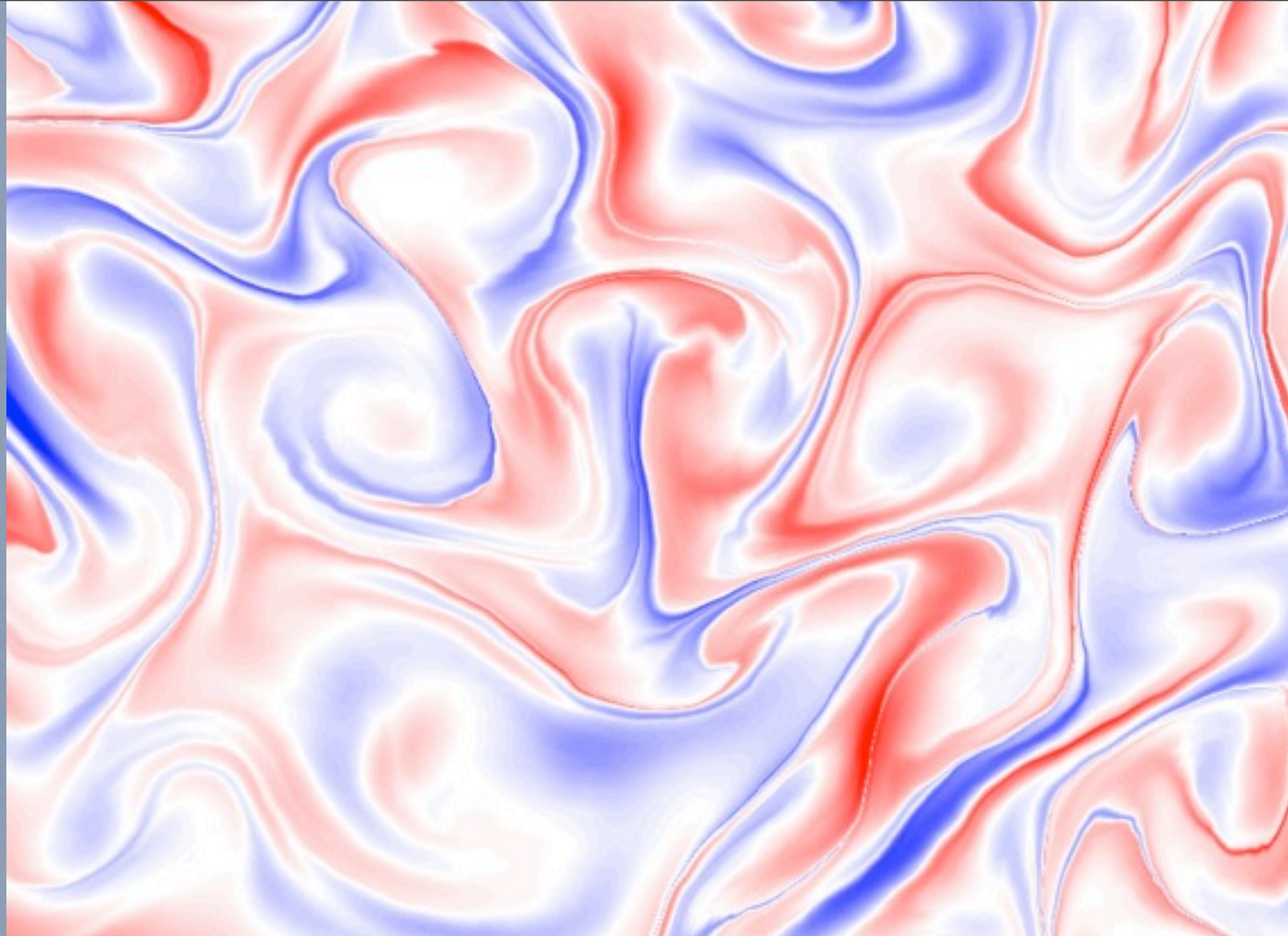
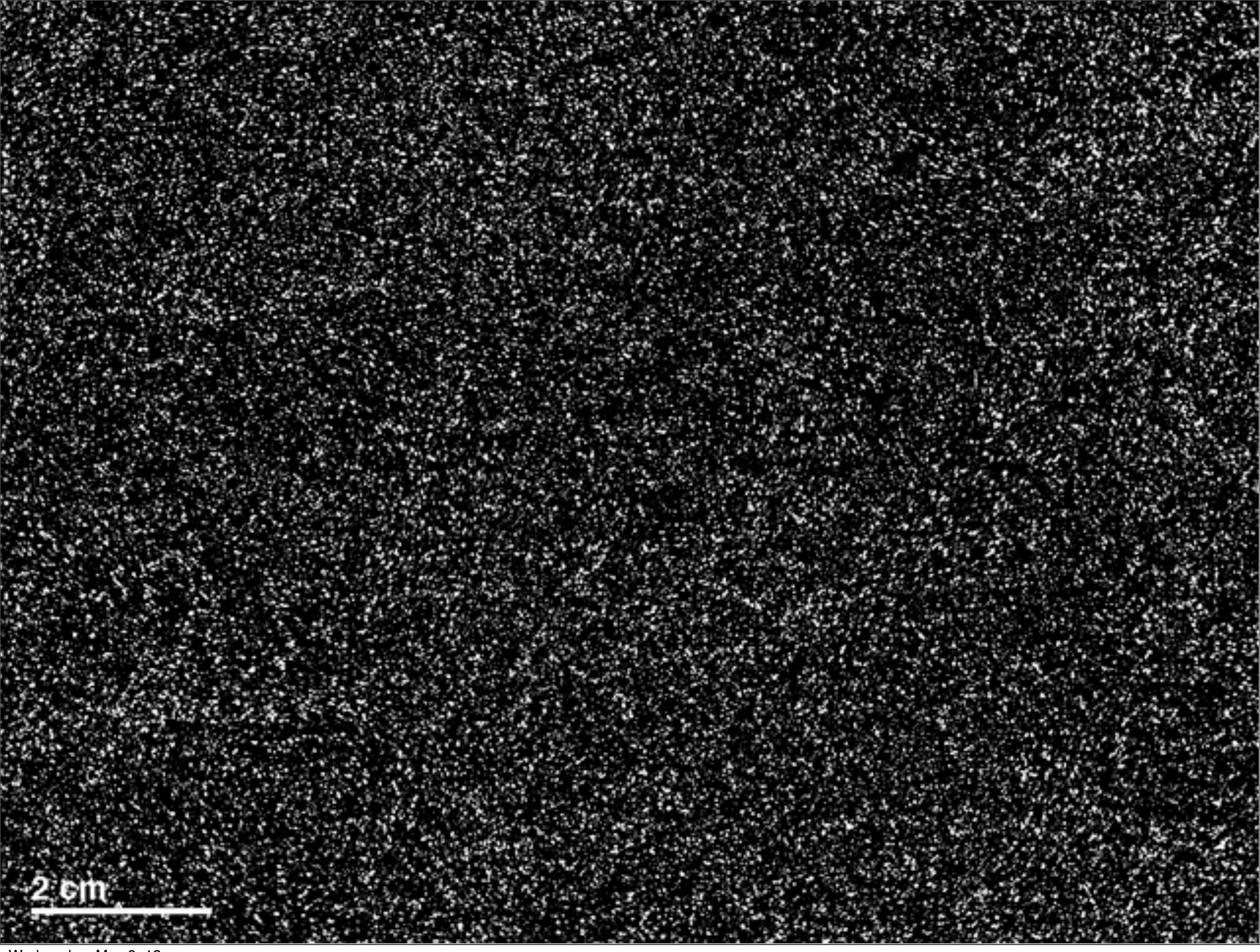


Physical Transport of Spectral Properties in 2D Turbulence

N.T. Ouellette

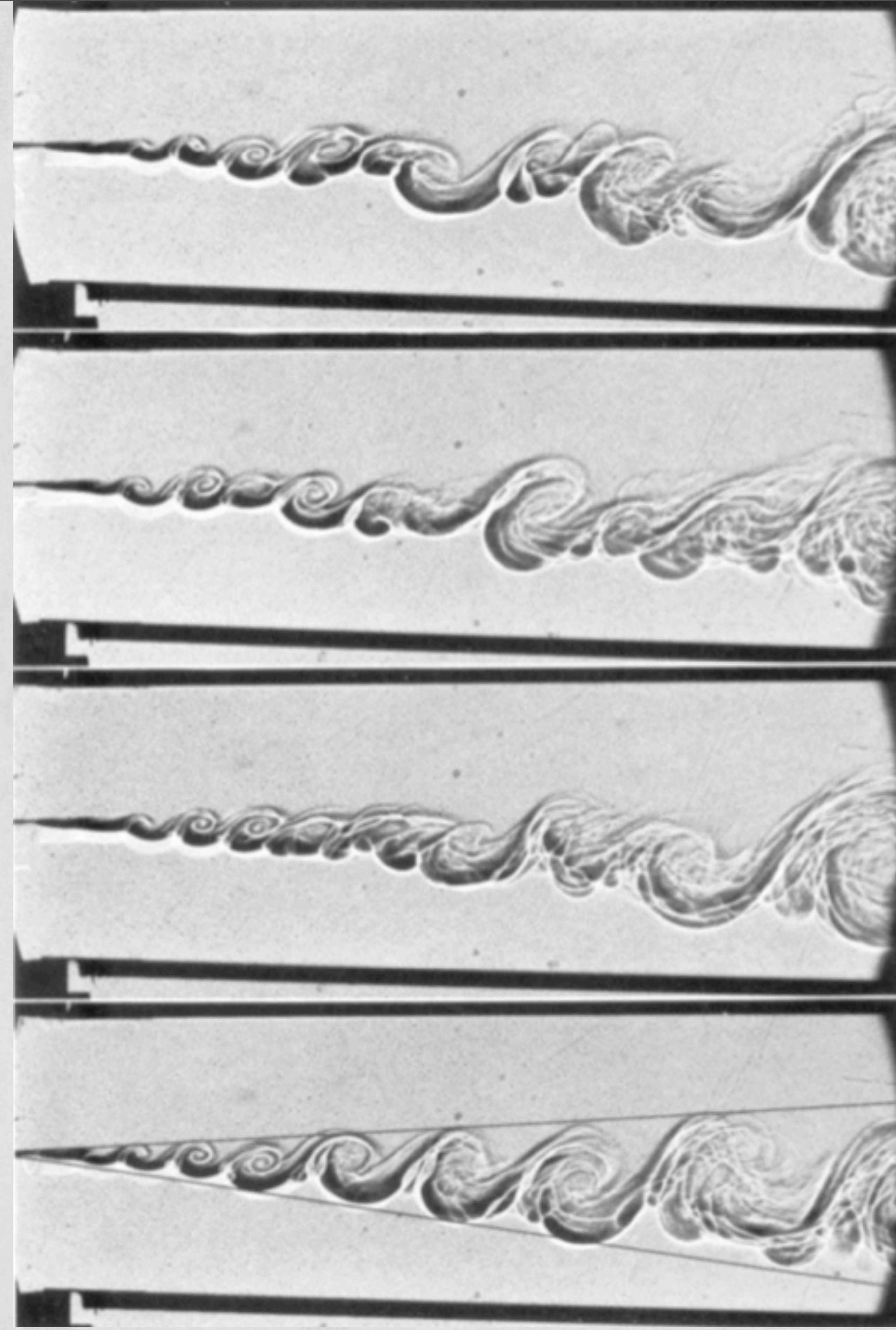
D.H. Kelley, Y. Liao


2 cm

Flow Structures

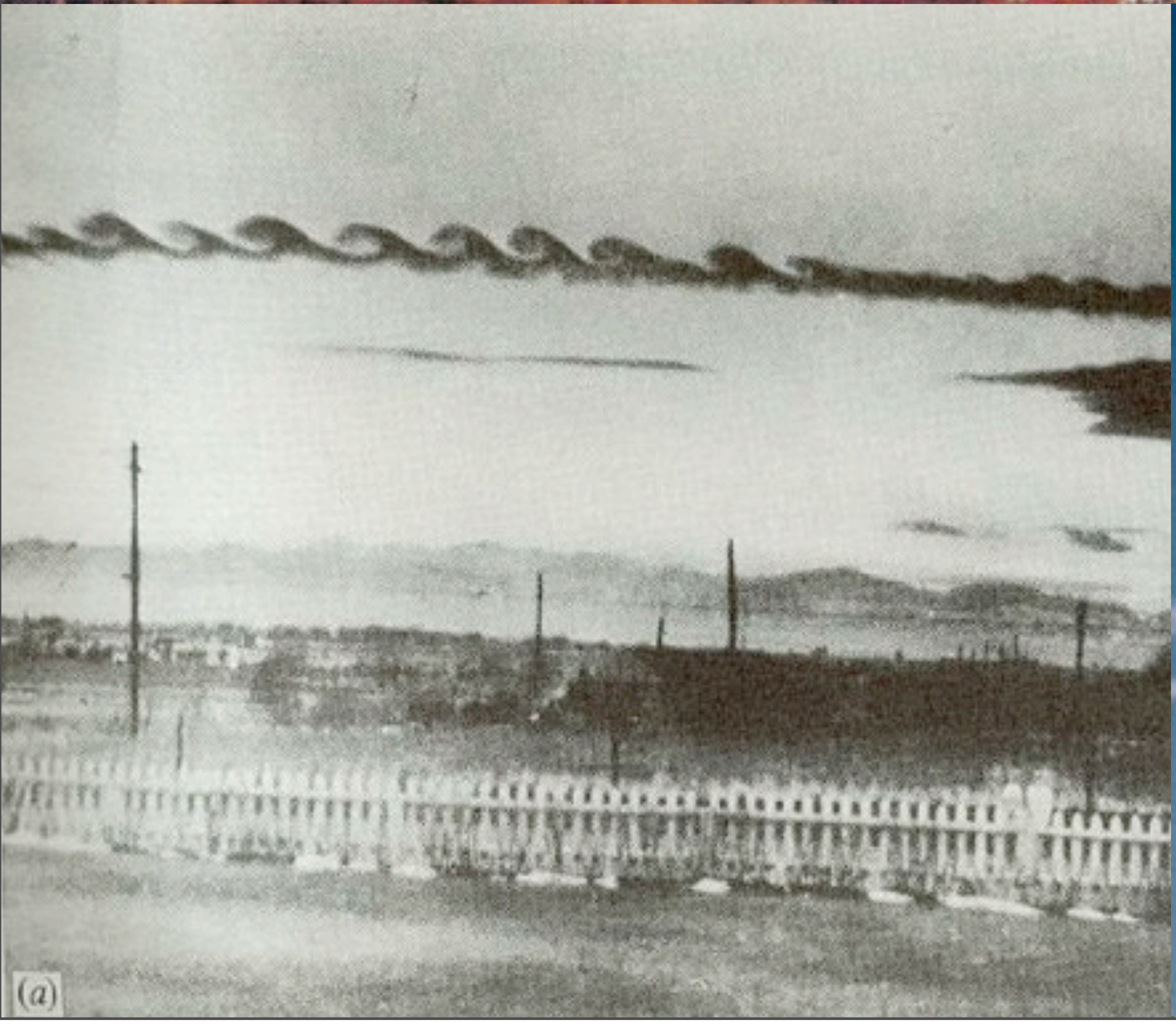
Fluid flows are coherent in space and time.

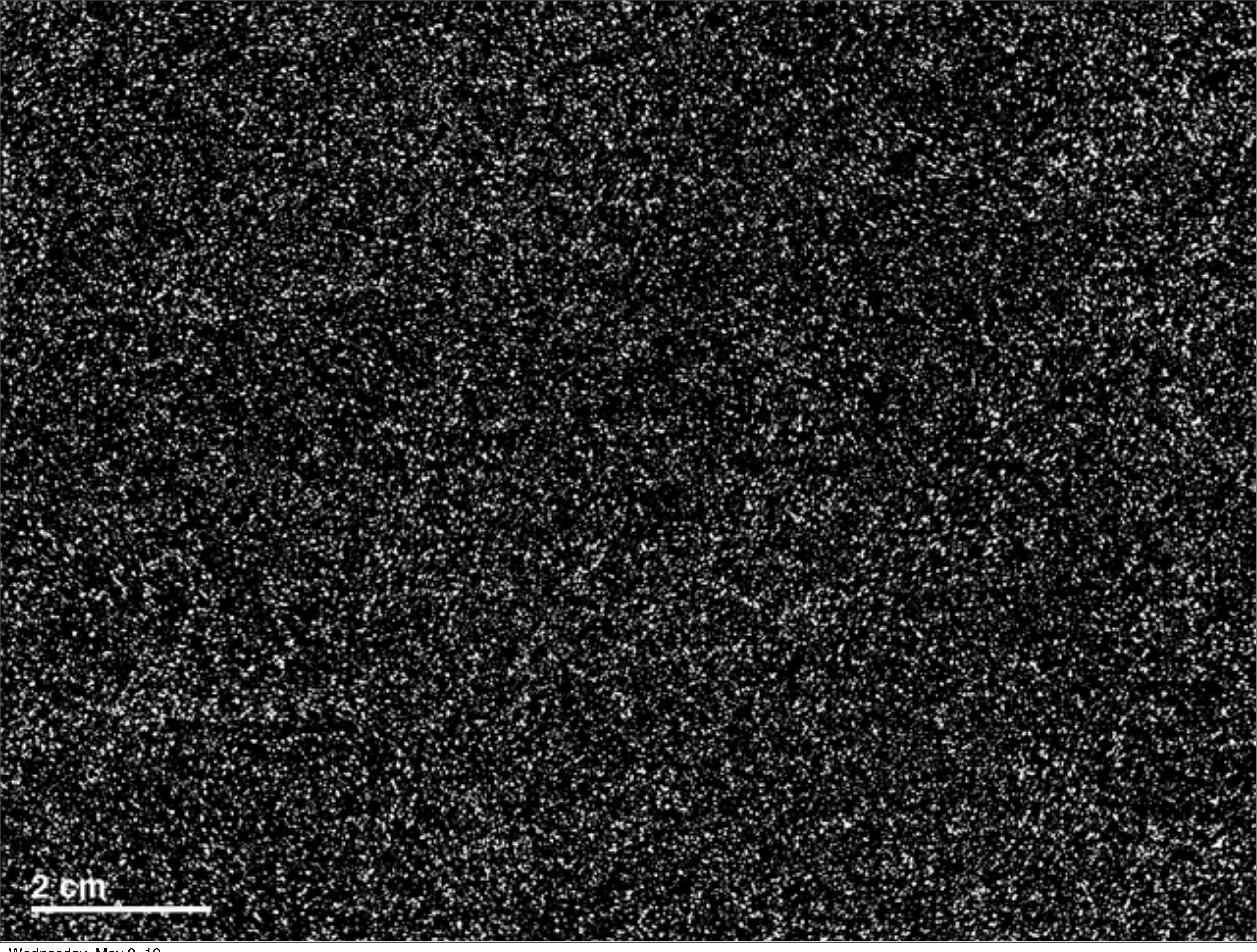
Nonlinearities generate structure on many scales.

