 10]
```

Out[17]= $\left\{\frac{1}{2}\,i0\,(1+i0)\right\}$

In[18]:=
```
Rate[1, 2, 6, 24]
```

Out[18]= {PProduct[1 + i1, {i1, 1, -1 + i0}]}

In[19]:=
```
Rate[1, 2, 5, 14, 42]
```

Out[19]= $\left\{\text{PProduct}\left[\frac{2\,(1+2\,i1)}{2+i1}, \{i1, 1, -1+i0\}\right]\right\}$

In[20]:=
```
Rate[1, 2, 7, 42, 429, 7436, 218 348, 10 850 216]
```

Out[20]= $\left\{\text{PProduct}\left[2\,\text{PProduct}\left[\frac{3\,(2+3\,i2)\,(4+3\,i2)}{4\,(1+2\,i2)\,(3+2\,i2)}, \{i2, 1, -1+i1\}\right], \{i1, 1, -1+i0\}\right]\right\}$

In[21]:=
```
Rate[2, 16, 84, 400, 1820, 8064, 35112]
```

Out[21]=
$$\left\{2\ \text{PProduct}\left[\frac{2\ (-1+2\ i1)\ (1+3\ i1)}{i1\ (-2+3\ i1)},\ \{i1,\ 1,\ -1+i0\}\right]\right\}$$

In[22]:=
```
Rate[1, 1, 2, 3, 5, 8, 13]
```

Out[22]=
```
{}
```

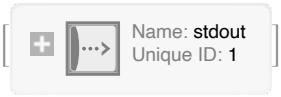
In[23]:=
```
<< RISC`Guess`
```

Intersection: Nonatomic expression expected at position 2 in {StandardForm, TraditionalForm}⋂FormatType.

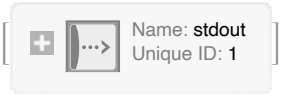
Intersection: Nonatomic expression expected at position 2 in {StandardForm, TraditionalForm}⋂FormatType.

Intersection: Nonatomic expression expected at position 2 in {StandardForm, TraditionalForm}⋂FormatType.

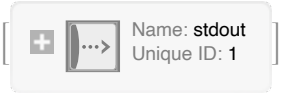
General: Further output of Intersection::normal will be suppressed during this calculation.

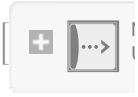
Options: FormatType is not a known option for OutputStream[

]⋂{OutputStream[]

Options: FormatType is not a known option for OutputStream[

]⋂{OutputStream[]

Options: FormatType is not a known option for OutputStream[

]⋂{OutputStream[]

General: Further output of Options::optnf will be suppressed during this calculation.

Intersection: Heads List and OutputStream at positions 2 and 1 are expected to be the same.

Intersection: Heads List and OutputStream at positions 2 and 1 are expected to be the same.

Intersection: Heads List and OutputStream at positions 2 and 1 are expected to be the same.

General: Further output of Intersection::heads will be suppressed during this calculation.

Delta: Symbol Delta appears in multiple contexts {RISC`RISCComb`, RISCComb`}; definitions in context RISC`RISCComb` may shadow or be shadowed by other definitions.

EquationSolver: Symbol EquationSolver appears in multiple contexts {RISC`RISCComb`, RISCComb`}; definitions in context RISC`RISCComb` may shadow or be shadowed by other definitions.

F: Symbol F appears in multiple contexts {RISC`RISCComb`, RISCComb`}; definitions in context RISC`RISCComb` may shadow or be shadowed by other definitions.

NumericCheck: Symbol NumericCheck appears in multiple contexts {RISC`RISCComb`, RISCComb`}; definitions in context RISC`RISCComb` may shadow or be shadowed by other definitions.

SUM: Symbol SUM appears in multiple contexts {RISC`RISCComb`, RISCComb`}; definitions in context RISC`RISCComb` may shadow or be shadowed by other definitions.

LinSolve: Symbol LinSolve appears in multiple contexts {RISC`LinSolve`, RISCComb`}; definitions in context RISC`LinSolve` may shadow or be shadowed by other definitions.

PivotDebug: Symbol PivotDebug appears in multiple contexts {RISC`LinSolve`, RISCComb`}; definitions in context RISC`LinSolve` may shadow or be shadowed by other definitions.

gfe: Symbol gfe appears in multiple contexts {RISC`LinSolve`, RISCComb`}; definitions in context RISC`LinSolve` may shadow or be shadowed by other definitions.

```
HolonomicFunctions Package version 1.7.3 (21-Mar-2017)
written by Christoph Koutschan
Copyright Research Institute for Symbolic Computation (RISC),
Johannes Kepler University, Linz, Austria
```

```
--> Type  ?HolonomicFunctions  for help.
```

Package GeneratingFunctions version 0.8 written by Christian Mallinger
Copyright Research Institute for Symbolic Computation (RISC),
Johannes Kepler University, Linz, Austria

Guess Package version 0.52
written by Manuel Kauers
Copyright Research Institute for Symbolic Computation (RISC),
Johannes Kepler University, Linz, Austria

In[24]:=
```
GuessMinRE[{1, 1, 2, 3, 5, 8, 13, 21, 34}, f[n]]
```
Out[24]=
$$-f[n]-f[1+n]+f[2+n]$$

In[25]:=
```
GuessMinRE[{1, 2, 6, 24, 120, 720}, f[n]]
```
Out[25]=
$$(-2-n)\,f[n]+f[1+n]$$

In[26]:=
```
GuessMinRE[{1, 1, 2, 4, 9, 21, 51, 127, 323, 835, 2188, 5798, 15511, 41835}, f[n]]
```
Out[26]=
$$(-3-3n)\,f[n]+(-5-2n)\,f[1+n]+(4+n)\,f[2+n]$$

LU - Factorisation: the Mills-Robbins-Rumsey Determinant (again)

In[27]:=
```
W[n_] :=
  Table[FunctionExpand[Binomial[mu + i + j, 2 i - j]], {i, 0, n - 1}, {j, 0, n - 1}];
