Schedule

Workshop on

"Dispersion and Integrability"

WPI, Vienna, 30. Sep – 04. Oct 2019

Monday, September 30

14:30 - 15:30 Rémi Carles: *Turbulent effects through quasi-rectification*

15:30 – 16:30 Nikola Stoilov: *Numerical study of the Davey-Stewartson equation*

16:30 – 17:00 *Coffee break*

17:00 – 18:00 Christian Klein: *Multi-domain spectral methods for dispersive PDEs*

Tuesday, October 1st

9:00 - 10:00 Patrick Gérard: *On the integrability of the Benjamin-Ono equation on the torus (Part 1)*

10:00 - 10:30 *Coffee break*

10:30 - 11:30 Thomas Kappeler: *On the integrability of the Benjamin-Ono equation on the torus (Part 2)*

11:30 - 12:30 Anton Arnold: Short- and long-time behavior in (hypo)coercive ODE-systems and Fokker-Planck equations

Wednesday, October 2

9:00 - 10:00 Vincent Duchêne: *On the Favrie-Gavrilyuk approximation to the Serre-Green-Naghdi system.*

10:00 - 10:30 *Coffee break*

10:30 - 11:30 Ricardo Barros: *Effect of variation in density on the stability of bilinear shear currents with a free surface.*

11:30 - 12:30 Valeria Banica: *On the energy of critical solutions of the binormal flow*

Schedule

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Thursday, October 3

9:00 - 10:00 **Miguel Rodrigues:** *Harmonic and solitary wave limits of periodic traveling waves*

10:00 - 10:30 *Coffee break*

10:30 - 11:30 **Guillaume Ferriere:** *Multi-solitons for the logarithmic Schrödinger equation*

11:30 - 12:30 **Corentin Audiard:** *Lifespan of solutions of the Euler-Korteweg system.*

Friday, October 4

9:00 - 10:00 **Thomas Alazard:** *Entropies and Lyapounov functionals for the Hele-Shaw equation.*

10:00 - 10:30 *Coffee break*

10:30 - 11:30 **Joackim Bernier:** *Long time behavior of the solutions of NLW on the d-dimensional torus*

Some news on nonlinear dispersive equations : modeling, theory and numerics
WPI, September 30-October 4

Thomas Alazard: ***Entropies and Lyapounov functionals for the Hele-Shaw equation.***

This lecture is devoted to the study of the Hele-Shaw equation, based on a joint work with Nicolas Meunier and Didier Smets. We introduce an approach inspired by the water-wave theory. Starting from a reduction to the boundary, introducing the Dirichlet to Neumann operator and exploiting various cancellations, we exhibit parabolic evolution equations for the horizontal and vertical traces of the velocity on the free surface. This allows to quasi-linearize the equations in a very simple way. By combining these exact identities with convexity inequalities, we prove the existence of hidden Lyapounov functions of different natures. We also deduce from these identities and previous works on the water wave problem a simple proof of the well-posedness of the Cauchy problem.

Anton Arnold: ***Short- and long-time behavior in (hypo)coercive ODE-systems and Fokker-Planck equations.***

We are concerned with the short- and large-time behavior of Fokker-Planck equations with linear drift, i.e. $\partial_t f = div(D\nabla_x f + Cxf)$. A coordinate transformation can normalize these equations such that the diffusion and drift matrices are linked as $D = C_s$, the symmetric part of C.

The first main result of this talk is the connection between normalized Fokker-Planck equations and their drift-ODE $\dot{x} = -Cx$: Their L^2-propagator norms actually coincide. This implies that optimal decay estimates on the drift-ODE (w.r.t. both the maximum exponential decay rate and the minimum multiplicative constant) carry over to sharp exponential decay estimates of the Fokker-Planck solution towards the steady state.

Secondly, we define an "index of hypocoercivity", both for ODEs and Fokker-Planck equations that describes the interplay between between the dissipative and conservative part of their generator. This index characterizes the polynomial decay of the propagator norm for short time.