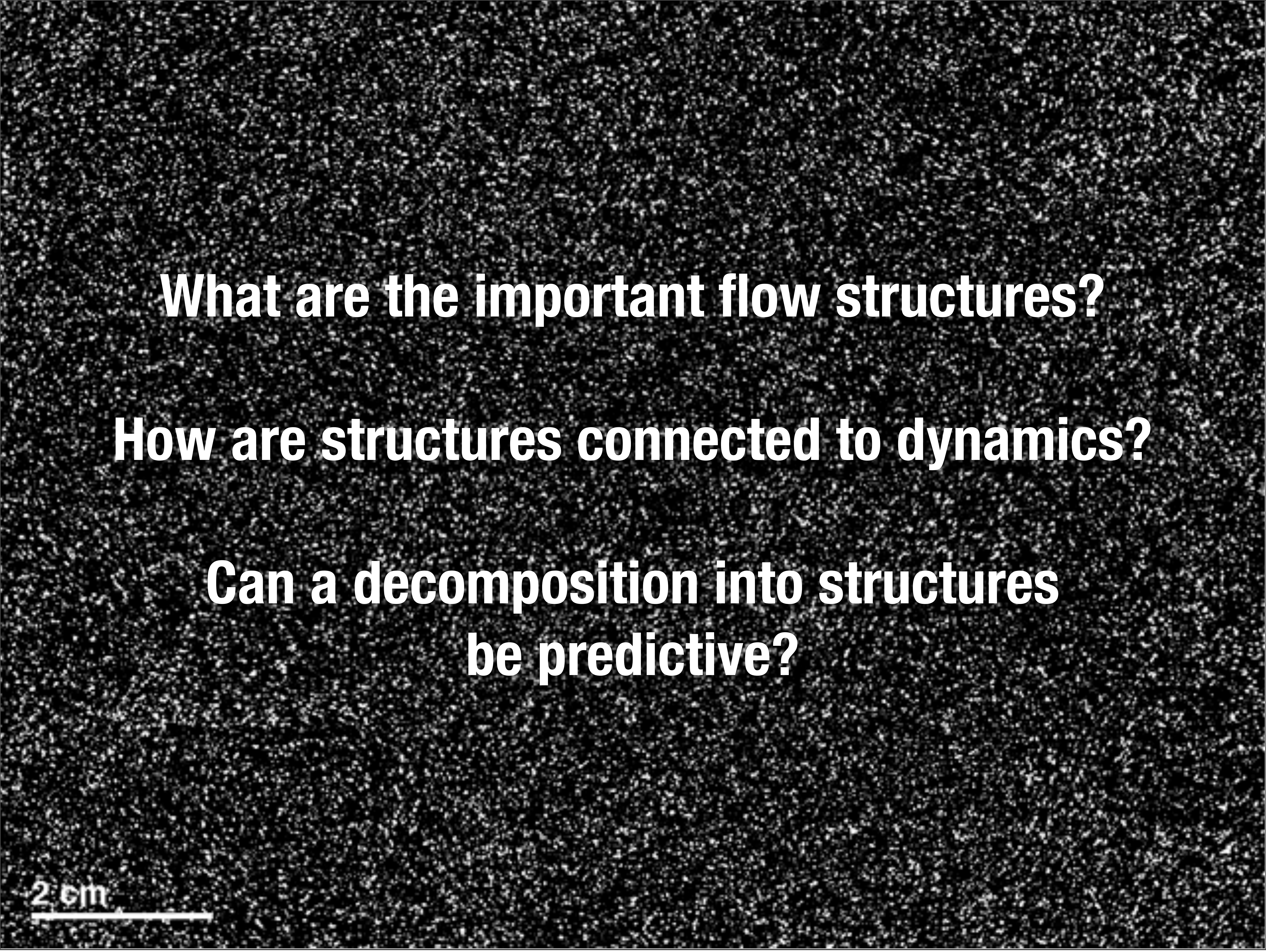


G.L. Brown & A. Roshko, J. Fluid Mech. (1974)

Pattern Formation




2 cm



What are the important flow structures?

How are structures connected to dynamics?

Can a decomposition into structures be predictive?

Defining "Dynamics"

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \boldsymbol{u}$$

Defining “Dynamics”

$$\frac{\partial \boldsymbol{u}}{\partial t} + \underline{\boldsymbol{u} \cdot \nabla \boldsymbol{u}} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u}$$

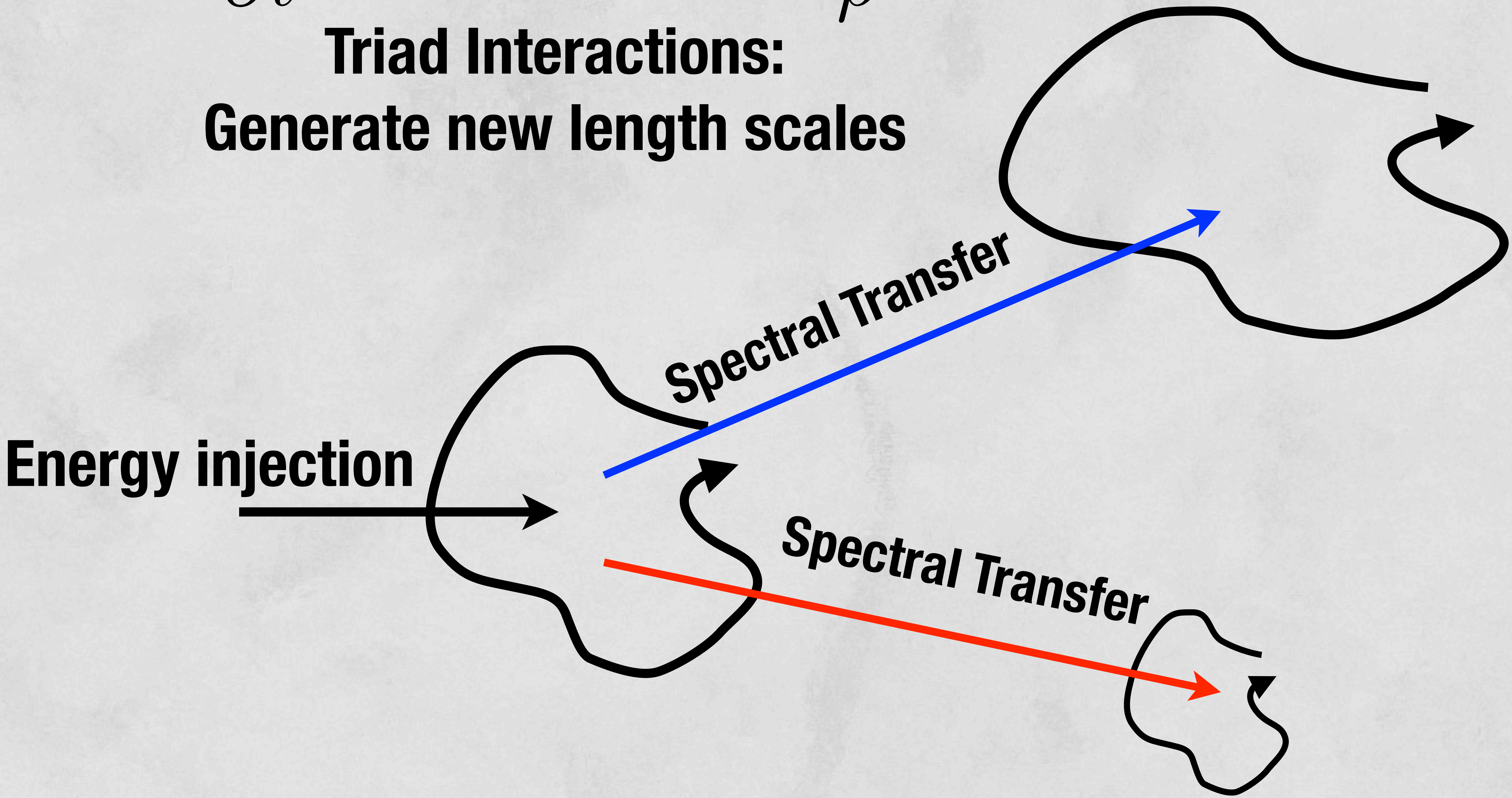
Triad Interactions:

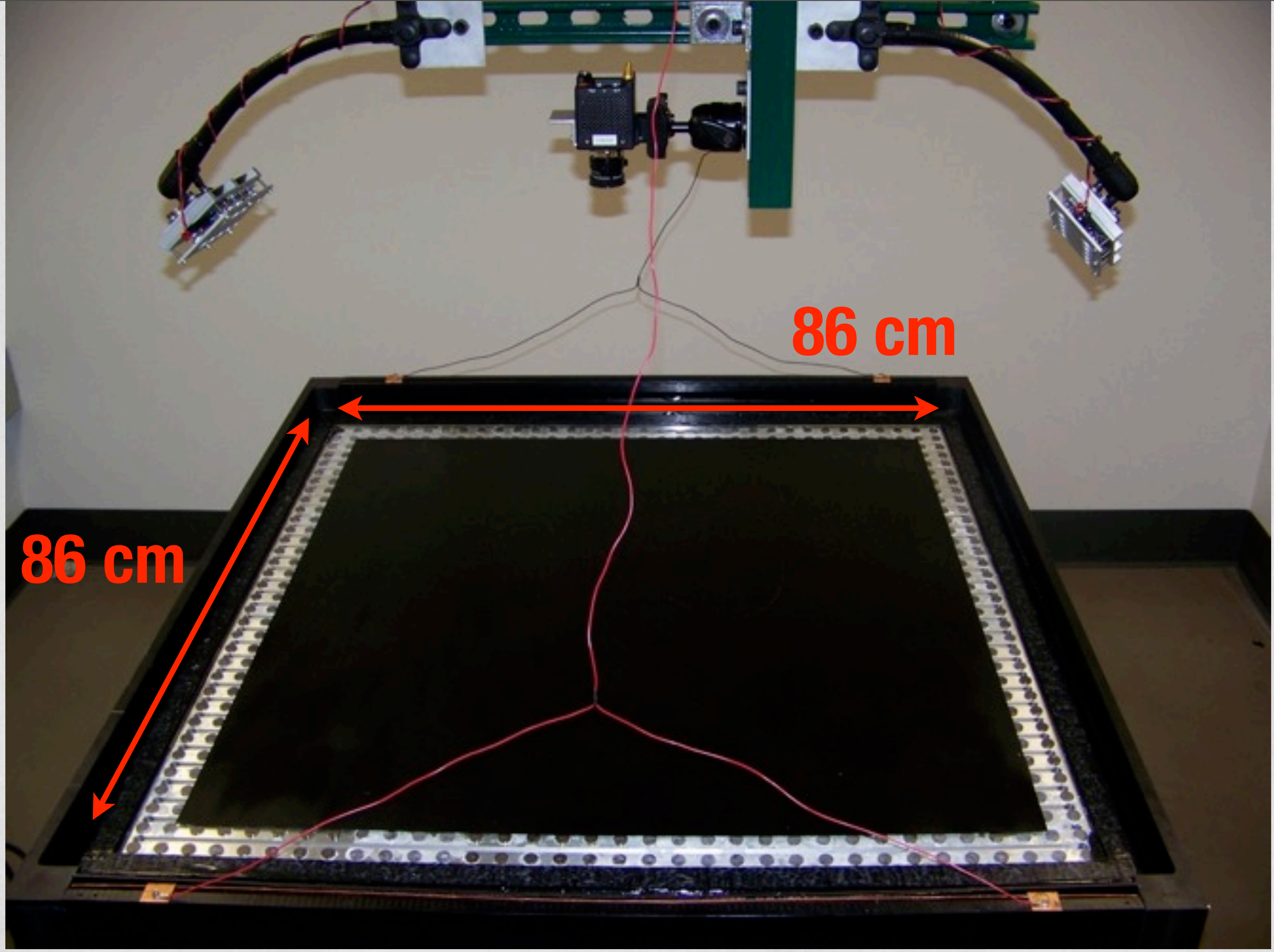
Generate new length scales

Defining "Dynamics"

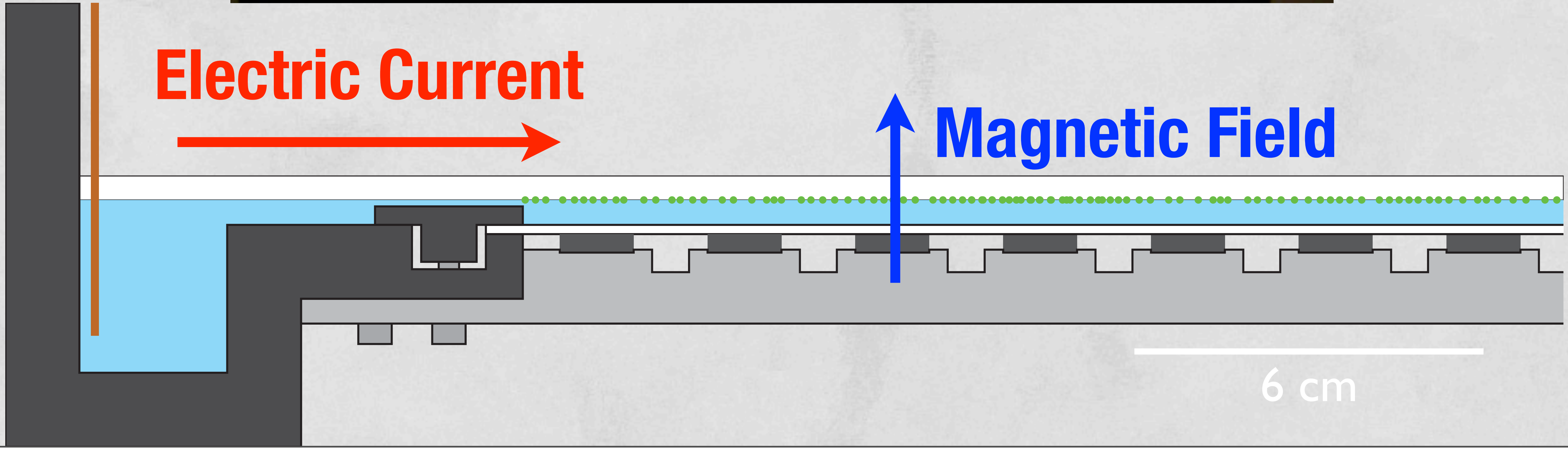
$$\frac{\partial \boldsymbol{u}}{\partial t} + \underline{\boldsymbol{u} \cdot \nabla \boldsymbol{u}} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u}$$