```
Tabelle | multipliziere Funktio··· | Binomialkoeffizient

In[28]:=
```
LU2[12]
```

```
Your upper triangular matrix is
```

$$\left\{\{1,0,0,0,0,0,0,0,0,0,0,0\},\left\{0,1,-\frac{1}{2+\text{mu}},\frac{6\,(5+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(11+2\,\text{mu})},\right.\right.$$
$$-\frac{30\,(6+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(15+2\,\text{mu})},\frac{420\,(7+\text{mu})\,(8+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(17+2\,\text{mu})\,(19+2\,\text{mu})},$$
$$-\frac{3780\,(8+\text{mu})\,(9+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(21+2\,\text{mu})\,(23+2\,\text{mu})},$$
$$\left.\frac{83\,160\,(9+\text{mu})\,(10+\text{mu})\,(11+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,(23+2\,\text{mu})\,(25+2\,\text{mu})\,(27+2\,\text{mu})},\right.$$

$$-\frac{1\,081\,080\,(10+\mu)\,(11+\mu)\,(12+\mu)}{(2+\mu)\,(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(27+2\mu)\,(29+2\mu)\,(31+2\mu)},$$

$$(32\,432\,400\,(11+\mu)\,(12+\mu)\,(13+\mu)\,(14+\mu))\,/\,((2+\mu)\,(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(29+2\mu)\,(31+2\mu)\,(33+2\mu)\,(35+2\mu)),$$

$$-((551\,350\,800\,(12+\mu)\,(13+\mu)\,(14+\mu)\,(15+\mu))\,/\,((2+\mu)\,(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(33+2\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu))),$$

$$(20\,951\,330\,400\,(13+\mu)\,(14+\mu)\,(15+\mu)\,(16+\mu)\,(17+\mu))\,/\,((2+\mu)\,(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu))\Big\},$$

$$\Big\{0,\,0,\,1,\,-\frac{6\,(5+\mu)}{(3+\mu)\,(11+2\mu)},\,\frac{30\,(6+\mu)}{(3+\mu)\,(4+\mu)\,(15+2\mu)},$$

$$-\frac{420\,(7+\mu)\,(8+\mu)}{(3+\mu)\,(4+\mu)\,(5+\mu)\,(17+2\mu)\,(19+2\mu)},$$

$$\frac{3780\,(8+\mu)\,(9+\mu)}{(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(21+2\mu)\,(23+2\mu)},$$

$$-\frac{83\,160\,(9+\mu)\,(10+\mu)\,(11+\mu)}{(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(23+2\mu)\,(25+2\mu)\,(27+2\mu)},$$

$$\frac{1\,081\,080\,(10+\mu)\,(11+\mu)\,(12+\mu)}{(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(27+2\mu)\,(29+2\mu)\,(31+2\mu)},$$

$$-((32\,432\,400\,(11+\mu)\,(12+\mu)\,(13+\mu)\,(14+\mu))\,/\,((3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(29+2\mu)\,(31+2\mu)\,(33+2\mu)\,(35+2\mu))),$$

$$(551\,350\,800\,(12+\mu)\,(13+\mu)\,(14+\mu)\,(15+\mu))\,/\,((3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(33+2\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)),$$

$$-((20\,951\,330\,400\,(13+\mu)\,(14+\mu)\,(15+\mu)\,(16+\mu)\,(17+\mu))\,/\,((3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu)))\Big\},$$

$$\Big\{0,\,0,\,0,\,1,\,-\frac{6\,(13+2\mu)}{(4+\mu)\,(15+2\mu)},\,\frac{90\,(8+\mu)\,(15+2\mu)}{(4+\mu)\,(5+\mu)\,(17+2\mu)\,(19+2\mu)},$$

$$-\frac{840\,(9+\mu)\,(17+2\mu)}{(4+\mu)\,(5+\mu)\,(6+\mu)\,(21+2\mu)\,(23+2\mu)},$$

$$\frac{18\,900\,(10+\mu)\,(11+\mu)\,(19+2\mu)}{(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(23+2\mu)\,(25+2\mu)\,(27+2\mu)},$$

$$-\frac{249\,480\,(11+\mu)\,(12+\mu)\,(21+2\mu)}{(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(27+2\mu)\,(29+2\mu)\,(31+2\mu)},$$

$$\frac{7\,567\,560\,(12+\mu)\,(13+\mu)\,(14+\mu)\,(23+2\mu)}{(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(29+2\mu)\,(31+2\mu)\,(33+2\mu)\,(35+2\mu)},$$

$$-((129\,729\,600\,(13+\mu)\,(14+\mu)\,(15+\mu)\,(25+2\mu))\,/\,((4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(33+2\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu))),$$

$$(4\,962\,157\,200\,(14+\mu)\,(15+\mu)\,(16+\mu)\,(17+\mu)\,(27+2\mu))\,/\,((4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu))\Big\},$$

$$\Big\{0,\,0,\,0,\,0,\,1,\,-\frac{20\,(8+\mu)}{(5+\mu)\,(19+2\mu)},\,\frac{210\,(9+\mu)\,(19+2\mu)}{(5+\mu)\,(6+\mu)\,(21+2\mu)\,(23+2\mu)},$$

$$-\frac{5040\,(10+\mu)\,(11+\mu)\,(21+2\mu)}{(5+\mu)\,(6+\mu)\,(7+\mu)\,(23+2\mu)\,(25+2\mu)\,(27+2\mu)},$$

$$\frac{69\,300\,(11+\mu)\,(12+\mu)\,(23+2\mu)}{(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(27+2\mu)\,(29+2\mu)\,(31+2\mu)},$$

$$-\frac{2\,162\,160\,(12+\mu)\,(13+\mu)\,(14+\mu)\,(25+2\mu)}{(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(29+2\mu)\,(31+2\mu)\,(33+2\mu)\,(35+2\mu)},$$

$$\frac{37\,837\,800\,(13+\mu)\,(14+\mu)\,(15+\mu)\,(27+2\mu)}{(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(33+2\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)}$$

$$, -\left(\left(1\,470\,268\,800\,(14+\mu)\,(15+\mu)\,(16+\mu)\,(17+\mu)\,(29+2\mu)\right)/\left((5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu)\right)\right)\Big\},$$