Corentin Audiard: ***Lifespan of solutions of the Euler-Korteweg system.***

The Euler-Korteweg system is a dispersive perturbation of the usual compressible Euler equations that includes the effect of capillary forces. For small irrotational initial data, global well-posedness is known to hold in dimension at least three. In this talk we discuss the case of small initial data with non zero vorticity, where the dispersive system becomes a coupled dispersive-transport system. The main result is that the time of existence only depends on the size of the initial vorticity.

Valeria Banica: ***On the energy of critical solutions of the binormal flow.***

The binormal flow is a model for the dynamics of a vortex filament in a 3-D inviscid incompressible fluid. The flow is also related with the classical continuous Heisenberg model in ferromagnetism, and the 1-D cubic Schrödinger equation. We consider a class of solutions at the critical level of regularity that generate singularities in finite time. One of our main results presented in this talk is to prove the existence of a natural energy associated to these solutions. This energy remains constant except at the time of the formation of the singularity when it has a jump discontinuity. When interpreting this conservation law in the framework of fluid mechanics, it involves the amplitude of the Fourier modes of the variation of the direction of the vorticity. This is a joint work with Luis Vega.

Ricardo Barros: ***Effect of variation in density on the stability of bilinear shear currents with a free surface.***

The linear stability of homogenous shear flows between two rigid walls is a classical problem that goes back to Rayleigh (1880). Among other things, Rayleigh was able to show that a shear flow with no inflection points is linearly stable. The generalisation of this stability criterion to the free-surface setting is not straightforward and was established much later by Yih (1971) (under certain restrictions) and, more recently, Hur & Lin (2008). In the case when a shear flow with a free surface is modelled by constant vorticity layers, no stability criterion is known. As a first step in this direction we consider the stability analysis of a bilinear shear current and establish a criterion for the stability of the flow. The effect of density stratification on the stability of the flow will also be investigated.

Joackim Bernier: ***Long time behavior of the solutions of NLW on the d-dimensional torus.***

I will present a new normal form transformation decomposing the dynamics of some nonlinear Hamiltonian systems into low and high frequencies with weak interactions. While the low part of the dynamics can be put under classical Birkhoff normal form, the high modes evolves according to a time dependent linear Hamiltonian system. We then control the global dynamics by using polynomial growth estimates for high modes and the preservation of Sobolev norms for the low modes. We will see how this procedure allows us to prove that, for almost any mass, small and smooth solutions of the nonlinear wave equation on $\mathbb{T}^d$ of high Sobolev indices are stable up to arbitrary long times with respect to the size of the initial data. This is a joint work with Erwan Faou and Benoît Grébert.

Rémi Carles: ***Turbulent effects through quasi-rectification.***

Tis is a joint work with Christophe Cheverry. We study high frequency solutions of nonlinear hyperbolic equations for time scales at which dispersive and nonlinear effects can be present in the leading term of the solution, on a model stemming from strongly magnetized plasmas or nuclear magnetic resonance experiments. We show how the produced waves can accumulate during long times to produce constructive and destructive interferences which, in the above contexts, are part of turbulent effects.

Vincent Duchêne: ***On the Favrie-Gavrilyuk approximation to the Serre-Green-Naghdi system.***

The Serre-Green-Naghdi system is a fully nonlinear and weakly dispersive model for the propagation of surface gravity waves. It enjoys many good theoretical properties, including a robust well-posedness theory for the initial-value problem, and a Hamiltonian structure. It is however not so suitable for practical use, as standard numerical strategies involve the costly inversion of an elliptic operator at each time step. N. Favrie and S. Gavrilyuk proposed a novel strategy for efficiently producing approximate solutions, by introducing a "relaxed" first-order quasilinear system of balance laws, depending on additional unknows and a free parameter. The claim is that in the singular limit when the parameter goes to infinity, solutions of the relaxed system approach solutions of the Serre-Green-Naghdi system. We will discuss a rigorous analysis. It differs from standard results due to the presence of an additional parameter (describing the shallowness of the flow) and order-zero source terms which become dominant when the shallowness parameter goes to zero.