Triad Interactions:
Generate new length scales

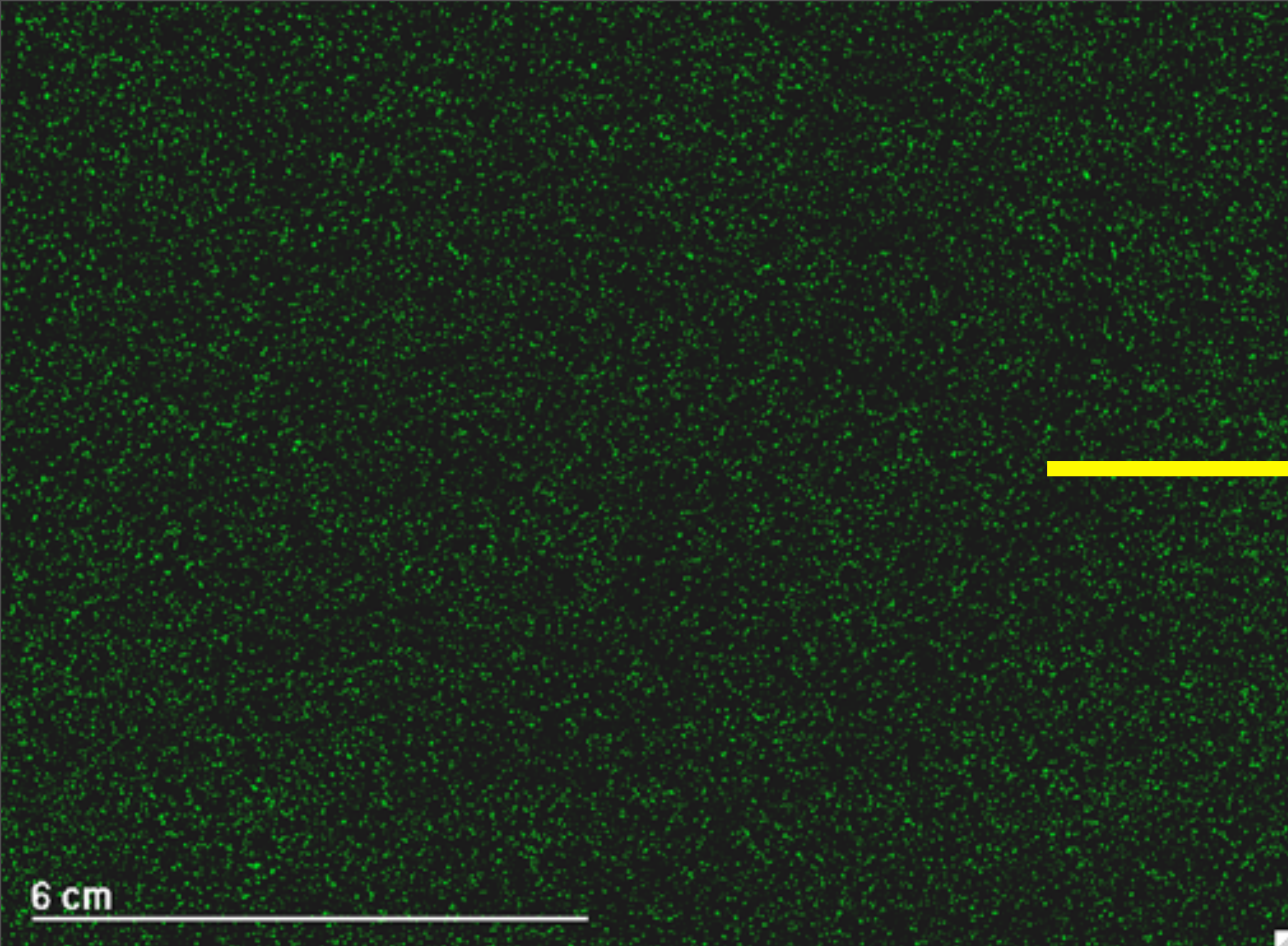


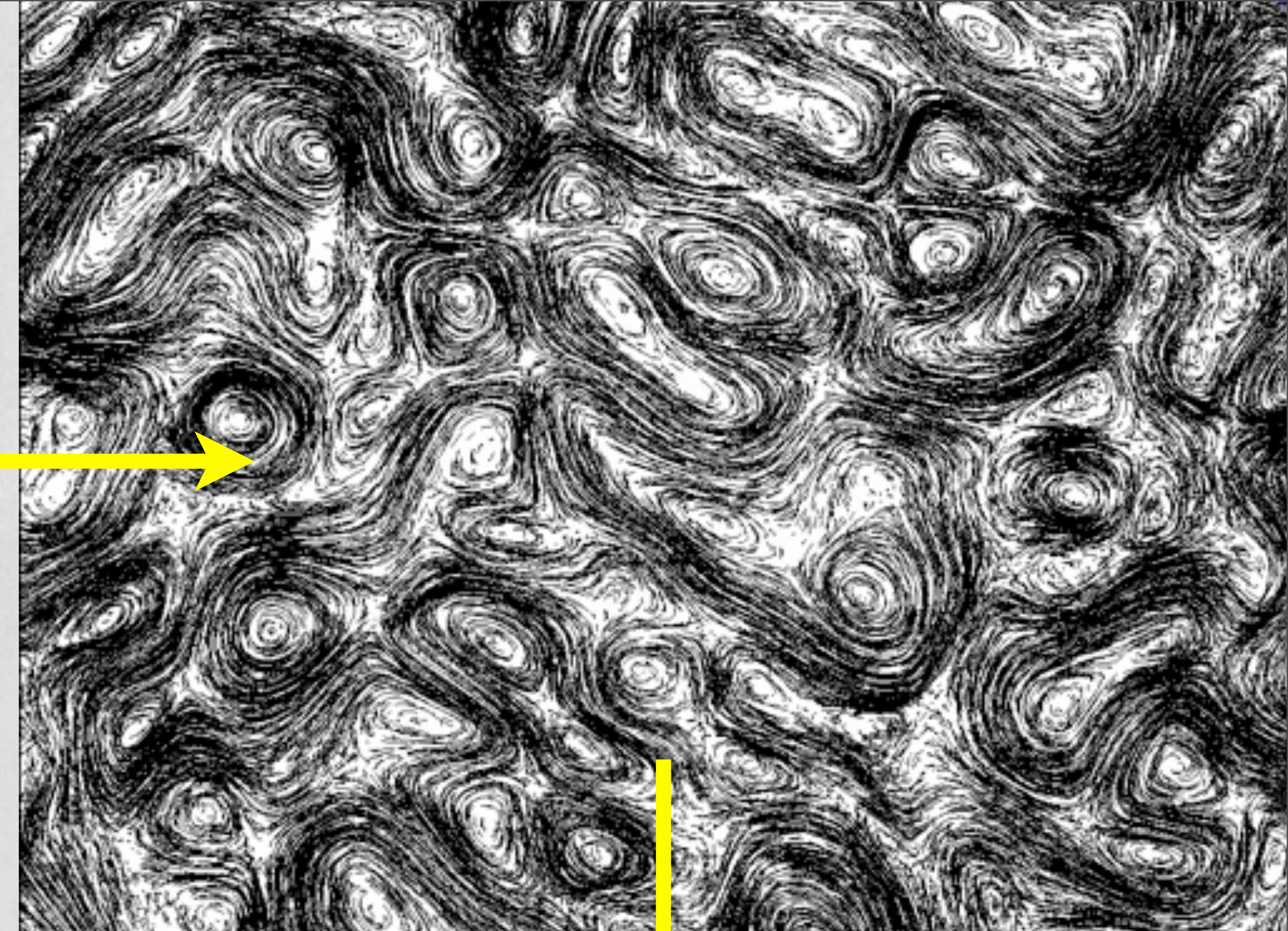


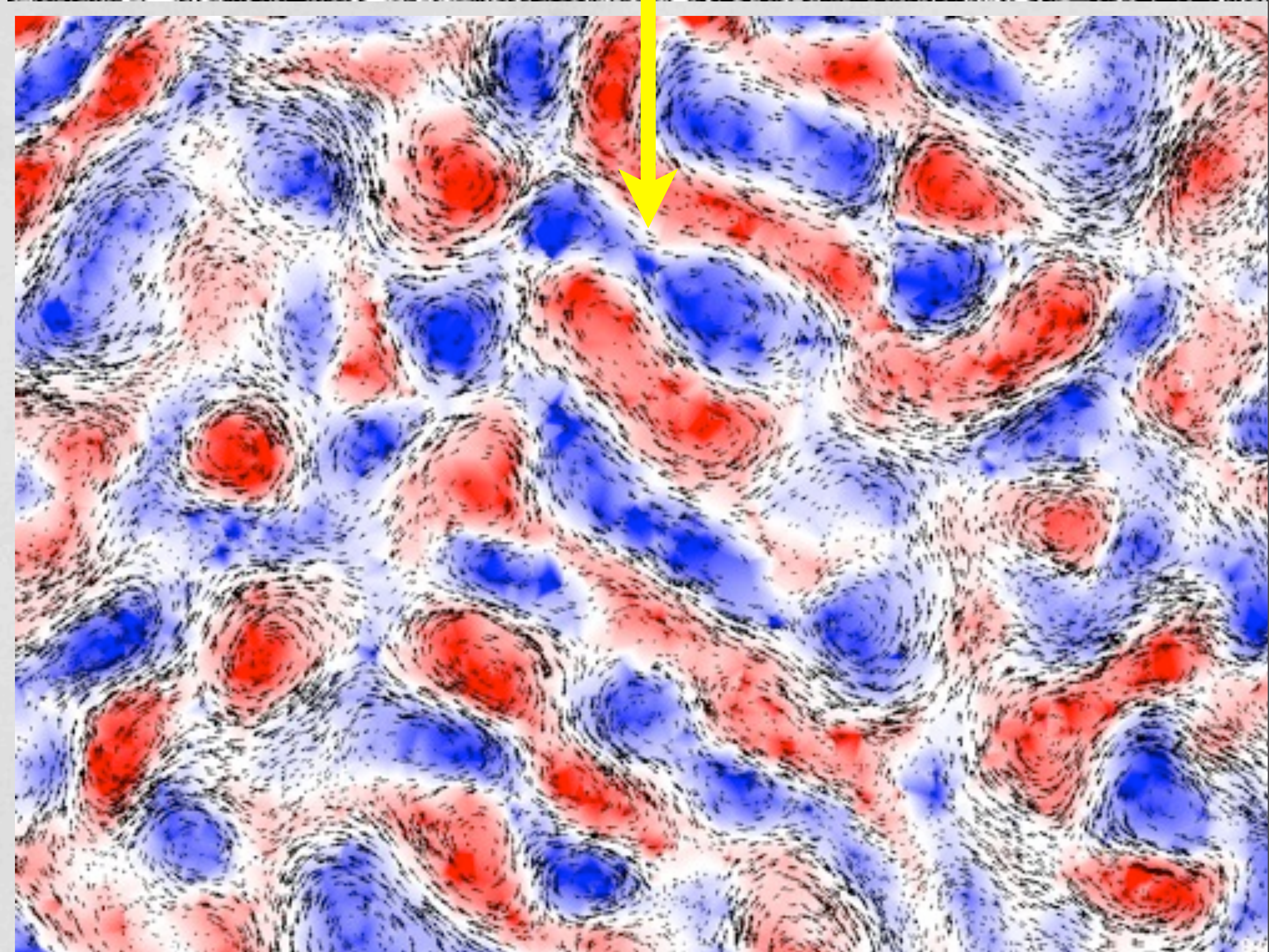
86 cm
86 cm



Electric Current
Magnetic Field
6 cm







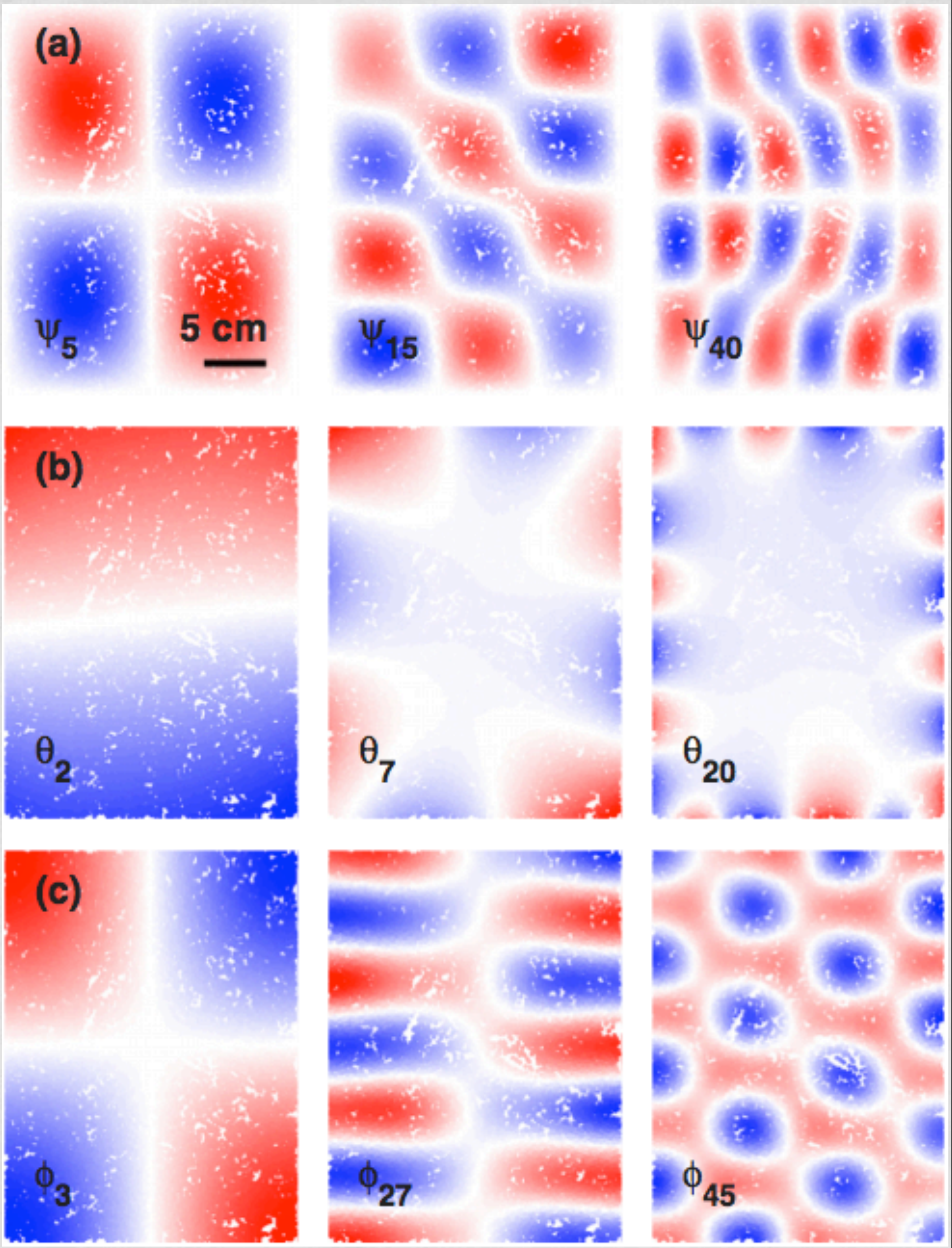
Measure velocity field with PTV

50 µm particles, ~35k per frame

Advect virtual particles through field

NTO, H. Xu, & E. Bodenschatz, Exp. Fluids (2006)
NTO, P.J.J. O'Malley, & J.P. Gollub, Phys. Rev. Lett. (2008)
S.T. Merrifield, D.H. Kelley, & NTO, Phys. Rev. Lett. (2010)

Field Conditioning



Ensure velocity field is 2D by projecting onto basis modes

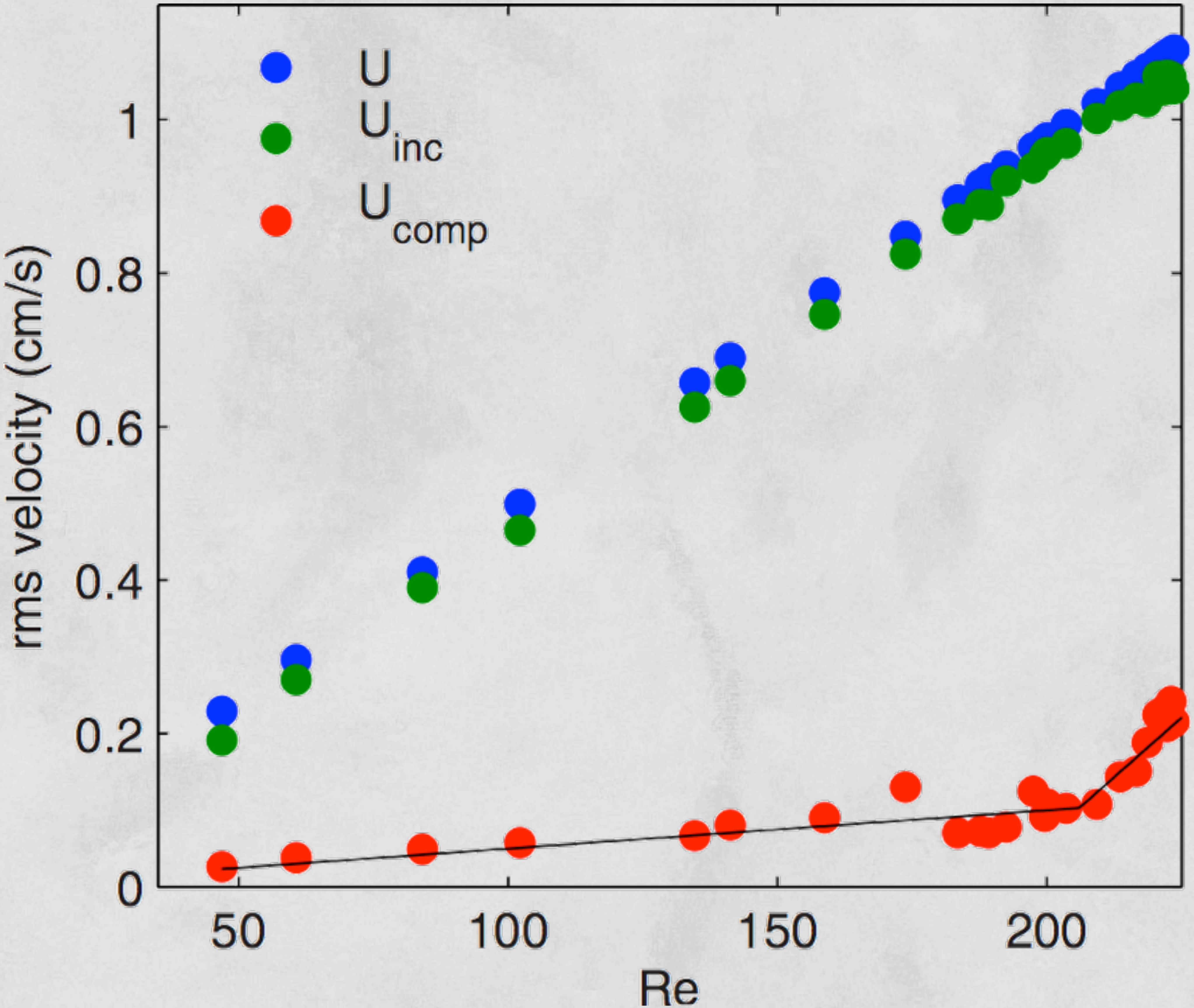
Define three sets of modes:

- **ψ: streamfunction**
- **θ: boundary**
- **Φ: potential**

Lekien et al., J. Geophys. Res. (2004)

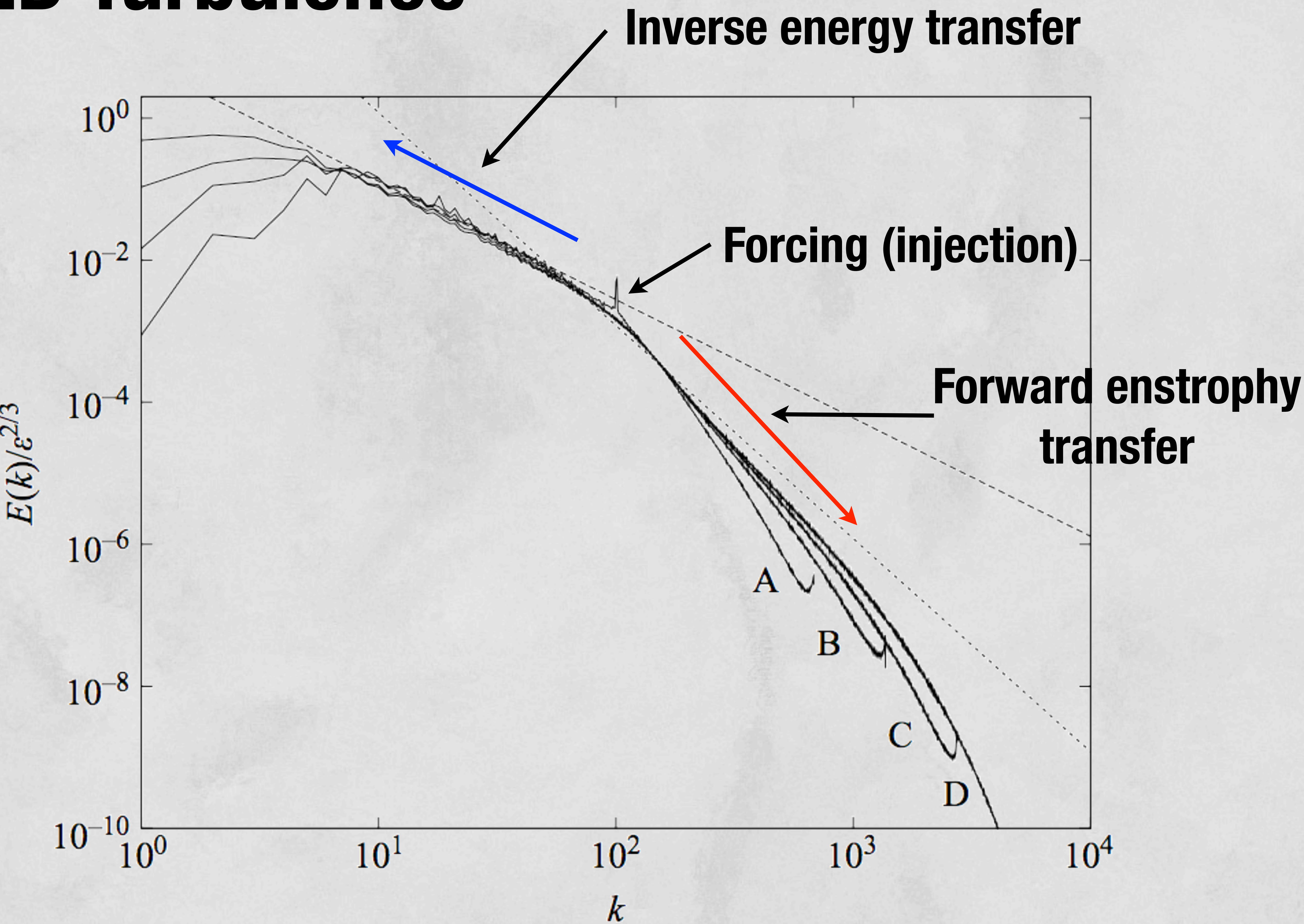
D.H. Kelley & NTO, Phys. Fluids (2011)