$$\Big\{0,0,0,0,0,1,-\frac{15\,(19+2\mu)}{(6+\mu)\,(23+2\mu)},\frac{420\,(11+\mu)\,(21+2\mu)}{(6+\mu)\,(7+\mu)\,(25+2\mu)\,(27+2\mu)},$$

$$-\frac{6300\,(12+\mu)\,(23+2\mu)\,(25+2\mu)}{(6+\mu)\,(7+\mu)\,(8+\mu)\,(27+2\mu)\,(29+2\mu)\,(31+2\mu)},$$

$$\frac{207\,900\,(13+\mu)\,(14+\mu)\,(25+2\mu)\,(27+2\mu)}{(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(29+2\mu)\,(31+2\mu)\,(33+2\mu)\,(35+2\mu)},$$

$$-\frac{3\,783\,780\,(14+\mu)\,(15+\mu)\,(27+2\mu)\,(29+2\mu)}{(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(33+2\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)},$$

$$\left(151\,351\,200\,(15+\mu)\,(16+\mu)\,(17+\mu)\,(29+2\mu)\,(31+2\mu)\right)/\left((6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu)\right)\Big\},$$

$$\Big\{0,0,0,0,0,0,1,-\frac{42\,(11+\mu)}{(7+\mu)\,(27+2\mu)},\frac{756\,(12+\mu)\,(25+2\mu)}{(7+\mu)\,(8+\mu)\,(29+2\mu)\,(31+2\mu)},$$

$$-\frac{27\,720\,(13+\mu)\,(14+\mu)\,(27+2\mu)}{(7+\mu)\,(8+\mu)\,(9+\mu)\,(31+2\mu)\,(33+2\mu)\,(35+2\mu)},$$

$$\frac{540\,540\,(14+\mu)\,(15+\mu)\,(29+2\mu)\,(31+2\mu)}{(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(33+2\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)},$$

$$-\left(\left(22\,702\,680\,(15+\mu)\,(16+\mu)\,(17+\mu)\,(31+2\mu)\,(33+2\mu)\right)/\left((7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu)\right)\right)\Big\},$$

$$\Big\{0,0,0,0,0,0,0,1,-\frac{28\,(25+2\mu)}{(8+\mu)\,(31+2\mu)},\frac{1260\,(14+\mu)\,(27+2\mu)}{(8+\mu)\,(9+\mu)\,(33+2\mu)\,(35+2\mu)},$$

$$-\frac{27\,720\,(15+\mu)\,(29+2\mu)\,(31+2\mu)}{(8+\mu)\,(9+\mu)\,(10+\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)},$$

$$\frac{1\,261\,260\,(16+\mu)\,(17+\mu)\,(31+2\mu)\,(33+2\mu)}{(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\,(37+2\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu)}\Big\},$$

$$\Big\{0,0,0,0,0,0,0,0,1,-\frac{72\,(14+\mu)}{(9+\mu)\,(35+2\mu)},\frac{1980\,(15+\mu)\,(31+2\mu)}{(9+\mu)\,(10+\mu)\,(37+2\mu)\,(39+2\mu)},$$

$$-\frac{102\,960\,(16+\mu)\,(17+\mu)\,(33+2\mu)}{(9+\mu)\,(10+\mu)\,(11+\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu)}\Big\},$$

$$\Big\{0,0,0,0,0,0,0,0,0,1,-\frac{45\,(31+2\mu)}{(10+\mu)\,(39+2\mu)},$$

$$\frac{2970\,(17+\mu)\,(33+2\mu)}{(10+\mu)\,(11+\mu)\,(41+2\mu)\,(43+2\mu)}\Big\},$$

$$\Big\{0,0,0,0,0,0,0,0,0,0,1,-\frac{110\,(17+\mu)}{(11+\mu)\,(43+2\mu)}\Big\},$$

$$\{0,0,0,0,0,0,0,0,0,0,0,1\}\Big\}$$

Your lower triangular matrix is

$$\Big\{\{1,0,0,0,0,0,0,0,0,0,0,0\},\Big\{\frac{1}{2}\,\mu\,(1+\mu),2+\mu,0,0,0,0,0,0,0,0,0,0\Big\},$$

$$\Big\{\frac{1}{24}\,(-1+\mu)\,\mu\,(1+\mu)\,(2+\mu),\frac{1}{6}\,(1+\mu)\,(2+\mu)\,(3+\mu),\frac{1}{6}\,(3+\mu)\,(11+2\mu),$$

$$0,0,0,0,0,0,0,0,0\Big\},\Big\{\frac{1}{720}\,(-2+\mu)\,(-1+\mu)\,\mu\,(1+\mu)\,(2+\mu)\,(3+\mu),$$

$$\frac{1}{120}\,\mu\,(1+\mu)\,(2+\mu)\,(3+\mu)\,(4+\mu),\frac{1}{60}\,(3+\mu)\,(4+\mu)\left(25+17\,\mu+2\,\mu^2\right),$$

$$\frac{(4+\mu)\,(5+\mu)\,(15+2\mu)\,(17+2\mu)}{30\,(11+2\mu)},0,0,0,0,0,0,0,0\Big\},$$

$$\left\{\frac{(-3+\text{mu})\,(-2+\text{mu})\,(-1+\text{mu})\,\text{mu}\,(1+\text{mu})\,(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})}{40\,320},\right.$$
$$\frac{(-1+\text{mu})\,\text{mu}\,(1+\text{mu})\,(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})}{5040},$$
$$\frac{(1+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,\left(28+19\,\text{mu}+2\,\text{mu}^2\right)}{1680},$$
$$\frac{(4+\text{mu})\,(5+\text{mu})\,(17+2\,\text{mu})\,\left(273+180\,\text{mu}+35\,\text{mu}^2+2\,\text{mu}^3\right)}{420\,(11+2\,\text{mu})},$$
$$\left.\frac{(5+\text{mu})\,(6+\text{mu})\,(19+2\,\text{mu})\,(21+2\,\text{mu})\,(23+2\,\text{mu})}{420\,(15+2\,\text{mu})},\,0,\,0,\,0,\,0,\,0,\,0,\,0\right\},$$
$$\left\{\frac{(-4+\text{mu})\,(-3+\text{mu})\,(-2+\text{mu})\,(-1+\text{mu})\,\text{mu}\,(1+\text{mu})\,(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})}{3\,628\,800},\right.$$
$$\frac{(-2+\text{mu})\,(-1+\text{mu})\,\text{mu}\,(1+\text{mu})\,(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})}{362\,880},$$
$$\frac{\text{mu}\,(1+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,\left(31+21\,\text{mu}+2\,\text{mu}^2\right)}{90\,720},$$
$$\frac{(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,\left(11\,088+14\,071\,\text{mu}+6450\,\text{mu}^2+1315\,\text{mu}^3+120\,\text{mu}^4+4\,\text{mu}^5\right)}{15\,120\,(11+2\,\text{mu})},$$
$$\frac{(5+\text{mu})\,(6+\text{mu})\,(21+2\,\text{mu})\,(23+2\,\text{mu})\,\left(576+295\,\text{mu}+45\,\text{mu}^2+2\,\text{mu}^3\right)}{7560\,(15+2\,\text{mu})},$$
