

The d-bar-Neumann operator and related topics

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(1) Wider research context.

This project is located at the intersection of several different fields : complex analysis, partial differential equations, functional analysis, operator theory, spectral analysis, potential theory, differential geometry and mathematical physics. . Many modern techniques in Several Complex Variables have their roots in the analysis of the d-bar-Neumann problem.

(2) Research questions.

We will concentrate on the problem of compactness of the d-bar-Neumann operator using an approach which has the advantage that it covers both bounded pseudoconvex domains as well as unbounded domains with weights and which can therefore be used to handle unsolved problems for unbounded domains. Properties of commutators between the Bergman projection and certain multiplication operators provide a powerful tool to investigate compactness of the d-bar-Neumann operator. It is of interest to clarify if or to what extent compactness of the restriction to forms with holomorphic coefficients already implies compactness of the original solution operator to d-bar.

(3) Methods.

Spectral analysis and results for Schrödinger operators with magnetic field will be used to investigate compactness of the d-bar-Neumann operator for spaces with plurisubharmonic weight functions. Computation of the Szegő kernel on model domains and its applications to the Leray transform. Recently developed methods from differential geometry and microlocal analysis can be used get accurate estimates for the Bergman kernel even for more general weight functions.

(4) Innovation.

A somehow analogous situation appears if one considers the d-complex on the Segal Bargmann space, where the dual operator of differentiation is the multiplication operator both being unbounded densely defined operators on the Segal Bargmann space. The annihilation operator of quantum mechanics can be represented by the differentiation and its adjoint, the creation operator, by the multiplication operator. We will investigate the question under what conditions on a Hermitian metric and a weight function on a given complex manifold M , does the corresponding d-complex on the weighted Bergman spaces of $(p, 0)$ -forms possess a similar duality. In this context we will concentrate on differential geometric concepts like real holomorphic vector fields and holomorphic torsion. Replacing single derivatives by polynomial differential operators one gets the generalized d-complex on the Segal Bargmann space, where we will try to prove basic estimates which make the machinery of the d-bar-Neumann operator running. The basic estimates appear as generalization of the uncertainty principle.

(5) Primary researchers involved.

We plan to support a PhD student and Dr. Luke Edholm by a Postdoc position.