Field Conditioning



D.H. Kelley & NTO, Phys. Fluids (2011)

2D Turbulence

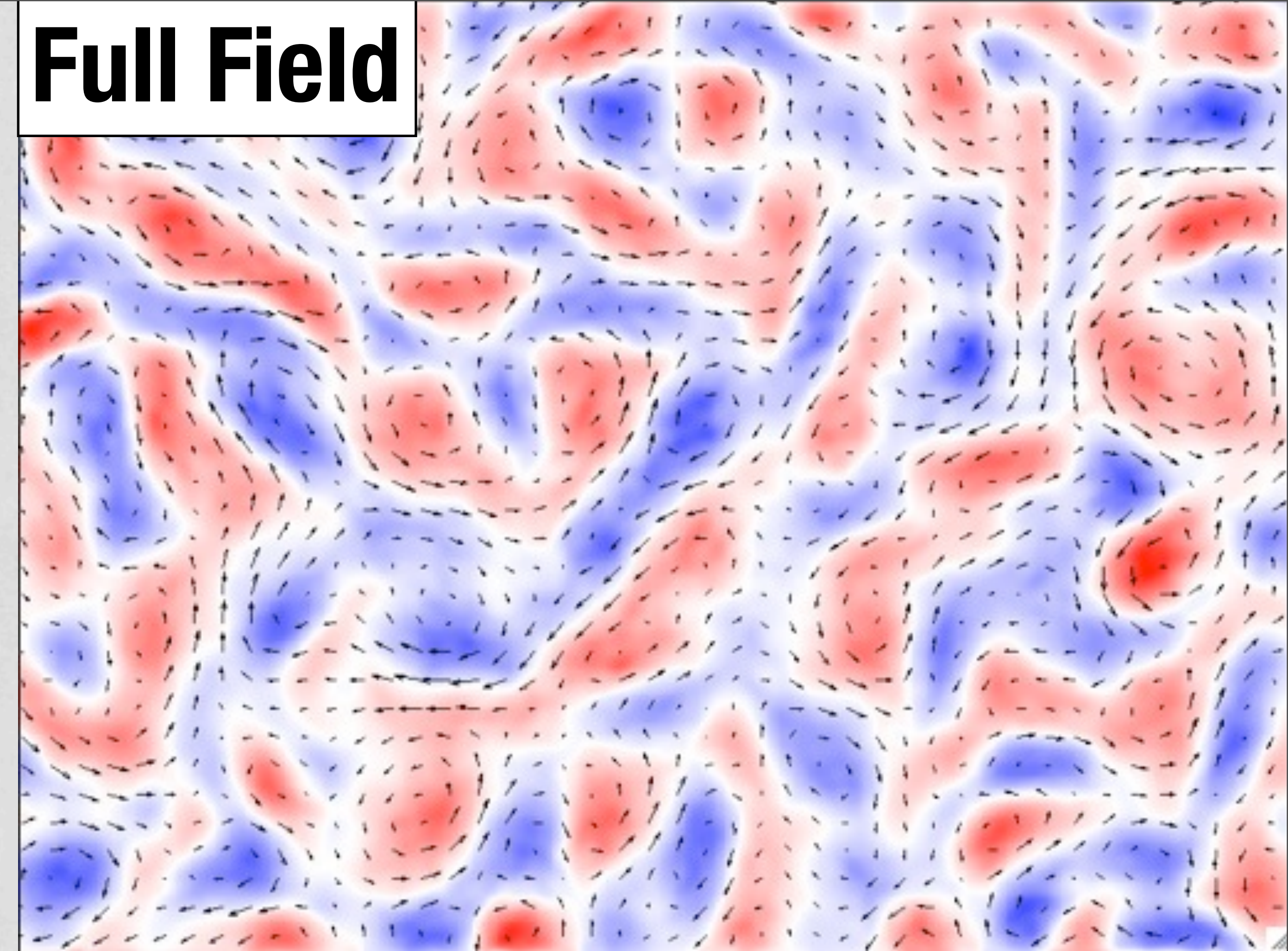


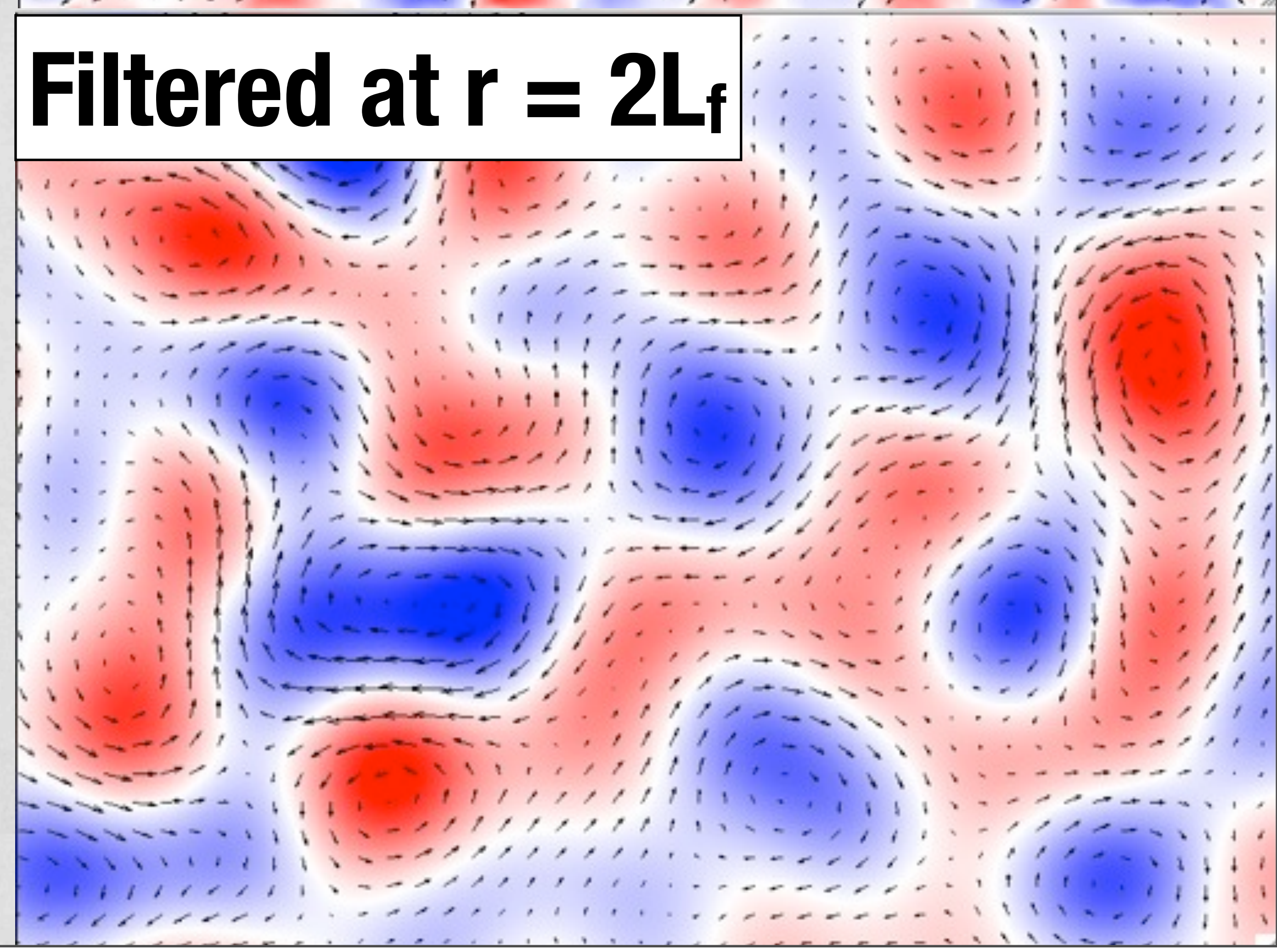
G. Boffetta, J. Fluid Mech. (2007)

Spatially Resolved Spectral Fluxes

Convolve velocity field with spectral low pass filter:

$$\boldsymbol{u}^{(r)} = \int G^{(r)}(\boldsymbol{x} - \boldsymbol{x}')\boldsymbol{u}(\boldsymbol{x})\mathrm{d}\boldsymbol{x}'$$





G.L. Eyink, J. Stat. Phys. (1995)
M.K. Rivera et al., Phys. Rev. Lett. (2003)

Write equation of motion for filtered energy:

$$\frac{\partial E^{(r)}}{\partial t} = -\frac{\partial J_i^{(r)}}{\partial x_i} - \nu \frac{\partial u_i^{(r)}}{\partial x_j}\frac{\partial u_i^{(r)}}{\partial x_j} - \Pi^{(r)}$$

G.L. Eyink, J. Stat. Phys. (1995)
M.K. Rivera et al., Phys. Rev. Lett. (2003)

Write equation of motion for filtered energy:

$$\frac{\partial E^{(r)}}{\partial t} = -\frac{\partial J_i^{(r)}}{\partial x_i} - \nu \frac{\partial u_i^{(r)}}{\partial x_j} \frac{\partial u_i^{(r)}}{\partial x_j} - \Pi^{(r)}$$

Change in energy at a point

G.L. Eyink, J. Stat. Phys. (1995)

M.K. Rivera et al., Phys. Rev. Lett. (2003)

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$$\frac{\partial E^{(r)}}{\partial t} = -\frac{\partial J_i^{(r)}}{\partial x_i} - \nu \frac{\partial u_i^{(r)}}{\partial x_j} \frac{\partial u_i^{(r)}}{\partial x_j} - \Pi^{(r)}$$

Change in energy at a point

Spatial transport

G.L. Eyink, J. Stat. Phys. (1995)

M.K. Rivera et al., Phys. Rev. Lett. (2003)

Write equation of motion for filtered energy:

$$\frac{\partial E^{(r)}}{\partial t} = -\frac{\partial J_i^{(r)}}{\partial x_i} - \nu \frac{\partial u_i^{(r)}}{\partial x_j}\frac{\partial u_i^{(r)}}{\partial x_j} - \Pi^{(r)}$$

Change in energy at a point

Spatial transport

Viscous dissipation

G.L. Eyink, J. Stat. Phys. (1995)

M.K. Rivera et al., Phys. Rev. Lett. (2003)

Write equation of motion for filtered energy:

$$\frac{\partial E^{(r)}}{\partial t} = -\frac{\partial J_i^{(r)}}{\partial x_i} - \nu \frac{\partial u_i^{(r)}}{\partial x_j}\frac{\partial u_i^{(r)}}{\partial x_j} - \Pi^{(r)}$$

Change in energy at a point

Spatial transport

Viscous dissipation

Coupling to other scales

G.L. Eyink, J. Stat. Phys. (1995)

M.K. Rivera et al., Phys. Rev. Lett. (2003)

Write equation of motion for filtered energy:

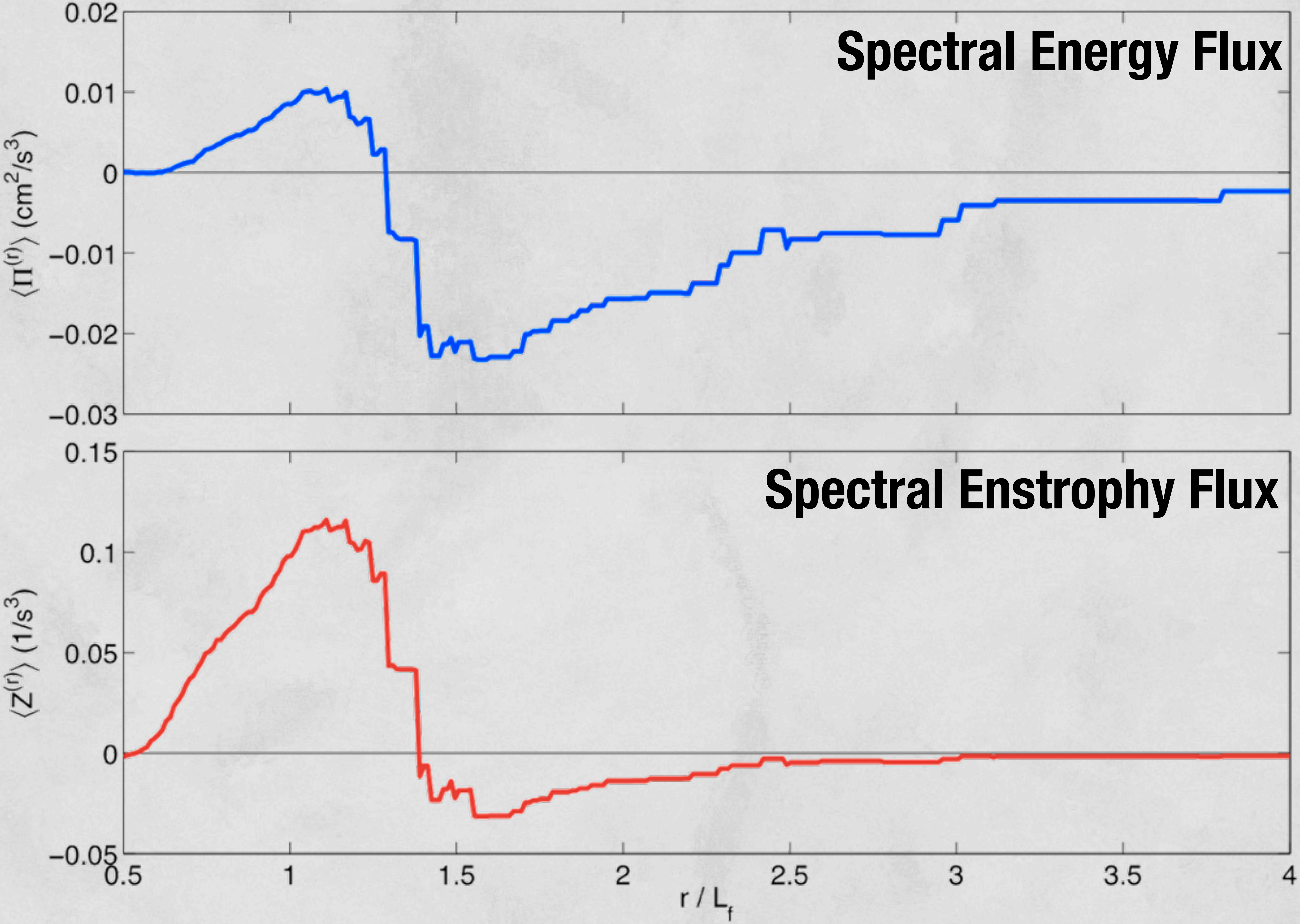
$$\frac{\partial E^{(r)}}{\partial t} = -\frac{\partial J_i^{(r)}}{\partial x_i} - \nu \frac{\partial u_i^{(r)}}{\partial x_j}\frac{\partial u_i^{(r)}}{\partial x_j} - \Pi^{(r)}$$

Change in energy at a point	Spatial transport	Viscous dissipation	Coupling to other scales

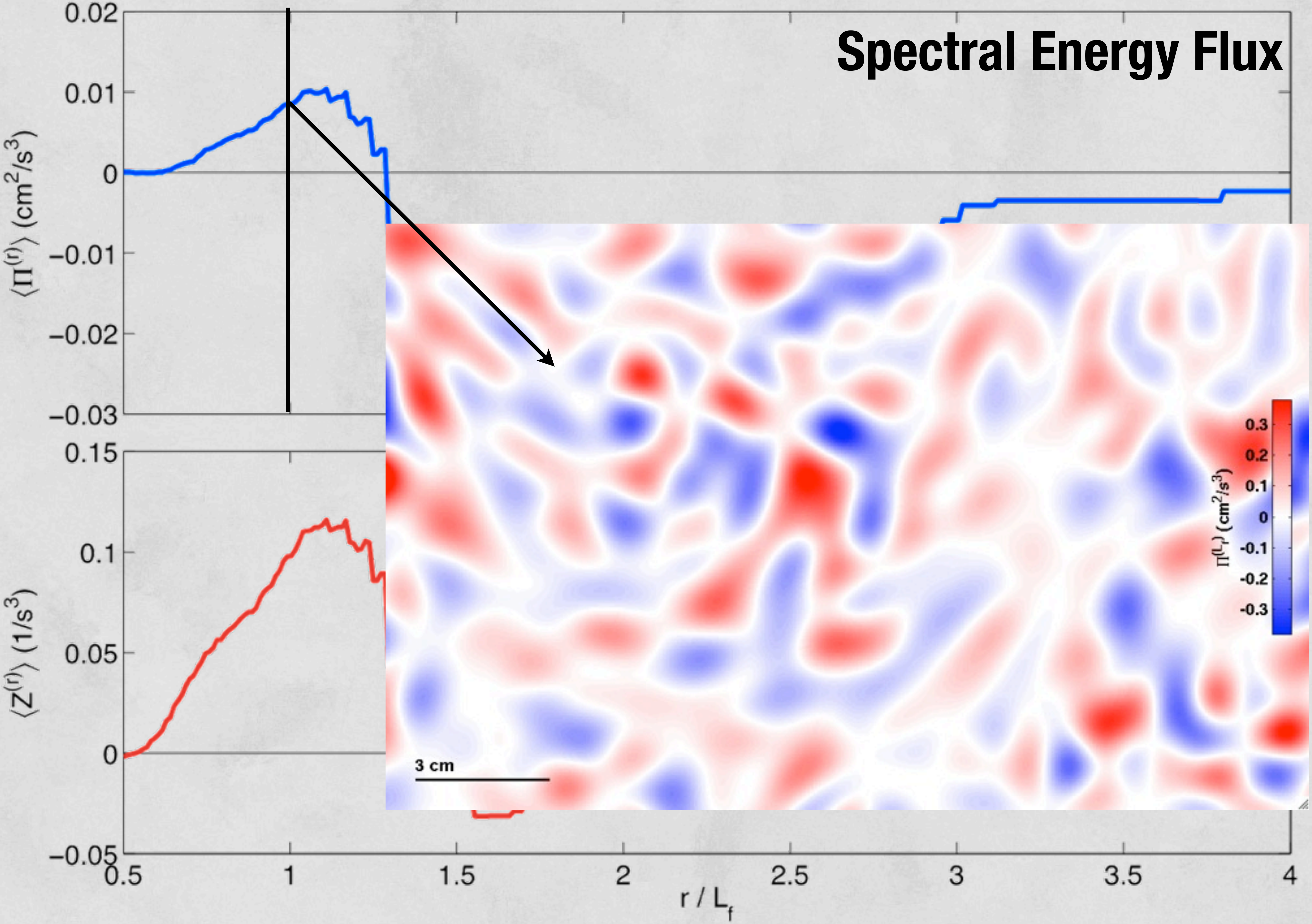
$$\Pi^{(r)} = -\left[(u_i u_j)^{(r)} - u_i^{(r)} u_j^{(r)}\right] \frac{\partial u_i^{(r)}}{\partial x_j}$$