$$\left.\frac{(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu})\,(23+2\,\text{mu})\,(25+2\,\text{mu})\,(27+2\,\text{mu})\,(29+2\,\text{mu})}{3780\,(17+2\,\text{mu})\,(19+2\,\text{mu})},\,0,\,0,\,0,\,0,\,0,\,0\right\},$$
$$\left\{\frac{1}{479\,001\,600}(-5+\text{mu})\,(-4+\text{mu})\,(-3+\text{mu})\,(-2+\text{mu})\,(-1+\text{mu})\right.$$
$$\text{mu}\,(1+\text{mu})\,(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu}),$$
$$\frac{(-3+\text{mu})\,(-2+\text{mu})\,(-1+\text{mu})\,\text{mu}\,(1+\text{mu})\,(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})}{39\,916\,800},$$
$$\frac{(-1+\text{mu})\,\text{mu}\,(1+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,\left(34+23\,\text{mu}+2\,\text{mu}^2\right)}{7\,983\,360},$$
$$\frac{1}{997\,920\,(11+2\,\text{mu})}(1+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})$$
$$\left(13\,068+16\,479\,\text{mu}+7456\,\text{mu}^2+1475\,\text{mu}^3+128\,\text{mu}^4+4\,\text{mu}^5\right),$$
$$\frac{(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,(23+2\,\text{mu})\,\left(45\,144+42\,018\,\text{mu}+14\,397\,\text{mu}^2+2231\,\text{mu}^3+156\,\text{mu}^4+4\,\text{mu}^5\right)}{332\,640\,(15+2\,\text{mu})},$$
$$\frac{(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu})\,(25+2\,\text{mu})\,(27+2\,\text{mu})\,(29+2\,\text{mu})\,\left(1045+438\,\text{mu}+55\,\text{mu}^2+2\,\text{mu}^3\right)}{83\,160\,(17+2\,\text{mu})\,(19+2\,\text{mu})},$$
$$\left.\frac{(7+\text{mu})\,(8+\text{mu})\,(9+\text{mu})\,(27+2\,\text{mu})\,(29+2\,\text{mu})\,(31+2\,\text{mu})\,(33+2\,\text{mu})\,(35+2\,\text{mu})}{83\,160\,(21+2\,\text{mu})\,(23+2\,\text{mu})},\,0,\,0,\,0,\right.$$
$$\left.0,\,0\right\},\,\left\{\frac{1}{87\,178\,291\,200}(-6+\text{mu})\,(-5+\text{mu})\,(-4+\text{mu})\,(-3+\text{mu})\,(-2+\text{mu})\,(-1+\text{mu})\,\text{mu}\,(1+\text{mu})\right.$$
$$(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu}),\,\frac{1}{6\,227\,020\,800}(-4+\text{mu})\,(-3+\text{mu})$$
$$(-2+\text{mu})\,(-1+\text{mu})\,\text{mu}\,(1+\text{mu})\,(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu}),$$
$$\frac{1}{1\,037\,836\,800}(-2+\text{mu})\,(-1+\text{mu})\,\text{mu}\,(1+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})$$
$$(8+\text{mu})\,\left(37+25\,\text{mu}+2\,\text{mu}^2\right),\,\frac{1}{103\,783\,680\,(11+2\,\text{mu})}\text{mu}\,(1+\text{mu})\,(4+\text{mu})\,(5+\text{mu})$$
$$(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu})\,\left(15\,222+19\,077\,\text{mu}+8534\,\text{mu}^2+1643\,\text{mu}^3+136\,\text{mu}^4+4\,\text{mu}^5\right),$$
$$\frac{1}{25\,945\,920\,(15+2\,\text{mu})}(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu})$$
$$\left(2\,548\,260+3\,842\,964\,\text{mu}+2\,277\,207\,\text{mu}^2+682\,586\,\text{mu}^3+111\,021\,\text{mu}^4+9830\,\text{mu}^5+444\,\text{mu}^6+8\,\text{mu}^7\right),$$

$$\Big((6+\mu)(7+\mu)(8+\mu)(27+2\mu)(29+2\mu)\big(1\,069\,640+923\,720\,\mu+315\,393\,\mu^2+54\,176\,\mu^3+4923\,\mu^4+224\,\mu^5+4\,\mu^6\big)\Big)\Big/\big(4\,324\,320(17+2\mu)(19+2\mu)\big),\ \Big((7+\mu)(8+\mu)(9+\mu)(11+2\mu)(29+2\mu)(31+2\mu)(33+2\mu)(35+2\mu)\big(156+27\,\mu+\mu^2\big)\Big)\Big/\big(2\,162\,160(21+2\mu)(23+2\mu)\big),$$
$$\big((8+\mu)(9+\mu)(10+\mu)(11+\mu)(31+2\mu)(33+2\mu)(35+2\mu)(37+2\mu)(39+2\mu)(41+2\mu)\big)/\big(1\,081\,080(23+2\mu)(25+2\mu)(27+2\mu)\big),0,0,0,0\Big\},$$
$$\Big\{\frac{1}{20\,922\,789\,888\,000}(-7+\mu)(-6+\mu)(-5+\mu)(-4+\mu)(-3+\mu)(-2+\mu)(-1+\mu)\,\mu(1+\mu)(2+\mu)(3+\mu)(4+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu),\ \frac{1}{1\,307\,674\,368\,000}(-5+\mu)(-4+\mu)(-3+\mu)(-2+\mu)(-1+\mu)\,\mu(1+\mu)(2+\mu)(3+\mu)(4+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu),$$
$$\frac{1}{186\,810\,624\,000}(-3+\mu)(-2+\mu)(-1+\mu)\,\mu(1+\mu)(3+\mu)(4+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)\big(40+27\,\mu+2\,\mu^2\big),$$
$$\frac{1}{15\,567\,552\,000(11+2\mu)}(-1+\mu)\,\mu(1+\mu)(4+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)\big(17\,550+21\,865\,\mu+9684\,\mu^2+1819\,\mu^3+144\,\mu^4+4\,\mu^5\big),$$
$$\frac{1}{3\,113\,510\,400(15+2\mu)}(1+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)\big(3\,088\,800+4\,630\,140\,\mu+2\,717\,592\,\mu^2+802\,067\,\mu^3+127\,155\,\mu^4+10\,850\,\mu^5+468\,\mu^6+8\,\mu^7\big),$$