Guillaume Ferriere: ***Multi-solitons for the logarithmic Schrödinger equation.***

In this presentation, we consider the *nonlinear Schrödinger equation with logarithmic nonlinearity* (logNLS in short). We mostly focus on the *focusing* case which presents a very special Gaussian stationary solution, called *Gausson*, which is orbitally stable. In fact, more generally, it has been shown that every Gaussian data remains Gaussian through the flow of logNLS, and this feature gives rise to (almost) periodic solutions in the focusing case, called *breathers*. The main result of this talk addresses the existence of multi-solitons, i.e. solutions to logNLS which behaves like the sum of several solitons (i.e. Gaussons here) for large times, in dimension 1. This kind of result is rather usual for dispersive equations with polynomial-like nonlinearity, and our proof is directly inspired from the usual proof with *energy techniques*. The main difficulty is the fact that the energy cannot be linearized as one would want, at least not everywhere. Furthermore, some new and surprising features appear in this result: the convergence is in H^1 and $\mathcal{F}(H^1)$ with a rate faster than exponential, and there is no need for a large enough relative speed (non-zero is sufficient).

Patrick Gérard: ***On Birkhoff coordinates of the Benjamin-Ono equation on the torus and applications to solutions with negative Sobolev regularity. Part 1.***

This is a jointwork with Thomas Kappeler. Using the Lax pair structure for the Benjamin-Ono equation with periodic boundary conditions, we construct a global system of Birkhoff coordinates on the phase space of real valued square integrable functions with average 0 on the torus, including a characterisation of finite gap potentials. Among consequences, we infer almost periodicity of all trajectories, identification of traveling waves and construction of periodic in time solutions with low regularity.

Thomas Kappeler: ***On Birkhoff coordinates of the Benjamin-Ono equation on the torus and applications to solutions with negative Sobolev regularity. Part 2.***

In this talk I report on joint work with Patrick Gérard and Peter Topalov concerning properties of the flow map of the Benjamin-Ono equation on the torus. The main result says that the flow map, introduced in our previous work on the space $L^2_{r,0}$ of real valued, $2\pi-$periodic L^2-integrable functions with mean 0, can be continuously extended to the Sobolev spaces $H^{-s}_{r,0}$ for $0 < s < 1/2$. The key ingredient is a corresponding extension of the Birkhoff coordinates to these Sobolev spaces.

Christian Klein: ***Multi-domain spectral methods for dispersive PDEs.***

We discuss numerical methods to construct solutions to nonlinear dispersive PDEs on the whole real line, and this for initial data which are slowly decreasing towards infinity or just bounded there. As an example we discuss the transverse stability of the Peregrine solution in the 2d nonlinear Schrödinger equation.

Miguel Rodrigues: ***Harmonic and solitary wave limits of periodic traveling waves.***

In a series of papers with Sylvie Benzoni-Gavage (and, depending on papers, Pascal Noble or Colin Mietka), we have studied both co-periodic stability and modulation systems for periodic traveling waves of a rather large class of Hamiltonian partial differential equations that includes quasilinear generalizations of the Korteweg–de Vries equation and dispersive perturbations of the Euler equations for compressible fluids, either in Lagrangian or in Eulerian coordinates.

All characterizations are derived in terms of the Hessian matrix of the action integral of profile equations, a finite-dimensional object. In the present talk, with this in mind, we shall discuss the consequences of the recently obtained expansions of this matrix in two asymptotic regimes, namely the zero-amplitude and the zero-wavelength limits.

Nicolas Stoilov: ***Numerical study of the Davey-Stewartson equation.***

In this work we will look at the focusing Davey-Stewartson equation from two different angles, using advanced numerical tools.

As a nonlinear dispersive PDE and a generalisation of the non-linear Schrödinger equation, DS possesses solutions that develop a singularity in finite time. We numerically study the long time behaviour and potential blow-up of solutions to the focusing Davey-Stewartson II equation for various initial data and propose a conjecture describing the blow up rate and solution profiles near the singularity.

Secondly, DS is an integrable system and can be studied as an inverse scattering problem. Both the forward and inverse scattering transformation in this case are reduced to a d-bar system which plays the role that Riemann-Hilbert problems play in one dimensional problems. We will present numerical solutions for Schwartzian and compactly supported potentials. Further, to complement numerics, we will discuss analytical considerations to handle asymptotic behaviour. In all studied cases we use spectral methods and achieve machine precision.

Based on joint works with Christian Klein and Ken McLaughlin