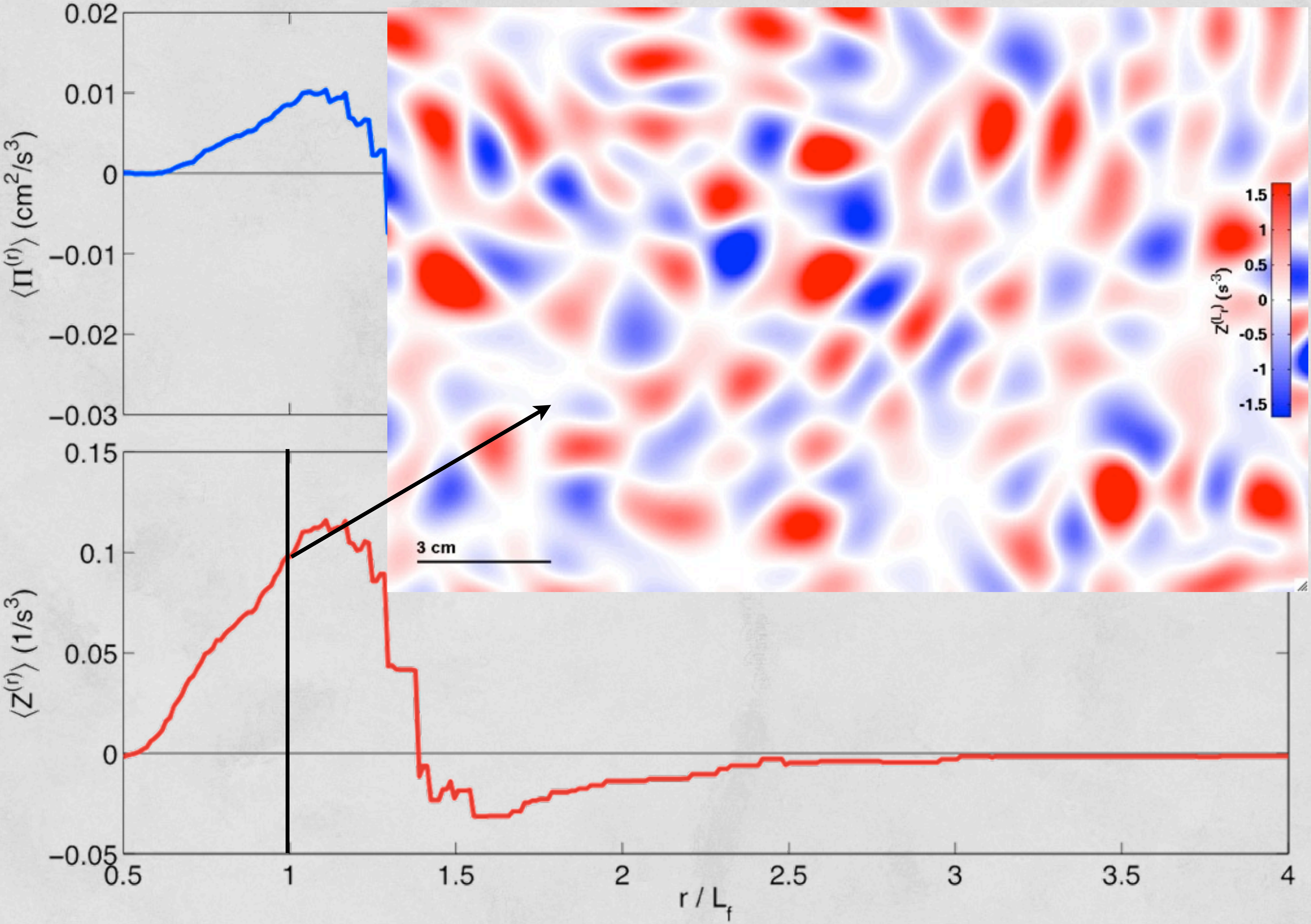
G.L. Eyink, J. Stat. Phys. (1995)

M.K. Rivera et al., Phys. Rev. Lett. (2003)

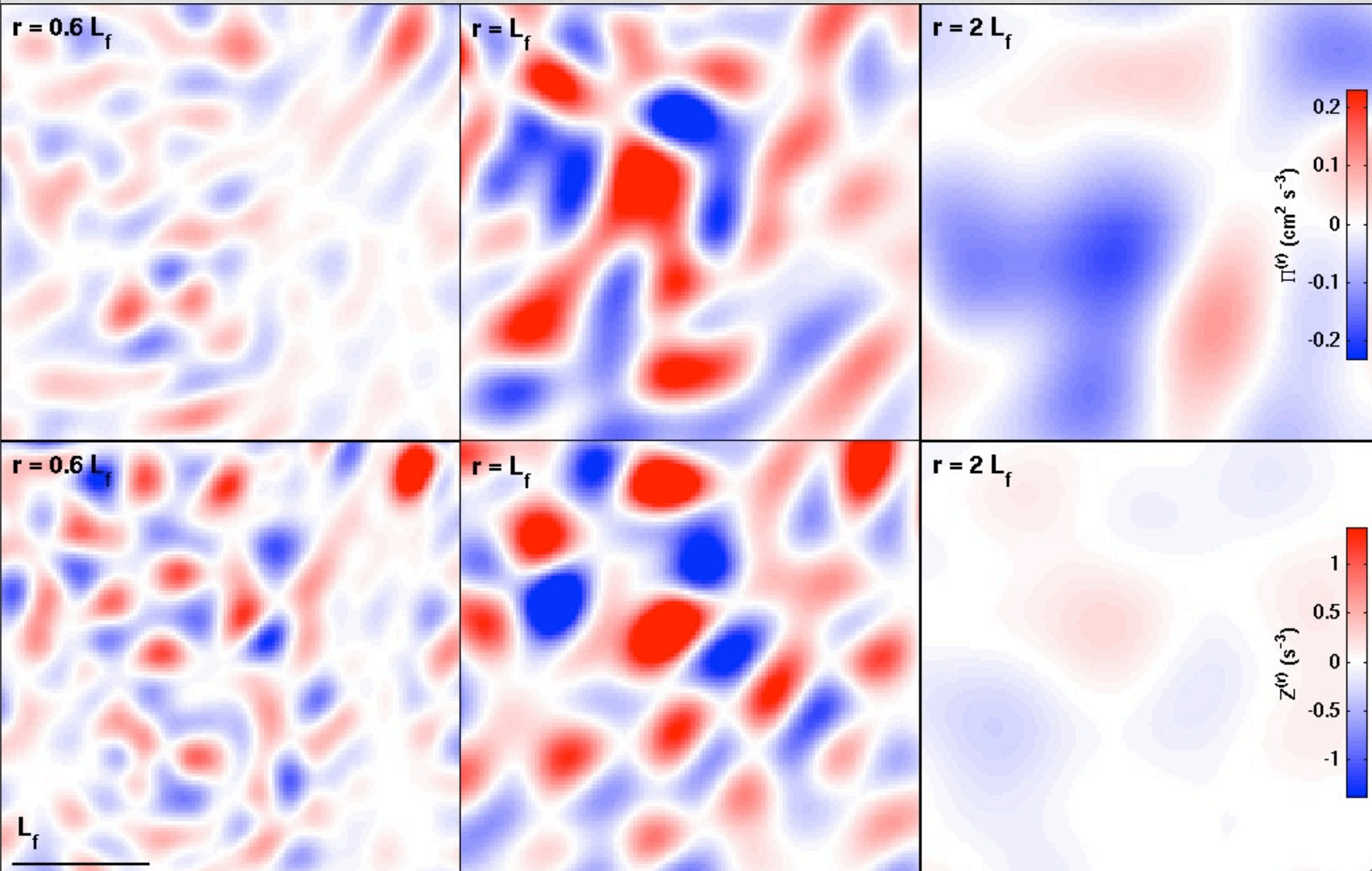

Spectral Energy Flux
Spectral Enstrophy Flux
⟨Π^(r)⟩ (cm^2/s^3)
⟨Z^(r)⟩ (1/s^3)
r / L_f
0.02
0.01
0
−0.01
−0.02
−0.03
0.15
0.1
0.05
0
−0.05
0.5
1
1.5
2
2.5
3
3.5
4

Spectral Energy Flux



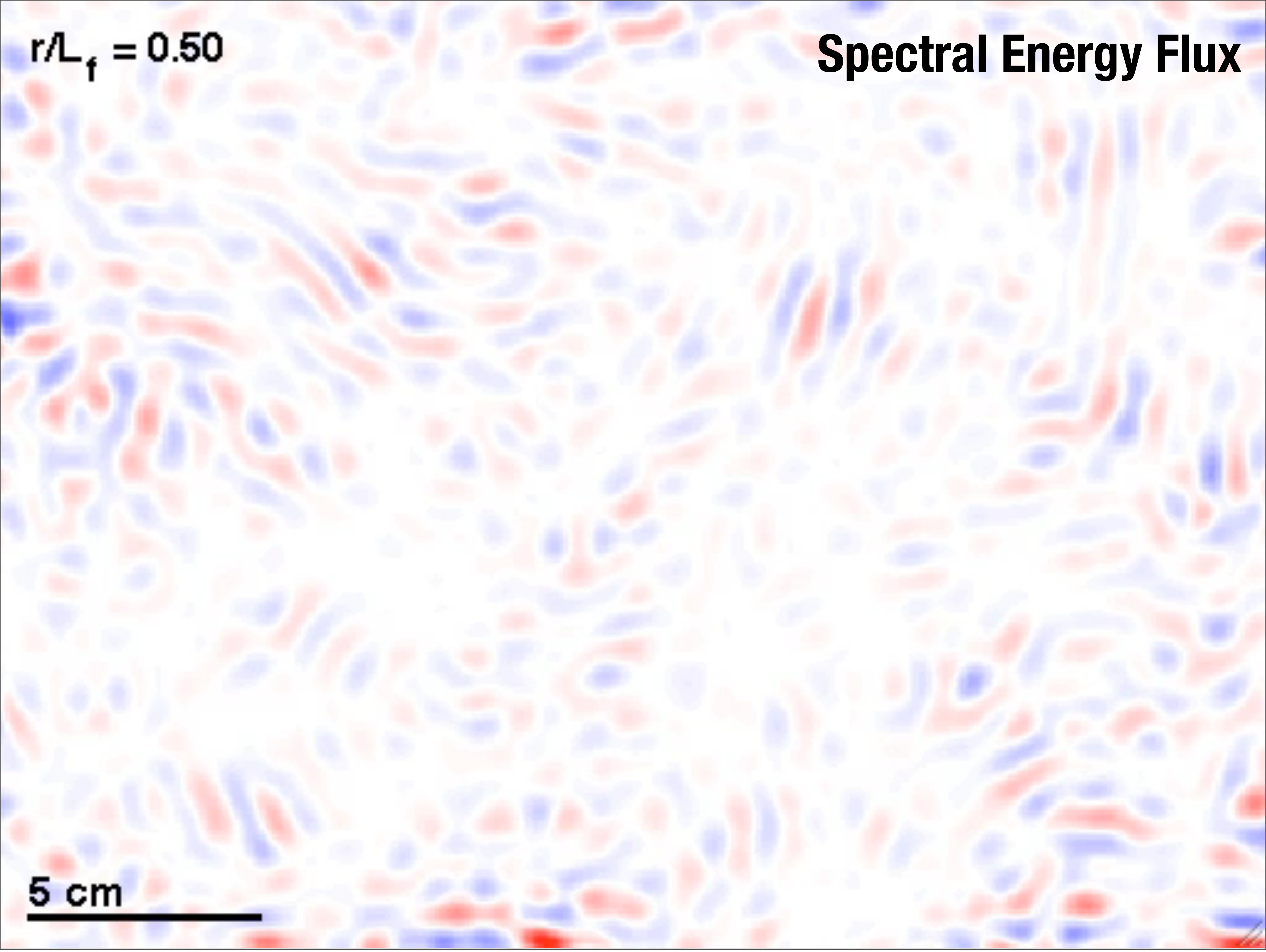

0.02
0.01
0
−0.01
−0.02
−0.03
⟨Π(r)⟩ (cm2/s3)
0.15
0.1
0.05
0
−0.05
⟨Z(r)⟩ (1/s3)
0.5
1
1.5
2
2.5
3
3.5
4
r / Lf
3 cm
1.5
1
0.5
0
-0.5
-1
-1.5
Z(Lf) (s-3)

Energy →



Enstrophy →

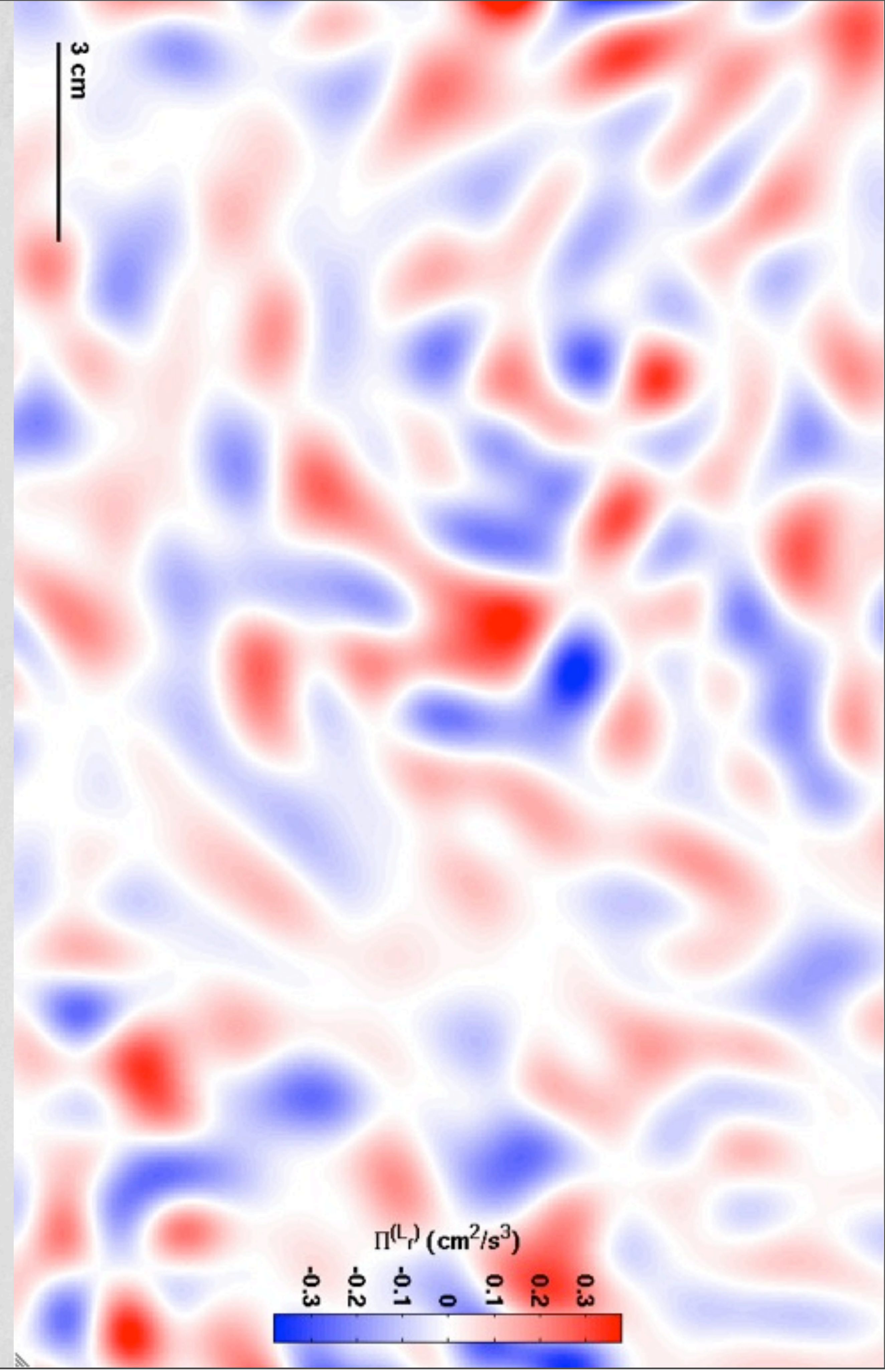
Spectral Energy Flux



r/L f = 0.50
Spectral Energy Flux
5 cm

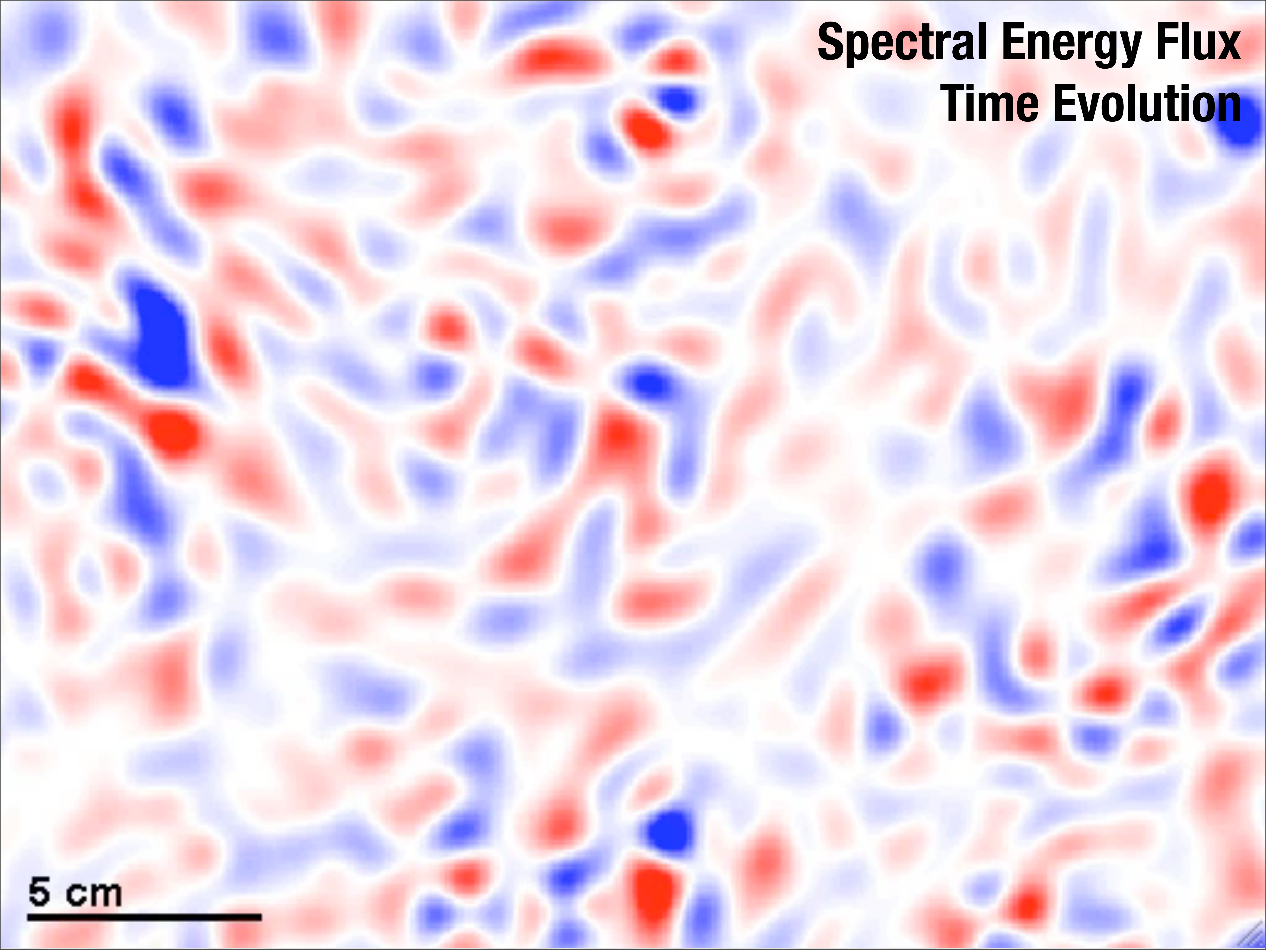
Spectral transfer is not constant in time!