$$\Big((6+\mu)(7+\mu)(8+\mu)(9+\mu)(29+2\mu)\big(109\,395\,000+137\,933\,880\,\mu+72\,023\,041\,\mu^2+20\,265\,333\,\mu^3+3\,349\,977\,\mu^4+332\,835\,\mu^5+19\,434\,\mu^6+612\,\mu^7+8\,\mu^8\big)\Big)\Big/\big(389\,188\,800(17+2\mu)(19+2\mu)\big),\ \Big((7+\mu)(8+\mu)(9+\mu)(31+2\mu)(33+2\mu)(35+2\mu)\big(2\,925\,000+2\,118\,876\,\mu+609\,751\,\mu^2+88\,650\,\mu^3+6835\,\mu^4+264\,\mu^5+4\,\mu^6\big)\Big)\Big/\big(129\,729\,600(21+2\mu)(23+2\mu)\big),\ \Big((8+\mu)(9+\mu)(10+\mu)(11+\mu)(33+2\mu)(35+2\mu)(37+2\mu)(39+2\mu)(41+2\mu)\big(2625+808\,\mu+75\,\mu^2+2\,\mu^3\big)\Big)\Big/\big(32\,432\,400(23+2\mu)(25+2\mu)(27+2\mu)\big),$$
$$\big((9+\mu)(10+\mu)(11+\mu)(12+\mu)(35+2\mu)(37+2\mu)(39+2\mu)(41+2\mu)(43+2\mu)(45+2\mu)(47+2\mu)\big)/\big(32\,432\,400(27+2\mu)(29+2\mu)(31+2\mu)\big),0,0,0\Big\},$$
$$\Big\{\frac{1}{6\,402\,373\,705\,728\,000}(-8+\mu)(-7+\mu)(-6+\mu)(-5+\mu)(-4+\mu)(-3+\mu)(-2+\mu)(-1+\mu)\,\mu(1+\mu)(2+\mu)(3+\mu)(4+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu),$$
$$\frac{1}{355\,687\,428\,096\,000}(-6+\mu)(-5+\mu)(-4+\mu)(-3+\mu)(-2+\mu)(-1+\mu)\,\mu(1+\mu)(2+\mu)(3+\mu)(4+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)(10+\mu),$$
$$\frac{1}{44\,460\,928\,512\,000}(-4+\mu)(-3+\mu)(-2+\mu)(-1+\mu)\,\mu(1+\mu)(3+\mu)(4+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)(10+\mu)\big(43+29\,\mu+2\,\mu^2\big),$$
$$\Big((-2+\mu)(-1+\mu)\,\mu(1+\mu)(4+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)(10+\mu)\big(20\,052+24\,843\,\mu+10\,906\,\mu^2+2003\,\mu^3+152\,\mu^4+4\,\mu^5\big)\Big)\Big/\big(3\,175\,780\,608\,000(11+2\mu)\big),$$
$$\frac{1}{529\,296\,768\,000(15+2\mu)}\mu(1+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)(10+\mu)\big(3\,702\,888+5\,519\,562\,\mu+3\,211\,203\,\mu^2+934\,472\,\mu^3+144\,657\,\mu^4+11\,918\,\mu^5+492\,\mu^6+8\,\mu^7\big),$$
$$\Big((6+\mu)(7+\mu)(8+\mu)(9+\mu)(10+\mu)\big(7\,914\,947\,040+14\,407\,804\,032\,\mu+11\,024\,458\,006\,\mu^2+4\,669\,904\,675\,\mu^3+1\,212\,051\,065\,\mu^4+201\,354\,741\,\mu^5+21\,712\,953\,\mu^6+1\,505\,400\,\mu^7+64\,520\,\mu^8+1552\,\mu^9+16\,\mu^{10}\big)\Big)\Big/\big(52\,929\,676\,800(17+2\mu)(19+2\mu)\big),\ \Big((7+\mu)(8+\mu)(9+\mu)(10+\mu)(33+2\mu)(35+2\mu)\big(467\,812\,800+481\,299\,084\,\mu+206\,742\,333\,\mu^2+48\,214\,501\,\mu^3+6\,650\,117\,\mu^4+554\,191\,\mu^5+27\,242\,\mu^6+724\,\mu^7+8\,\mu^8\big)\Big)\Big/\big(13\,232\,419\,200(21+2\mu)(23+2\mu)\big),$$
$$\big((8+\mu)(9+\mu)(10+\mu)(11+\mu)(35+2\mu)(37+2\mu)(39+2\mu)(41+2\mu)$$

$$\left(6\,897\,240 + 4\,309\,086\,\mu + 1\,072\,647\,\mu^2 + 135\,260\,\mu^3 + 9059\,\mu^4 + 304\,\mu^5 + 4\,\mu^6\right)\Big) \Big/ \left(2\,205\,403\,200\,(23+2\mu)\,(25+2\mu)\,(27+2\mu)\right),$$

$$\Big((9+\mu)\,(10+\mu)\,(11+\mu)\,(12+\mu)\,(37+2\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu)\,(45+2\mu)\,(47+2\mu)\left(3808+1035\,\mu+85\,\mu^2+2\,\mu^3\right)\Big) \Big/ \left(1\,102\,701\,600\,(27+2\mu)\,(29+2\mu)\,(31+2\mu)\right),$$

$$\left((10+\mu)\,(11+\mu)\,(12+\mu)\,(13+\mu)\,(14+\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu)\,(45+2\mu)\,(47+2\mu)\,(49+2\mu)\,(51+2\mu)\,(53+2\mu)\right) / \left(551\,350\,800\,(29+2\mu)\,(31+2\mu)\,(33+2\mu)\,(35+2\mu)\right), 0, 0\Big\},$$

$$\Big\{\frac{1}{2\,432\,902\,008\,176\,640\,000}(-9+\mu)\,(-8+\mu)\,(-7+\mu)\,(-6+\mu)\,(-5+\mu)\,(-4+\mu)\,(-3+\mu)\,(-2+\mu)\,(-1+\mu)\,\mu\,(1+\mu)\,(2+\mu)\,(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu),$$

$$\frac{1}{121\,645\,100\,408\,832\,000}(-7+\mu)\,(-6+\mu)\,(-5+\mu)\,(-4+\mu)\,(-3+\mu)\,(-2+\mu)\,(-1+\mu)\,\mu\,(1+\mu)\,(2+\mu)\,(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu),$$

$$\frac{1}{13\,516\,122\,267\,648\,000}(-5+\mu)\,(-4+\mu)\,(-3+\mu)\,(-2+\mu)\,(-1+\mu)\,\mu\,(1+\mu)\,(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\left(46+31\,\mu+2\,\mu^2\right),$$

$$\Big((-3+\mu)\,(-2+\mu)\,(-1+\mu)\,\mu\,(1+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\left(22\,728+28\,011\,\mu+12\,200\,\mu^2+2195\,\mu^3+160\,\mu^4+4\,\mu^5\right)\Big) \Big/ \left(844\,757\,641\,728\,000\,(11+2\mu)\right),$$

$$\Big((-1+\mu)\,\mu\,(1+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\left(4\,395\,168+6\,517\,548\,\mu+3\,761\,052\,\mu^2+1\,080\,449\,\mu^3+163\,575\,\mu^4+13\,034\,\mu^5+516\,\mu^6+8\,\mu^7\right)\Big) \Big/ \left(120\,679\,663\,104\,000\,(15+2\mu)\right),$$

$$\Big((1+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\left(9\,765\,194\,400+17\,689\,132\,320\,\mu+13\,444\,856\,422\,\mu^2+5\,643\,126\,859\,\mu^3+1\,446\,355\,421\,\mu^4+236\,219\,289\,\mu^5+24\,912\,537\,\mu^6+1\,680\,216\,\mu^7+69\,704\,\mu^8+1616\,\mu^9+16\,\mu^{10}\right)\Big) \Big/ \left(10\,056\,638\,592\,000\,(17+2\mu)\,(19+2\mu)\right),$$

$$\Big((7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\,(35+2\mu)\left(59\,044\,658\,400+83\,444\,445\,216\,\mu+50\,422\,834\,098\,\mu^2+17\,114\,934\,729\,\mu^3+3\,604\,597\,773\,\mu^4+491\,219\,119\,\mu^5+43\,839\,961\,\mu^6+2\,533\,096\,\mu^7+90\,952\,\mu^8+1840\,\mu^9+16\,\mu^{10}\right)\Big) \Big/ \left(2\,011\,327\,718\,400\,(21+2\mu)\,(23+2\mu)\right),$$

$$\Big((8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\,(37+2\mu)\,(39+2\mu)\,(41+2\mu)\left(17\,410\,604\,400+16\,765\,584\,570\,\mu+6\,933\,749\,595\,\mu^2+1\,612\,586\,374\,\mu^3+231\,910\,824\,\mu^4+21\,349\,290\,\mu^5+1\,256\,457\,\mu^6+45\,558\,\mu^7+924\,\mu^8+8\,\mu^9\right)\Big) \Big/ \left(251\,415\,964\,800\,(23+2\mu)\,(25+2\mu)\,(27+2\mu)\right),$$

$$\Big((9+\mu)\,(10+\mu)\,(11+\mu)\,(12+\mu)\,(39+2\mu)\,(41+2\mu)\,(43+2\mu)\,(45+2\mu)\,(47+2\mu)\left(14\,578\,928+8\,013\,194\,\mu+1\,758\,441\,\mu^2+195\,830\,\mu^3+11\,595\,\mu^4+344\,\mu^5+4\,\mu^6\right)\Big) \Big/ \left(83\,805\,321\,600\,(27+2\mu)\,(29+2\mu)\,(31+2\mu)\right),$$

$$\Big((10+\mu)\,(11+\mu)\,(12+\mu)\,(13+\mu)\,(14+\mu)\,(41+2\mu)\,(43+2\mu)\,(45+2\mu)\,(47+2\mu)\,(49+2\mu)\,(51+2\mu)\,(53+2\mu)\left(5301+1290\,\mu+95\,\mu^2+2\,\mu^3\right)\Big) \Big/ \left(20\,951\,330\,400\,(29+2\mu)\,(31+2\mu)\,(33+2\mu)\,(35+2\mu)\right),$$

$$\left((11+\mu)\,(12+\mu)\,(13+\mu)\,(14+\mu)\,(15+\mu)\,(43+2\mu)\,(45+2\mu)\,(47+2\mu)\,(49+2\mu)\,(51+2\mu)\,(53+2\mu)\,(55+2\mu)\,(57+2\mu)\,(59+2\mu)\right) / \left(20\,951\,330\,400\,(33+2\mu)\,(35+2\mu)\,(37+2\mu)\,(39+2\mu)\right), 0\Big\},$$

$$\Big\{\left((-10+\mu)\,(-9+\mu)\,(-8+\mu)\,(-7+\mu)\,(-6+\mu)\,(-5+\mu)\,(-4+\mu)\,(-3+\mu)\,(-2+\mu)\,(-1+\mu)\,\mu\,(1+\mu)\,(2+\mu)\,(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)\,(7+\mu)\,(8+\mu)\,(9+\mu)\,(10+\mu)\,(11+\mu)\right) / 1\,124\,000\,727\,777\,607\,680\,000,$$

$$\frac{1}{51\,090\,942\,171\,709\,440\,000}(-8+\mu)\,(-7+\mu)\,(-6+\mu)\,(-5+\mu)\,(-4+\mu)\,(-3+\mu)\,(-2+\mu)\,(-1+\mu)\,\mu\,(1+\mu)\,(2+\mu)\,(3+\mu)\,(4+\mu)\,(5+\mu)\,(6+\mu)$$

$$(7+\mu)(8+\mu)(9+\mu)(10+\mu)(11+\mu)(12+\mu),\ \frac{1}{5\,109\,094\,217\,170\,944\,000}$$
$$(-6+\mu)(-5+\mu)(-4+\mu)(-3+\mu)(-2+\mu)(-1+\mu)\,\mu\,(1+\mu)(3+\mu)(4+\mu)$$
$$(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)(10+\mu)(11+\mu)(12+\mu)\left(49+33\,\mu+2\,\mu^2\right),$$
$$\Big((-4+\mu)(-3+\mu)(-2+\mu)(-1+\mu)\,\mu\,(1+\mu)(4+\mu)(5+\mu)(6+\mu)$$
$$(7+\mu)(8+\mu)(9+\mu)(10+\mu)(11+\mu)(12+\mu)\left(25\,578+31\,369\,\mu+\right.$$
$$\left.13\,566\,\mu^2+2395\,\mu^3+168\,\mu^4+4\,\mu^5\right)\Big)\Big/(283\,838\,567\,620\,608\,000\,(11+2\,\mu)),$$
$$\Big((-2+\mu)(-1+\mu)\,\mu\,(1+\mu)(5+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)(10+\mu)$$
$$(11+\mu)(12+\mu)\left(5\,170\,284+7\,630\,416\,\mu+4\,370\,151\,\mu^2+1\,240\,646\,\mu^3+\right.$$
$$\left.183\,957\,\mu^4+14\,198\,\mu^5+540\,\mu^6+8\,\mu^7\right)\Big)\Big/(35\,479\,820\,952\,576\,000\,(15+2\,\mu)),$$
$$\Big(\mu\,(1+\mu)(6+\mu)(7+\mu)(8+\mu)(9+\mu)(10+\mu)(11+\mu)(12+\mu)$$
$$\left(11\,927\,286\,000+21\,507\,181\,920\,\mu+16\,245\,241\,546\,\mu^2+6\,760\,606\,755\,\mu^3+1\,712\,643\,245\,\mu^4+\right.$$
$$\left.275\,276\,925\,\mu^5+28\,426\,713\,\mu^6+1\,867\,320\,\mu^7+75\,080\,\mu^8+1680\,\mu^9+16\,\mu^{10}\right)\Big)\Big/$$
$$(2\,534\,272\,925\,184\,000\,(17+2\,\mu)(19+2\,\mu)),\ \Big((7+\mu)(8+\mu)(9+\mu)(10+\mu)$$
$$(11+\mu)(12+\mu)\left(5\,213\,071\,936\,800+10\,225\,168\,689\,960\,\mu+8\,614\,443\,665\,934\,\mu^2+\right.$$
$$\left.4\,125\,777\,834\,099\,\mu^3+1\,251\,336\,757\,562\,\mu^4+253\,169\,095\,230\,\mu^5+35\,041\,735\,942\,\mu^6+\right.$$
$$\left.3\,347\,369\,487\,\mu^7+219\,565\,506\,\mu^8+9\,679\,560\,\mu^9+273\,424\,\mu^{10}+4464\,\mu^{11}+32\,\mu^{12}\right)\Big)\Big/$$
$$(422\,378\,820\,864\,000\,(21+2\,\mu)(23+2\,\mu)),\ \Big((8+\mu)(9+\mu)(10+\mu)(11+\mu)$$
$$(12+\mu)(39+2\,\mu)(41+2\,\mu)\left(3\,341\,712\,780\,000+4\,202\,789\,049\,900\,\mu+\right.$$
$$\left.2\,315\,342\,912\,520\,\mu^2+736\,332\,258\,013\,\mu^3+149\,958\,823\,356\,\mu^4+20\,506\,874\,750\,\mu^5+\right.$$
$$\left.1\,919\,690\,892\,\mu^6+122\,977\,617\,\mu^7+5\,285\,328\,\mu^8+145\,304\,\mu^9+2304\,\mu^{10}+16\,\mu^{11}\right)\Big)\Big/$$
$$(42\,237\,882\,086\,400\,(23+2\,\mu)(25+2\,\mu)(27+2\,\mu)),$$
$$\Big((9+\mu)(10+\mu)(11+\mu)(12+\mu)(41+2\,\mu)(43+2\,\mu)(45+2\,\mu)(47+2\,\mu)$$
$$\left(54\,397\,272\,384+45\,769\,821\,804\,\mu+16\,586\,838\,372\,\mu^2+3\,389\,359\,609\,\mu^3+\right.$$
$$\left.429\,296\,889\,\mu^4+34\,874\,889\,\mu^5+1\,813\,791\,\mu^6+58\,170\,\mu^7+1044\,\mu^8+8\,\mu^9\right)\Big)\Big/$$