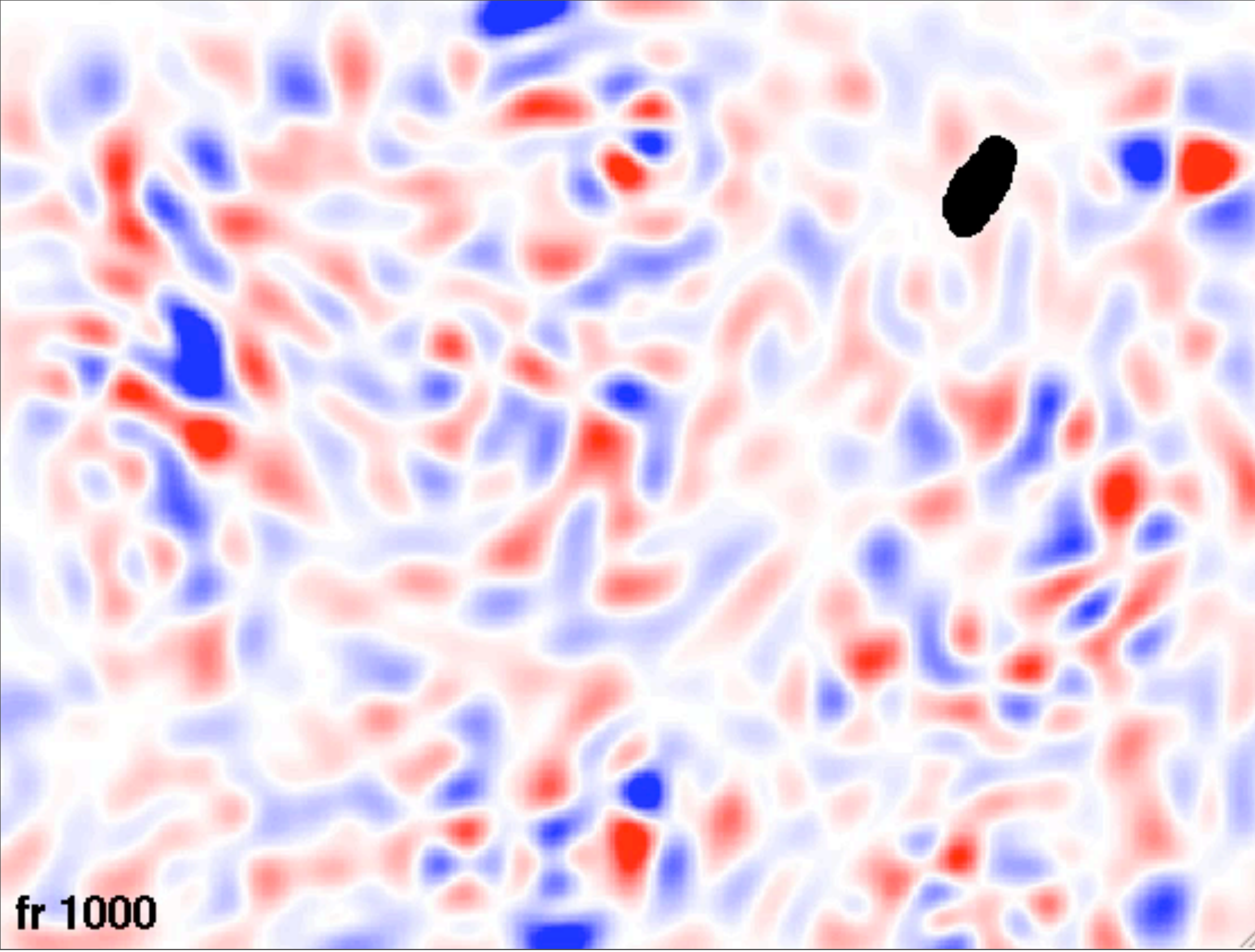
How does is change?
What are its dynamics?



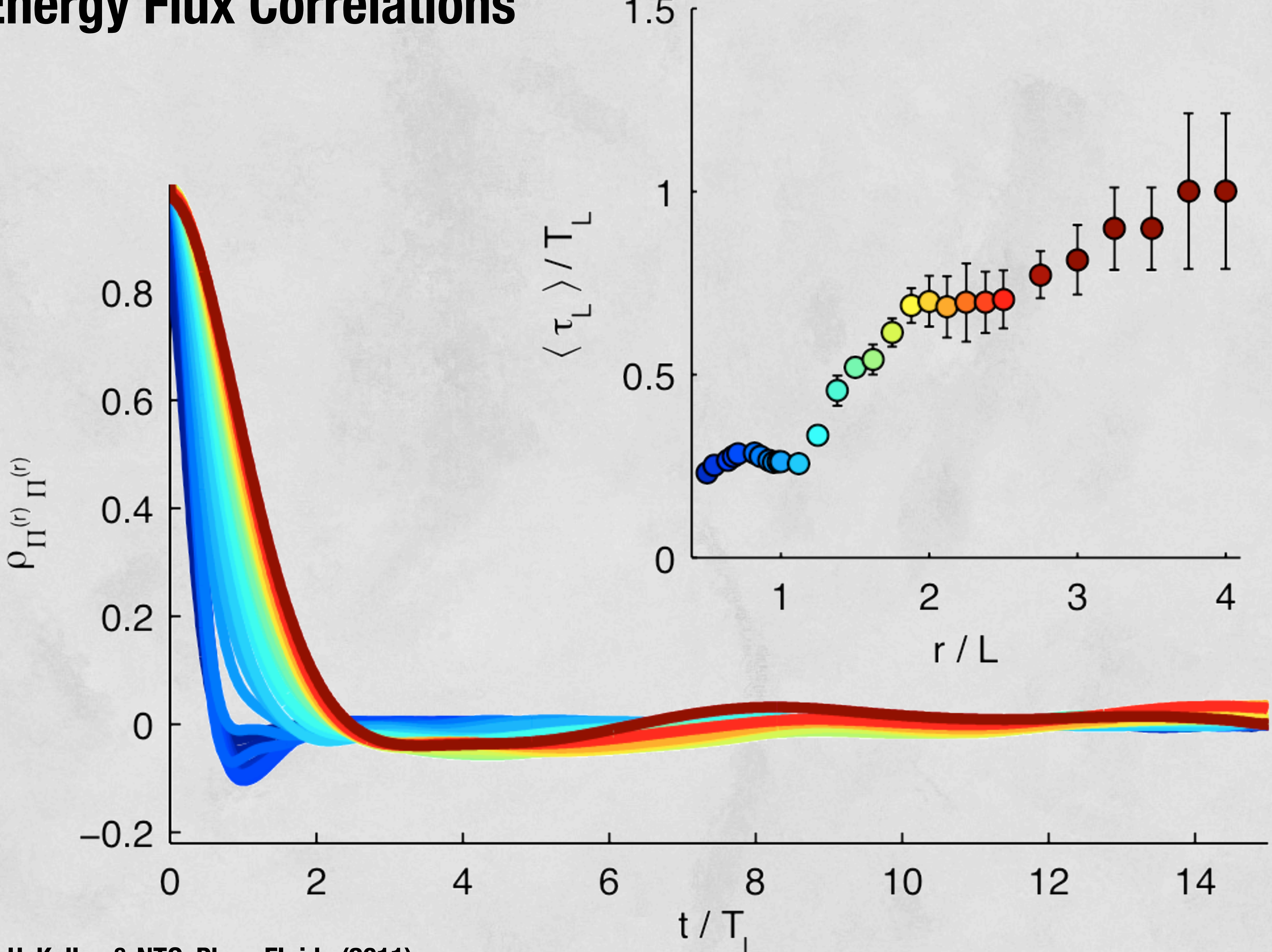
Spectral Energy Flux
Time Evolution

Spectral Energy Flux
Time Evolution



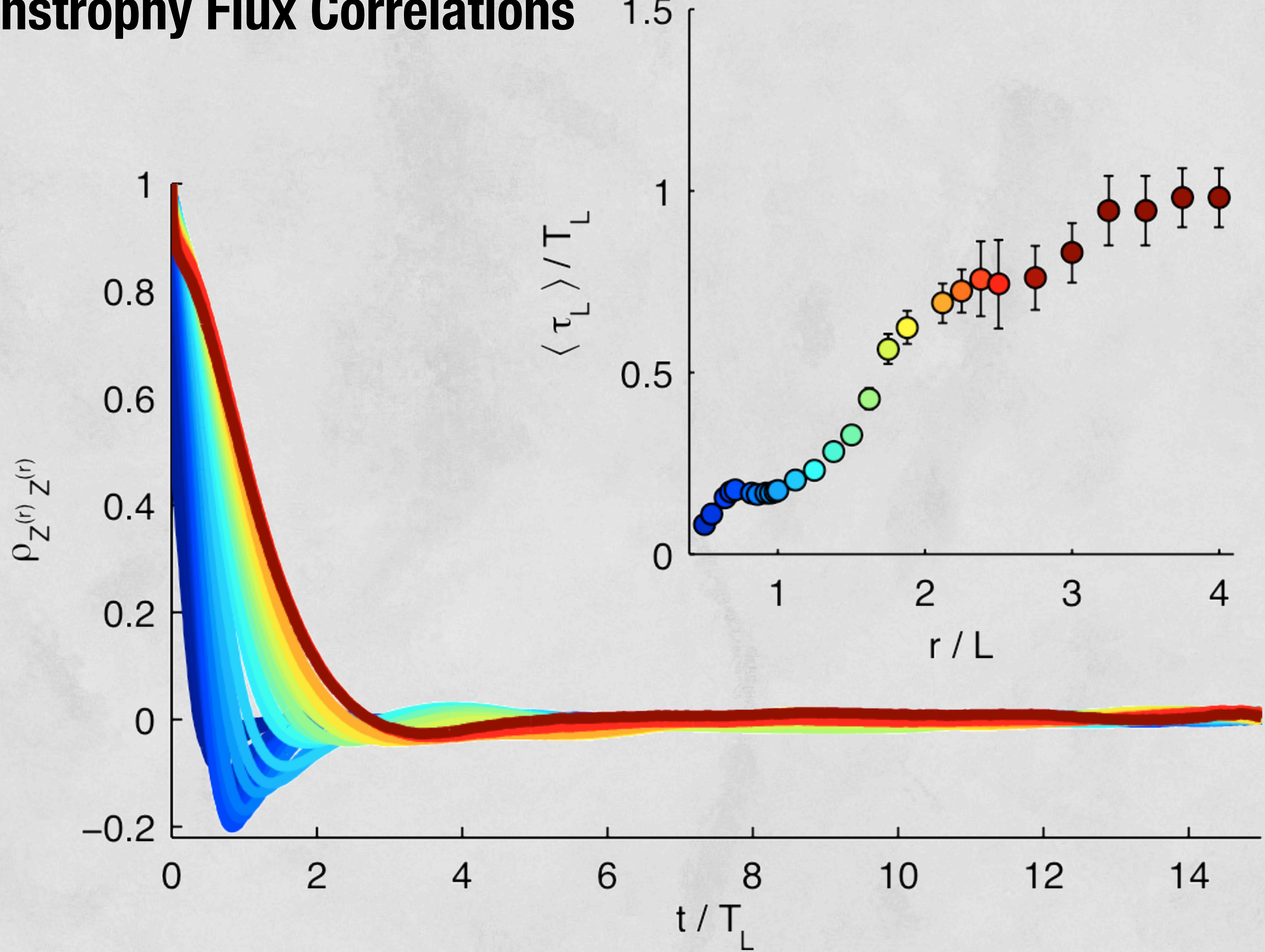

fr 1000

Energy Flux Correlations

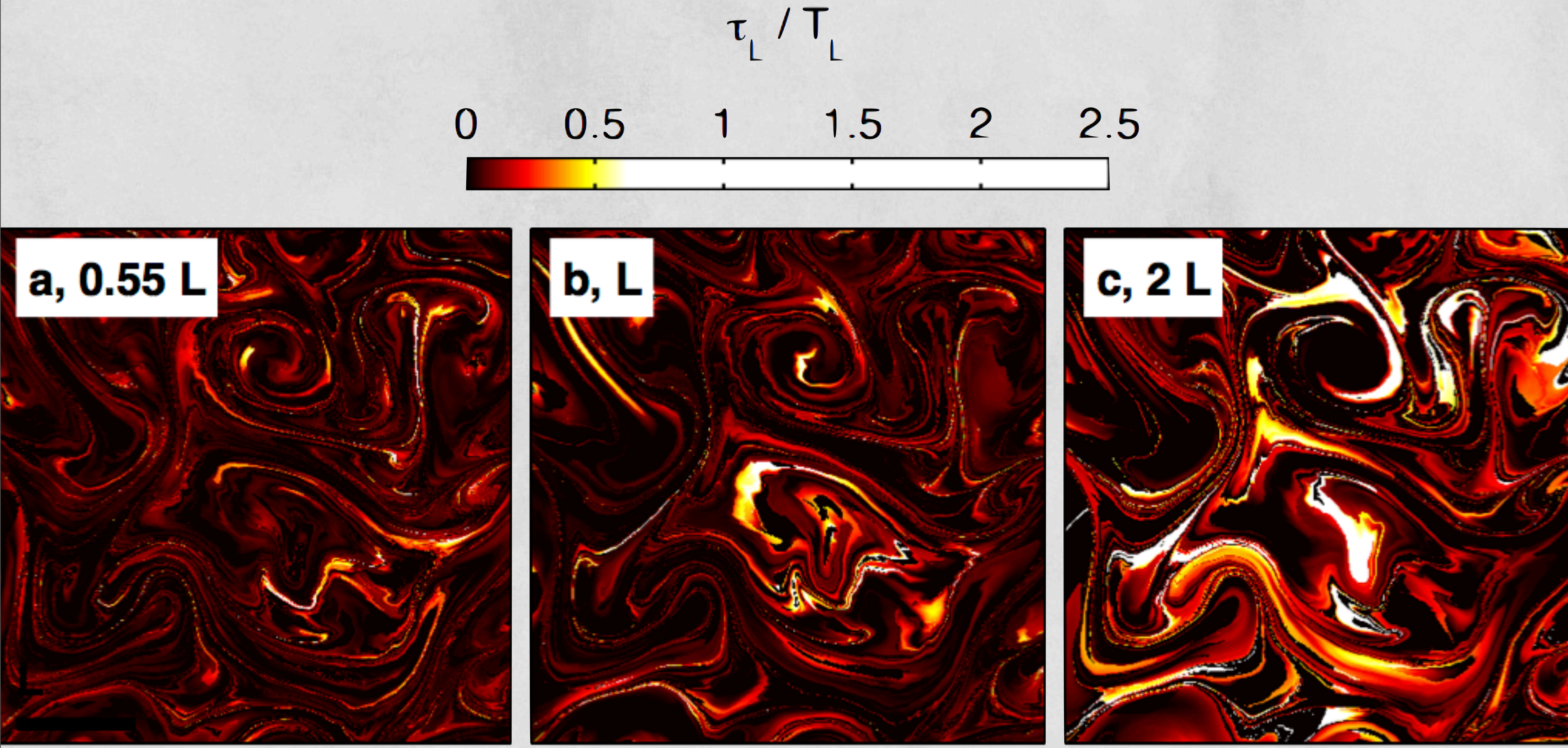


D.H. Kelley & NTO, Phys. Fluids (2011)

Enstrophy Flux Correlations

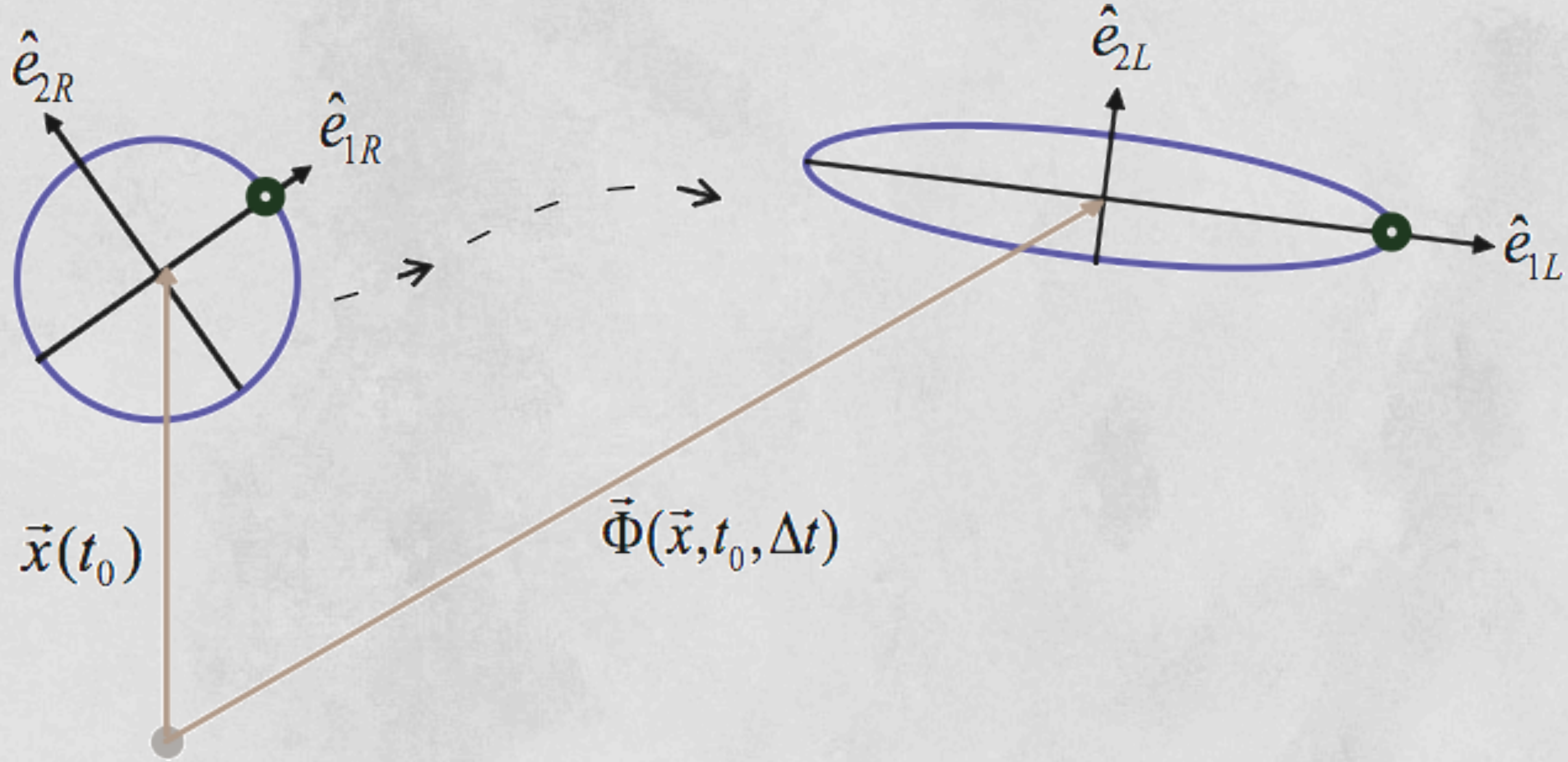


Spatial Dependence of Integral Times



D.H. Kelley & NTO, Phys. Fluids (2011)

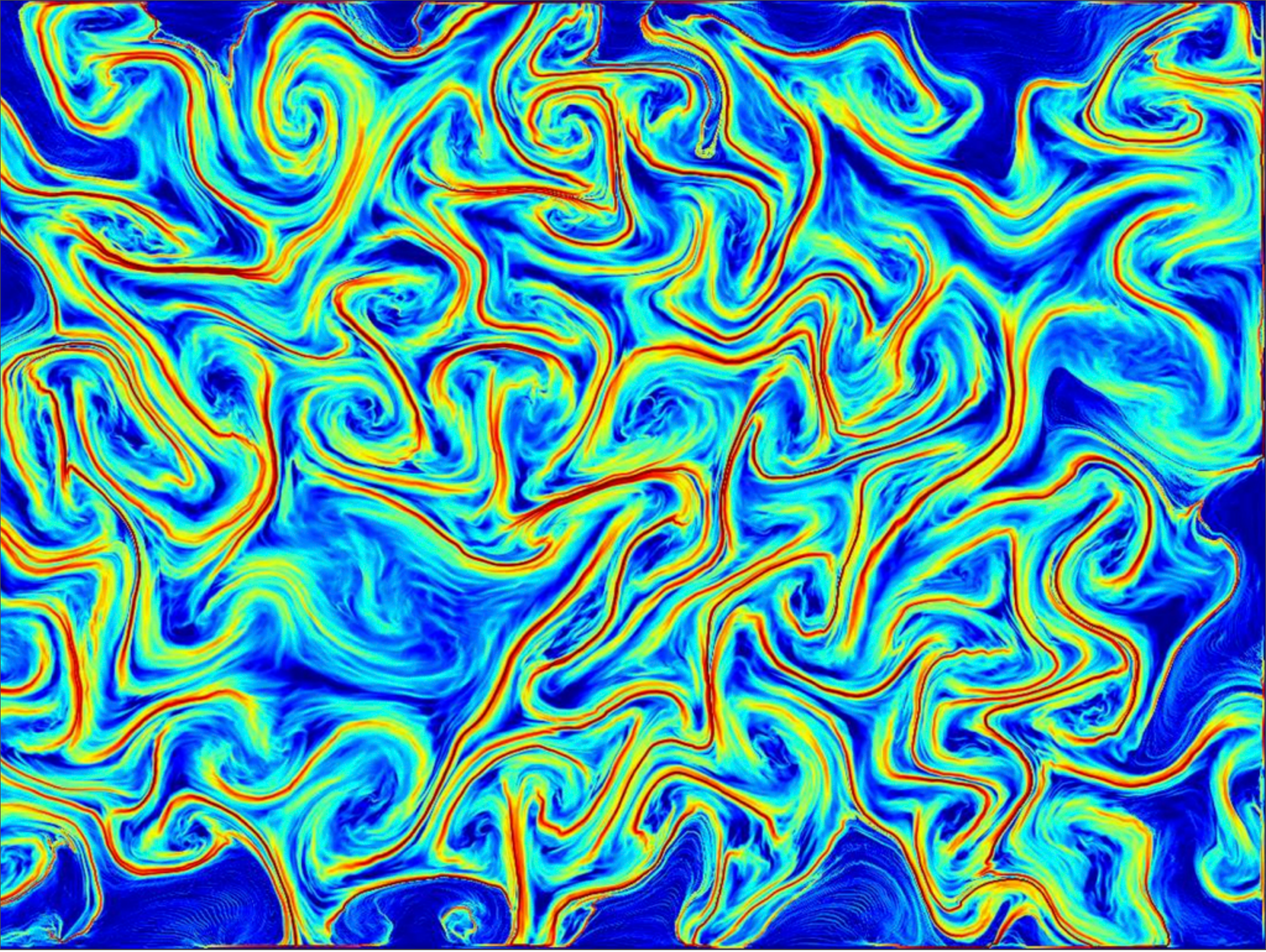
Aside: Lagrangian Coherent Structures



(Right) Cauchy-Green strain tensor: $C_{ij} = \frac{\partial \Phi_k}{\partial x_i} \frac{\partial \Phi_k}{\partial x_j}$

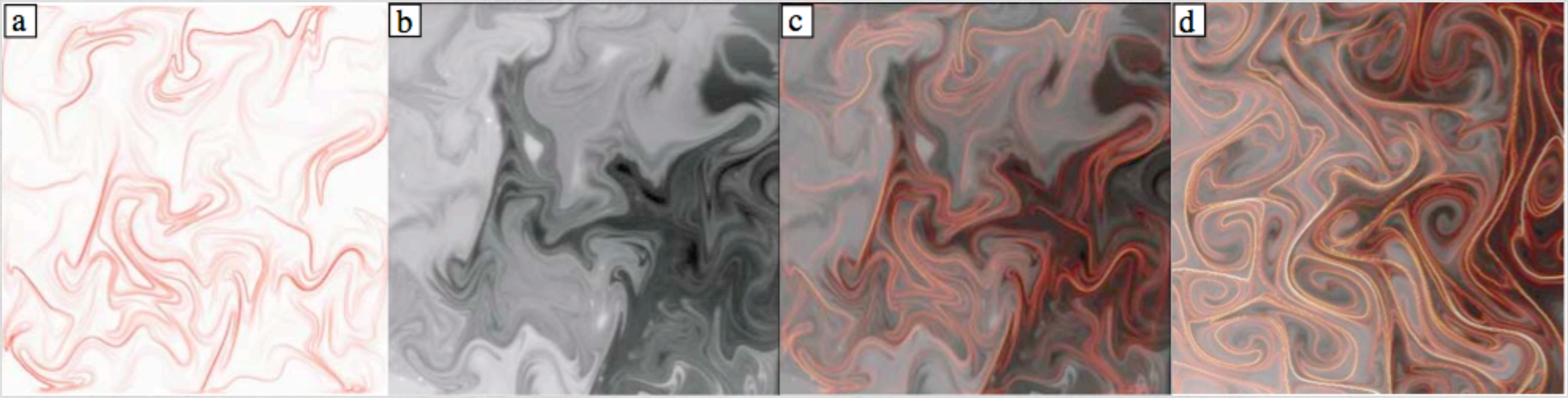
FTLE: $\sigma(\vec{x}, t_0, \Delta t) = \frac{1}{|T|} \ln \sqrt{\lambda_{\max}(C_{ij})}$

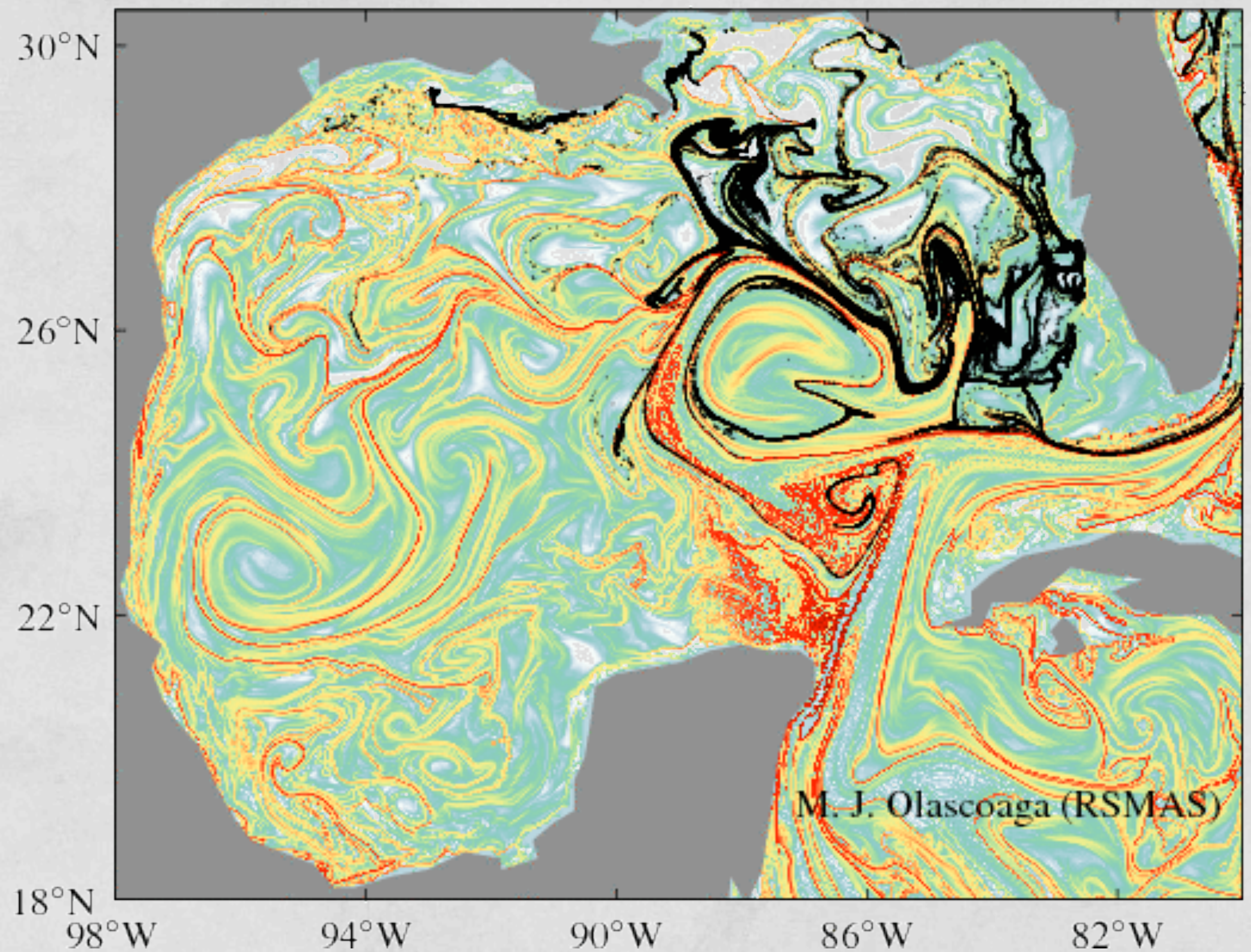
G. Haller & G. Yuan, Physica D (2000)
G.A. Voth et al., Phys. Rev. Lett. (2002)
S. Shadden, F. Lekien, & J. Marsden, Physica D (2005)



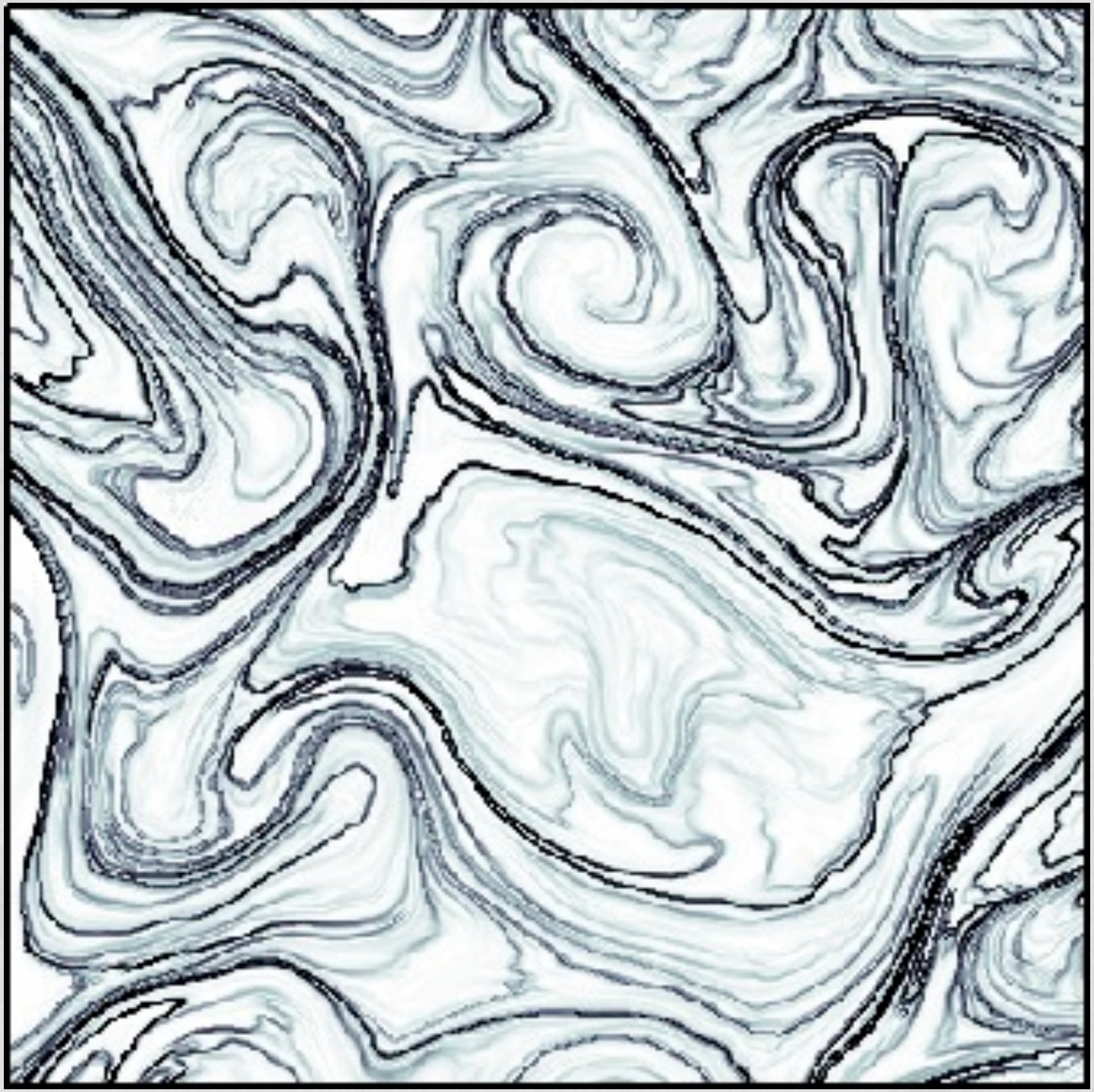
LCS Organize Mixing

G.A. Voth et al., Phys. Rev. Lett. (2002)