$$(10\,559\,470\,521\,600\,(27+2\,\mu)(29+2\,\mu)(31+2\,\mu)),\ \Big((10+\mu)(11+\mu)(12+\mu)$$
$$(13+\mu)(14+\mu)(43+2\,\mu)(45+2\,\mu)(47+2\,\mu)(49+2\,\mu)(51+2\,\mu)(53+2\,\mu)$$
$$\left(28\,325\,808+13\,904\,700\,\mu+2\,729\,077\,\mu^2+272\,184\,\mu^3+14\,443\,\mu^4+384\,\mu^5+4\,\mu^6\right)\Big)\Big/$$
$$(1\,759\,911\,753\,600\,(29+2\,\mu)(31+2\,\mu)(33+2\,\mu)(35+2\,\mu)),$$
$$\Big((11+\mu)(12+\mu)(13+\mu)(14+\mu)(15+\mu)(45+2\,\mu)(47+2\,\mu)(49+2\,\mu)(51+2\,\mu)$$
$$(53+2\,\mu)(55+2\,\mu)(57+2\,\mu)(59+2\,\mu)\left(7140+1573\,\mu+105\,\mu^2+2\,\mu^3\right)\Big)\Big/$$
$$(879\,955\,876\,800\,(33+2\,\mu)(35+2\,\mu)(37+2\,\mu)(39+2\,\mu)),$$
$$\Big((12+\mu)(13+\mu)(14+\mu)(15+\mu)(16+\mu)(17+\mu)(47+2\,\mu)(49+2\,\mu)(51+2\,\mu)$$
$$(53+2\,\mu)(55+2\,\mu)(57+2\,\mu)(59+2\,\mu)(61+2\,\mu)(63+2\,\mu)(65+2\,\mu)\Big)\Big/$$
$$(439\,977\,938\,400\,(35+2\,\mu)(37+2\,\mu)(39+2\,\mu)(41+2\,\mu)(43+2\,\mu))\Big\}\Big\}$$

In[29]:= `<< rate2.m`

Out[29]=

In[30]:= **Table[UU[[2, i + 1]], {i, 1, 11}]**
Tabelle

Out[30]=
$$\left\{1, -\frac{1}{2+\text{mu}}, \frac{6\,(5+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(11+2\,\text{mu})}, -\frac{30\,(6+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(15+2\,\text{mu})},\right.$$
$$\frac{420\,(7+\text{mu})\,(8+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(17+2\,\text{mu})\,(19+2\,\text{mu})},$$
$$-\frac{3780\,(8+\text{mu})\,(9+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(21+2\,\text{mu})\,(23+2\,\text{mu})},$$
$$\frac{83\,160\,(9+\text{mu})\,(10+\text{mu})\,(11+\text{mu})}{(2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,(23+2\,\text{mu})\,(25+2\,\text{mu})\,(27+2\,\text{mu})},$$
$$-\left((1\,081\,080\,(10+\text{mu})\,(11+\text{mu})\,(12+\text{mu}))\,/\,((2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\right.$$
$$\left.(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu})\,(27+2\,\text{mu})\,(29+2\,\text{mu})\,(31+2\,\text{mu}))\right),$$
$$(32\,432\,400\,(11+\text{mu})\,(12+\text{mu})\,(13+\text{mu})\,(14+\text{mu}))\,/\,((2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})$$
$$(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu})\,(9+\text{mu})\,(29+2\,\text{mu})\,(31+2\,\text{mu})\,(33+2\,\text{mu})\,(35+2\,\text{mu})),$$
$$-((551\,350\,800\,(12+\text{mu})\,(13+\text{mu})\,(14+\text{mu})\,(15+\text{mu}))\,/$$
$$((2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu})$$
$$(9+\text{mu})\,(10+\text{mu})\,(33+2\,\text{mu})\,(35+2\,\text{mu})\,(37+2\,\text{mu})\,(39+2\,\text{mu}))),$$
$$(20\,951\,330\,400\,(13+\text{mu})\,(14+\text{mu})\,(15+\text{mu})\,(16+\text{mu})\,(17+\text{mu}))\,/$$
$$((2+\text{mu})\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu})\,(9+\text{mu})\,(10+\text{mu})$$
$$(11+\text{mu})\,(35+2\,\text{mu})\,(37+2\,\text{mu})\,(39+2\,\text{mu})\,(41+2\,\text{mu})\,(43+2\,\text{mu}))\}$$

In[31]:= **Apply[Ratekurz, %]**
wende an

Out[31]=
$$\left\{\left(32\,768\,(3+\text{mu})\,(4+\text{mu})\,(5+\text{mu})\,(6+\text{mu})\,(7+\text{mu})\,(8+\text{mu})\,(9+\text{mu})\,(10+\text{mu})\right.\right.$$
$$(11+\text{mu})\,(12+\text{mu})\,(13+\text{mu})\,(14+\text{mu})\,(15+\text{mu})\,(16+\text{mu})\,(17+\text{mu})\,\text{PProduct}\left[\right.$$
$$\left.\left.-\frac{(-1+2\,\text{i1})\,(2+3\,\text{i1}+2\,\text{mu})\,(3+3\,\text{i1}+2\,\text{mu})\,(4+3\,\text{i1}+2\,\text{mu})}{2\,(1+\text{i1}+\text{mu})\,(2+\text{i1}+\text{mu})\,(1+4\,\text{i1}+2\,\text{mu})\,(3+4\,\text{i1}+2\,\text{mu})}, \{\text{i1}, 1, -1+\text{i0}\}\right]\right)\Big/$$
$$((6+2\,\text{mu})\,(8+2\,\text{mu})\,(10+2\,\text{mu})\,(12+2\,\text{mu})\,(14+2\,\text{mu})\,(16+2\,\text{mu})\,(18+2\,\text{mu})\,(20+2\,\text{mu})$$
$$(22+2\,\text{mu})\,(24+2\,\text{mu})\,(26+2\,\text{mu})\,(28+2\,\text{mu})\,(30+2\,\text{mu})\,(32+2\,\text{mu})\,(34+2\,\text{mu}))\}$$

Method 5: The holonomic Ansatz

In[32]:=
```
a[i_, j_] := Binomial[i + j, j];
```
Binomialkoeffizient
```
Clear[c]; c[n_, k_] := 0 /; n < k;
```
lösche
```
Module[{Var, cVar},
```
Modul
```
 For[n = 1, n < 21, n++,
```
For-Schleife
```
  c[n, n] = 1;
  cVar = Table[c[n, k], {k, 1, n - 1}];
```
Tabelle
```
  Var = Solve[Table[Sum[a[l, k] × c[n, k], {k, 1, n}] == 0, {l, 1, n - 1}], cVar];
```
löse Tabelle summiere
```
  For[k = 1, k < n, k++, c[n, k] = Var[[1, k, 2]]];
```
For-Schleife
```
 ]
]
```

In[35]:=
```
data = Table[c[n, k], {n, 1, 10}, {k, 1, 10}]
```
Tabelle

Out[35]=
$$\left\{\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{-\frac{3}{2}, 1, 0, 0, 0, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{2, -\frac{8}{3}, 1, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{-\frac{5}{2}, 5, -\frac{15}{4}, 1, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{3, -8, 9, -\frac{24}{5}, 1, 0, 0, 0, 0, 0\right\}, \left\{-\frac{7}{2}, \frac{35}{3}, -\frac{35}{2}, 14, -\frac{35}{6}, 1, 0, 0, 0, 0\right\},$$
$$\left\{4, -16, 30, -32, 20, -\frac{48}{7}, 1, 0, 0, 0\right\},$$
$$\left\{-\frac{9}{2}, 21, -\frac{189}{4}, 63, -\frac{105}{2}, 27, -\frac{63}{8}, 1, 0, 0\right\},$$
$$\left\{5, -\frac{80}{3}, 70, -112, \frac{350}{3}, -80, 35, -\frac{80}{9}, 1, 0\right\},$$
$$\left.\left\{-\frac{11}{2}, 33, -99, \frac{924}{5}, -231, 198, -\frac{231}{2}, 44, -\frac{99}{10}, 1\right\}\right\}$$

In[36]:=
```
TableForm[%]
```
Tabellendarstellung

Out[36]//TableForm=

1	0	0	0	0	0	0	0	0	0
$-\frac{3}{2}$	1	0	0	0	0	0	0	0	0
2	$-\frac{8}{3}$	1	0	0	0	0	0	0	0
$-\frac{5}{2}$	5	$-\frac{15}{4}$	1	0	0	0	0	0	0
3	−8	9	$-\frac{24}{5}$	1	0	0	0	0	0
$-\frac{7}{2}$	$\frac{35}{3}$	$-\frac{35}{2}$	14	$-\frac{35}{6}$	1	0	0	0	0
4	−16	30	−32	20	$-\frac{48}{7}$	1	0	0	0
$-\frac{9}{2}$	21	$-\frac{189}{4}$	63	$-\frac{105}{2}$	27	$-\frac{63}{8}$	1	0	0
5	$-\frac{80}{3}$	70	−112	$\frac{350}{3}$	−80	35	$-\frac{80}{9}$	1	0
$-\frac{11}{2}$	33	−99	$\frac{924}{5}$	−231	198	$-\frac{231}{2}$	44	$-\frac{99}{10}$	1

In[37]:=
```
Clear[n, k];
```
lösche
```
GuessMultRE[data, {f[n, k], f[n + 1, k],
  f[n + 1, k + 1], f[n + 1, k + 2], f[n, k + 1], f[n, k + 2]}, {n, k}, 1]
```

Out[37]=
$$\{-(3+k)\,f[n,1+k]+(5+k+n)\,f[n,2+k]+(2+n)\,f[1+n,2+k],\ (3+2k-n)\,f[n,k]-3(3+k)\,f[n,1+k]+(4+k)\,f[n,2+k]-(3+k)\,f[1+n,1+k]+(4+k)\,f[1+n,2+k],\ -(2+k)\,f[n,k]+(4+k+n)\,f[n,1+k]+(2+n)\,f[1+n,1+k]\}$$

In[38]:=
```
<< RISC`HolonomicFunctions`
```

HolonomicFunctions Package version 1.7.3 (21-Mar-2017)
written by Christoph Koutschan
Copyright Research Institute for Symbolic Computation (RISC),
Johannes Kepler University, Linz, Austria

--> Type ?HolonomicFunctions for help.

In[39]:=
```
ann1 = Annihilator[a[i, k], {S[k], S[n]}]
```

Out[39]=
$$\{S_n-1,\ (1+k)\,S_k+(-1-i-k)\}$$

In[•]:=
```
{-(2 + k) f[n, 1 + k] + (3 + k + n) f[n, 2 + k] + (1 + n) f[1 + n, 2 + k],
 (2 + 2 k - n) f[n, k] - 3 (2 + k) f[n, 1 + k] + (3 + k) f[n, 2 + k] - (2 + k) f[1 + n, 1 + k] +
  (3 + k) f[1 + n, 2 + k], -(1 + k) f[n, k] + (2 + k + n) f[n, 1 + k] + (1 + n) f[1 + n, 1 + k]}
```

In[40]:=
```
{-(2 + k) S[k] + (3 + k + n) S[k]^2 + (1 + n) S[n] × S[k]^2,
 (2 + 2 k - n) - 3 (2 + k) S[k] + (3 + k) S[k]^2 - (2 + k) S[n] × S[k] + (3 + k) S[n] × S[k]^2,
 -(1 + k) + (2 + k + n) S[k] + (1 + n) S[n] × S[k]}
```

Out[40]=
$$\{(-2-k)\,S[k]+(3+k+n)\,S[k]^2+(1+n)\,S[k]^2\,S[n],\ 2+2k-n-3(2+k)\,S[k]+(3+k)\,S[k]^2-(2+k)\,S[k]\times S[n]+(3+k)\,S[k]^2\,S[n],\ -1-k+(2+k+n)\,S[k]+(1+n)\,S[k]\times S[n]\}$$

In[41]:= `ToOrePolynomial[%]`

Out[41]= $\{(1+n)\,S_k^2\,S_n+(3+k+n)\,S_k^2+(-2-k)\,S_k,\ (3+k)\,S_k^2\,S_n+(3+k)\,S_k^2+(-2-k)\,S_k\,S_n+(-6-3k)\,S_k+(2+2k-n),\ (1+n)\,S_k\,S_n+(2+k+n)\,S_k+(-1-k)\}$

In[42]:= `ann2 = OreGroebnerBasis[%]`

Out[42]= $\{(-2k-3k^2-k^3)\,S_k+(k-k^2-n+3kn-k^2n-2n^2+2kn^2-n^3)\,S_n+(k+2k^2+k^3-n-k^2n-2n^2+kn^2-n^3),\ (-2+3k-k^2-3n+2kn-n^2)\,S_n^2+(-2+2k-4n+2kn-2n^2)\,S_n+(-n-n^2)\}$

In[43]:= `DFiniteTimes[ann1, ann2]`

Out[43]= $\{(2k+5k^2+4k^3+k^4)\,S_k+(-k-ik+ik^2+k^3+n+in-2kn-3ikn-2k^2n+ik^2n+k^3n+2n^2+2in^2-2ikn^2-2k^2n^2+n^3+in^3+kn^3)\,S_n+(-k-ik-3k^2-2ik^2-3k^3-ik^3-k^4+n+in+kn+k^2n+ik^2n+k^3n+2n^2+2in^2+kn^2-ikn^2-k^2n^2+n^3+in^3+kn^3),\ (2-3k+k^2+3n-2kn+n^2)\,S_n^2+(2-2k+4n-2kn+2n^2)\,S_n+(n+n^2)\}$

In[44]:= `CreativeTelescoping[%, S[k] - 1, {S[n]}]`

Out[44]= $\{\{(1+n)\,S_n+(-i+n)\},\ \{(1+n)\,S_n+(1+k+n)\}\}$

A Hankel Determinant of Bernoulli numbers

In[45]:= `HankB[n_] := Det[Table[BernoulliB[i + j + 2], {i, 0, n - 1}, {j, 0, n - 1}]]`

De··· Tabelle Bernoulli–Zahl oder –Polynom

In[46]:= `Table[HankB[n], {n, 1, 10}]`

Tabelle

Out[46]= $\left\{\frac{1}{6},-\frac{1}{180},-\frac{1}{10\,500},\frac{1}{643\,125},\frac{8}{207\,986\,625},-\frac{64}{30\,535\,209\,705},-\frac{2048}{5\,946\,804\,792\,929},\frac{9\,437\,184}{42\,189\,910\,011\,842\,593},\frac{70\,448\,201\,072\,640}{96\,679\,865\,527\,519\,203\,892\,963},-\frac{39\,441\,975\,230\,941\,102\,080\,000}{2\,678\,689\,850\,123\,372\,050\,237\,439\,080\,199}\right\}$

In[47]:= `Apply[Ratekurz, %]`

wende an

Out[47]= $\left\{\frac{1}{6}\,\text{PProduct}\left[-\frac{1}{30}\,\text{PProduct}\left[-\frac{(1+i2)\,(2+i2)^2\,(3+i2)}{4\,(3+2\,i2)\,(5+2\,i2)},\{i2,1,-1+i1\}\right],\{i1,1,-1+i0\}\right]\right\}$