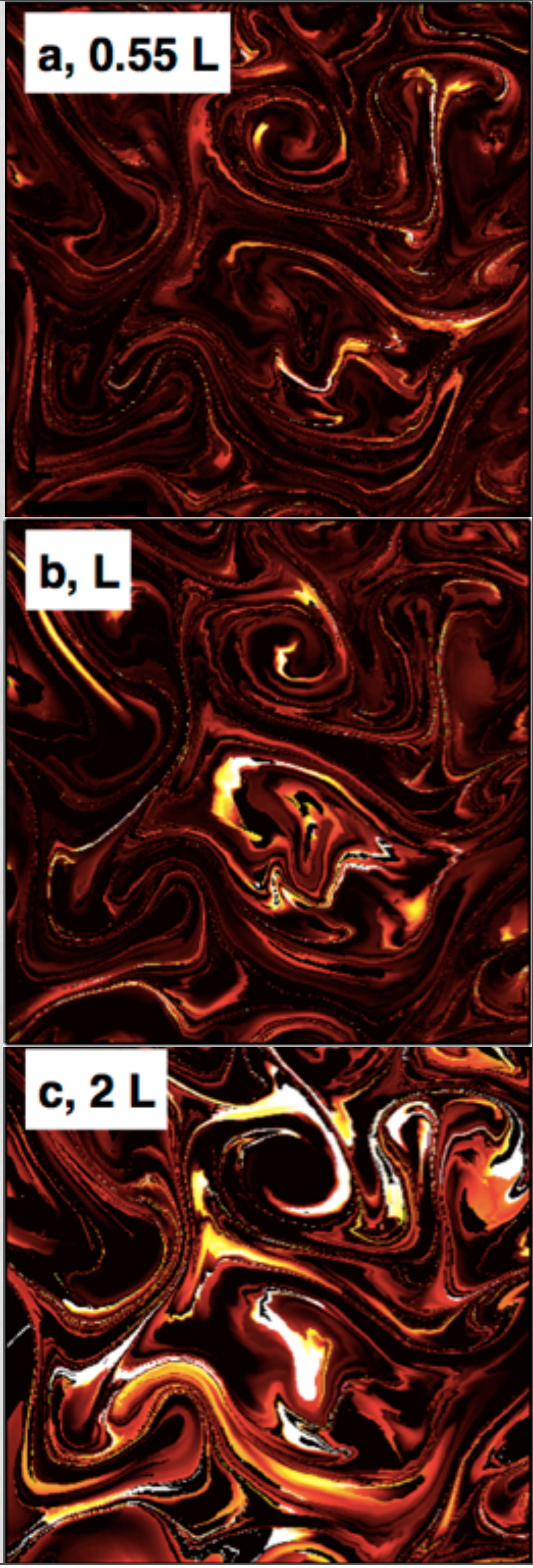




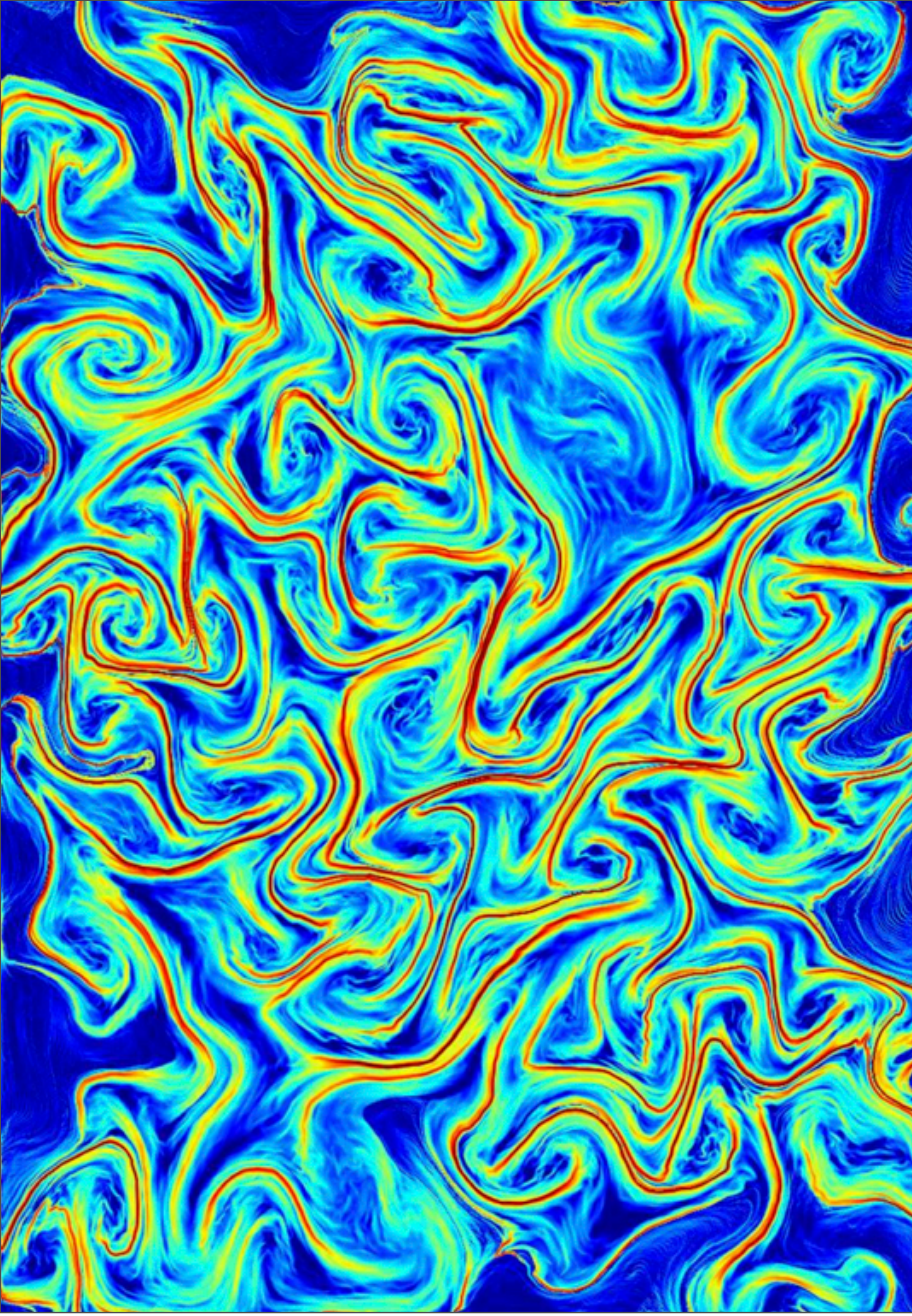
Lagrangian Coherent Structures

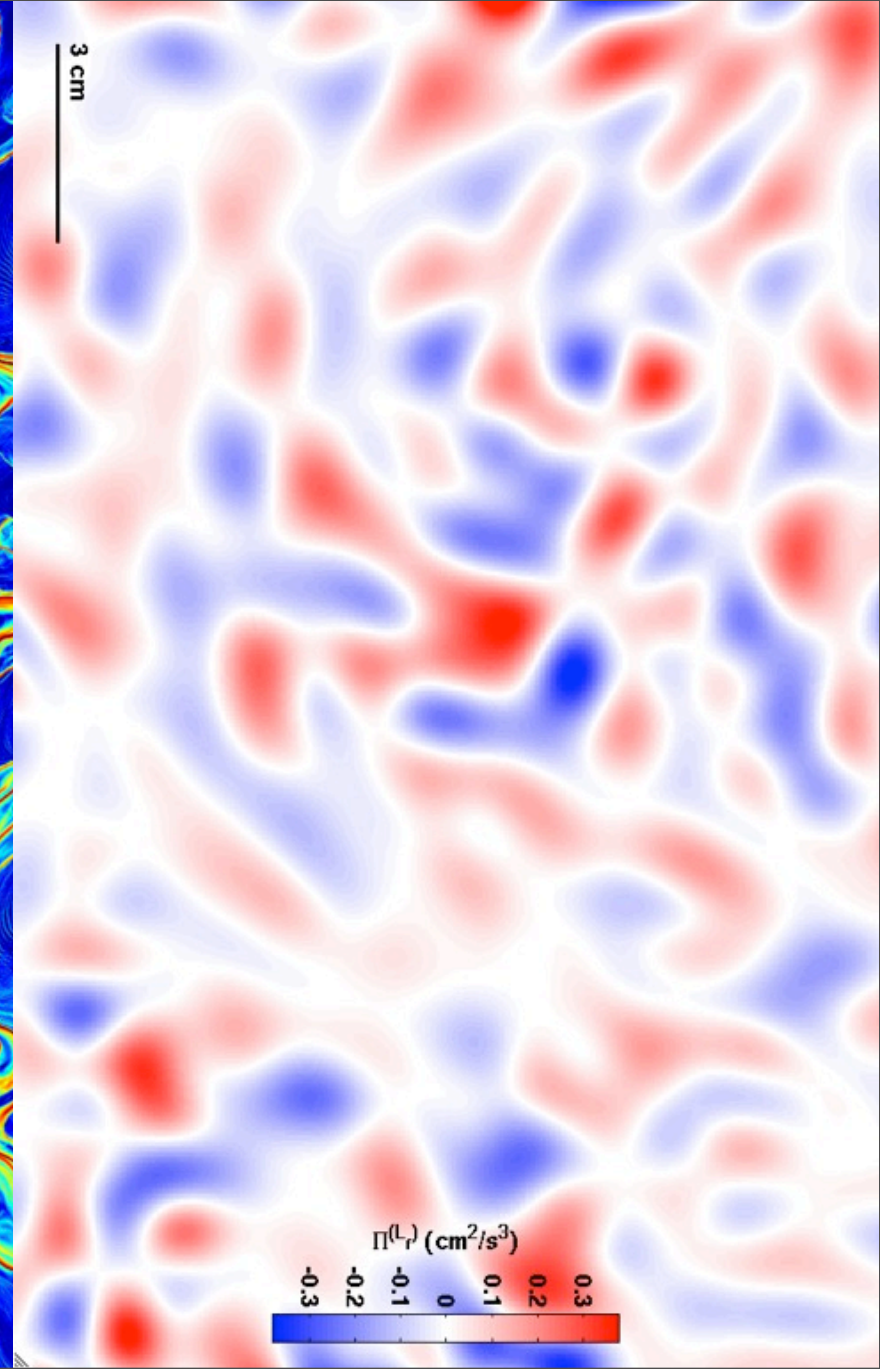


FTLE Field



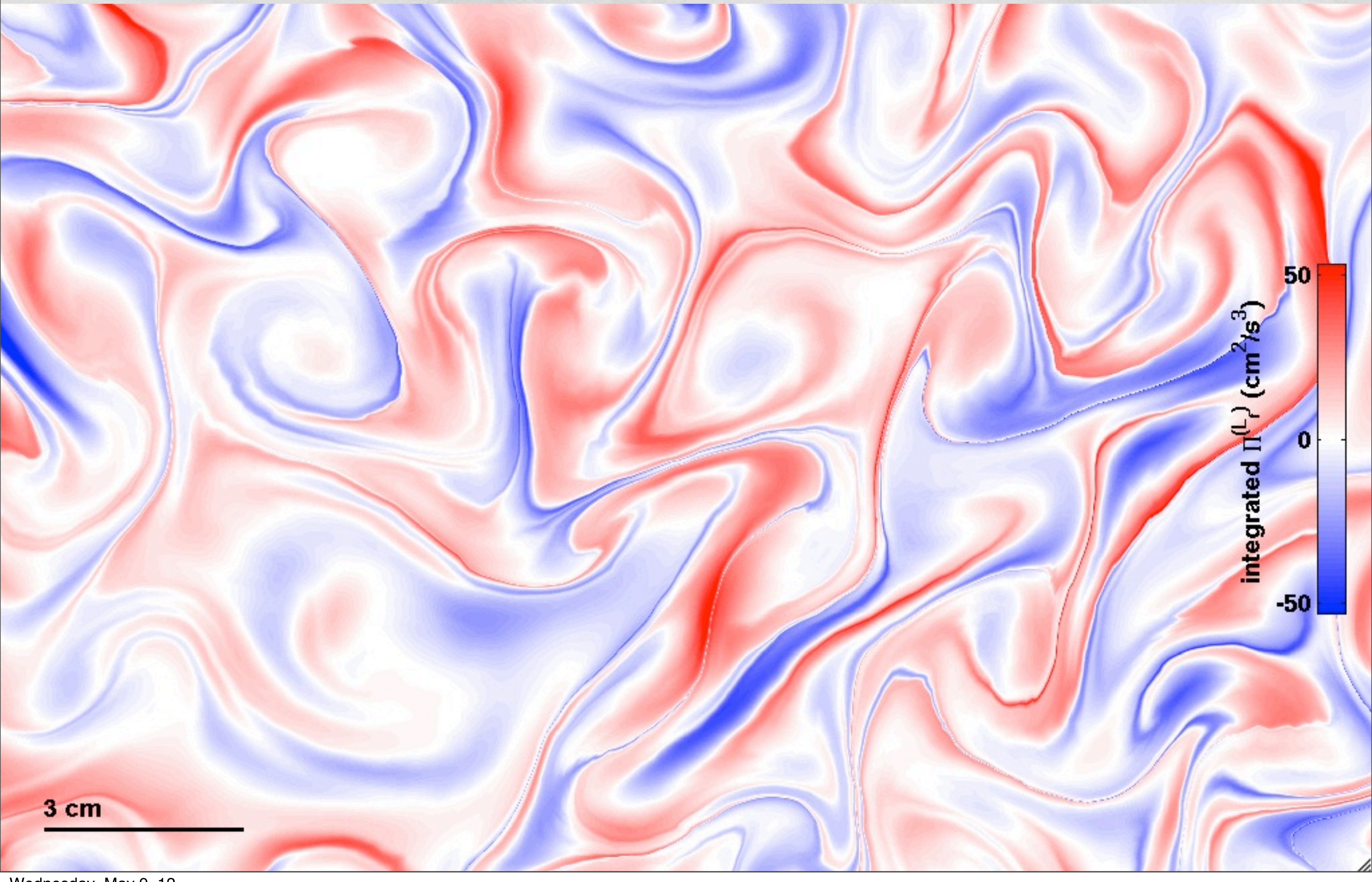
D.H. Kelley & NTO, Phys. Fluids (2011)

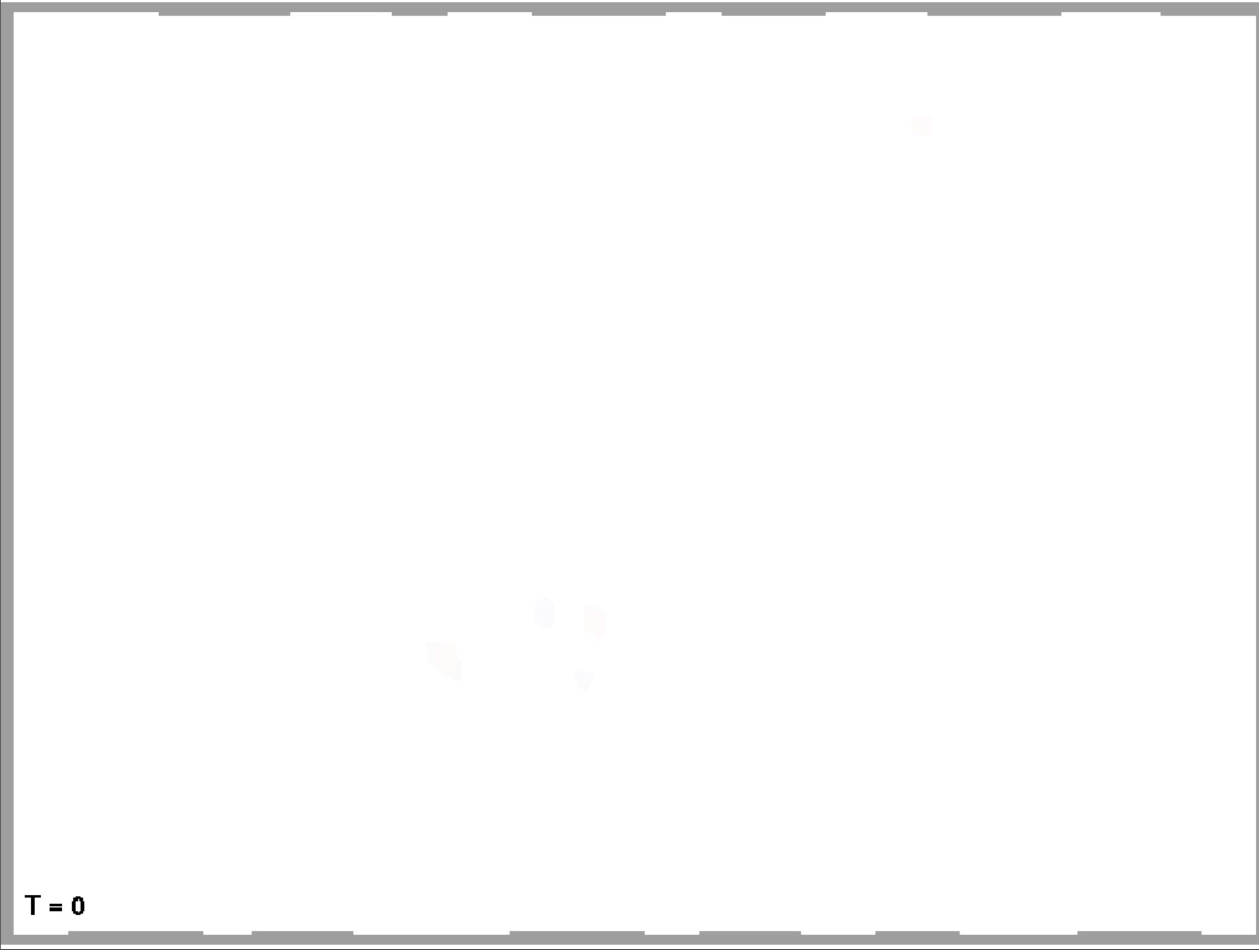



3 cm
Π(Lr) (cm2/s3)
-0.3
-0.2
-0.1
0
0.1
0.2
0.3

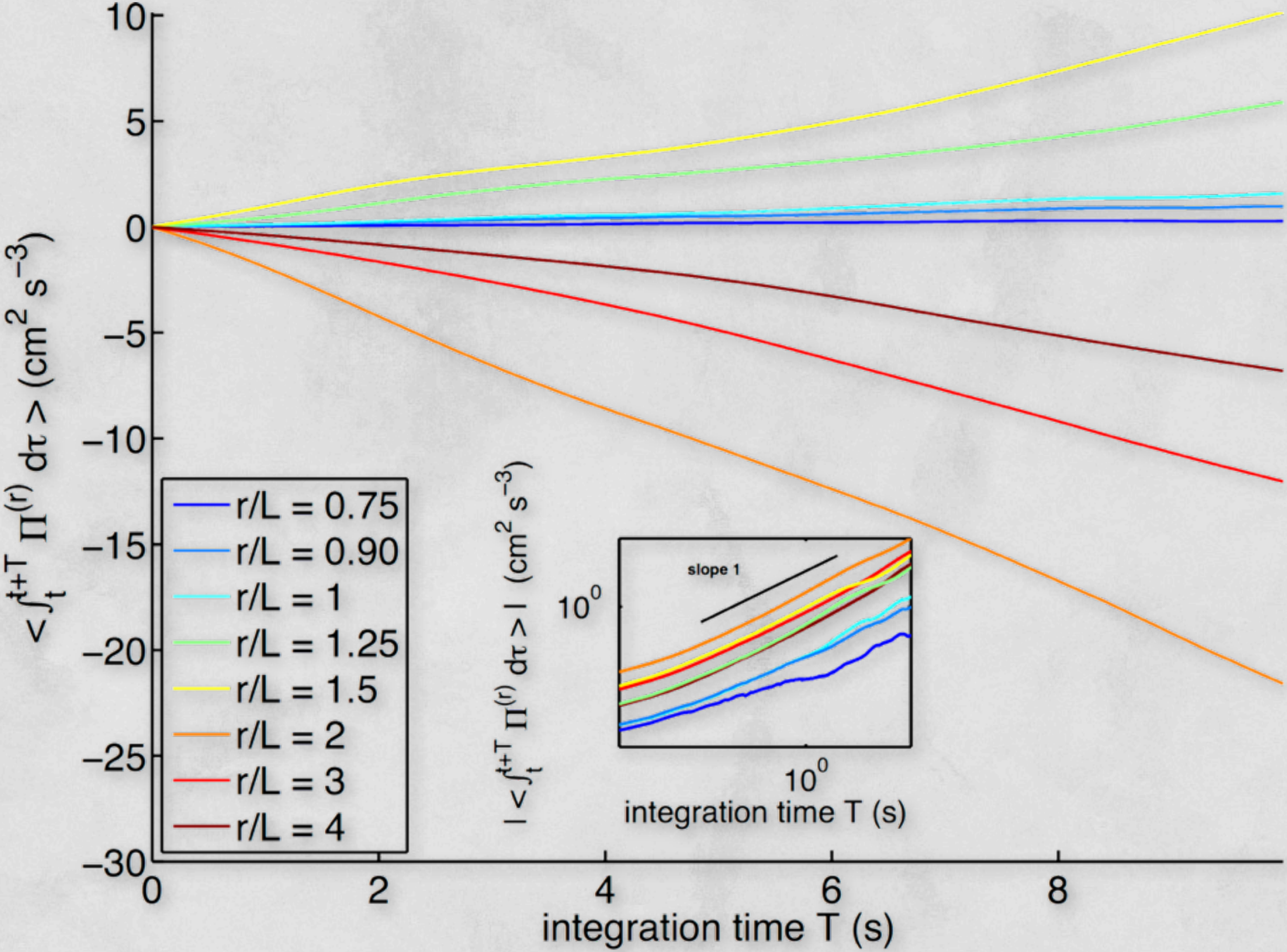
$$\int_t^{t+\tau} \Pi^{(r)}(t')\mathcal{D}\boldsymbol{x}(t')$$

$$\int_t^{t+\tau} \Pi^{(r)}(t')\mathcal{D}\boldsymbol{x}(t')$$

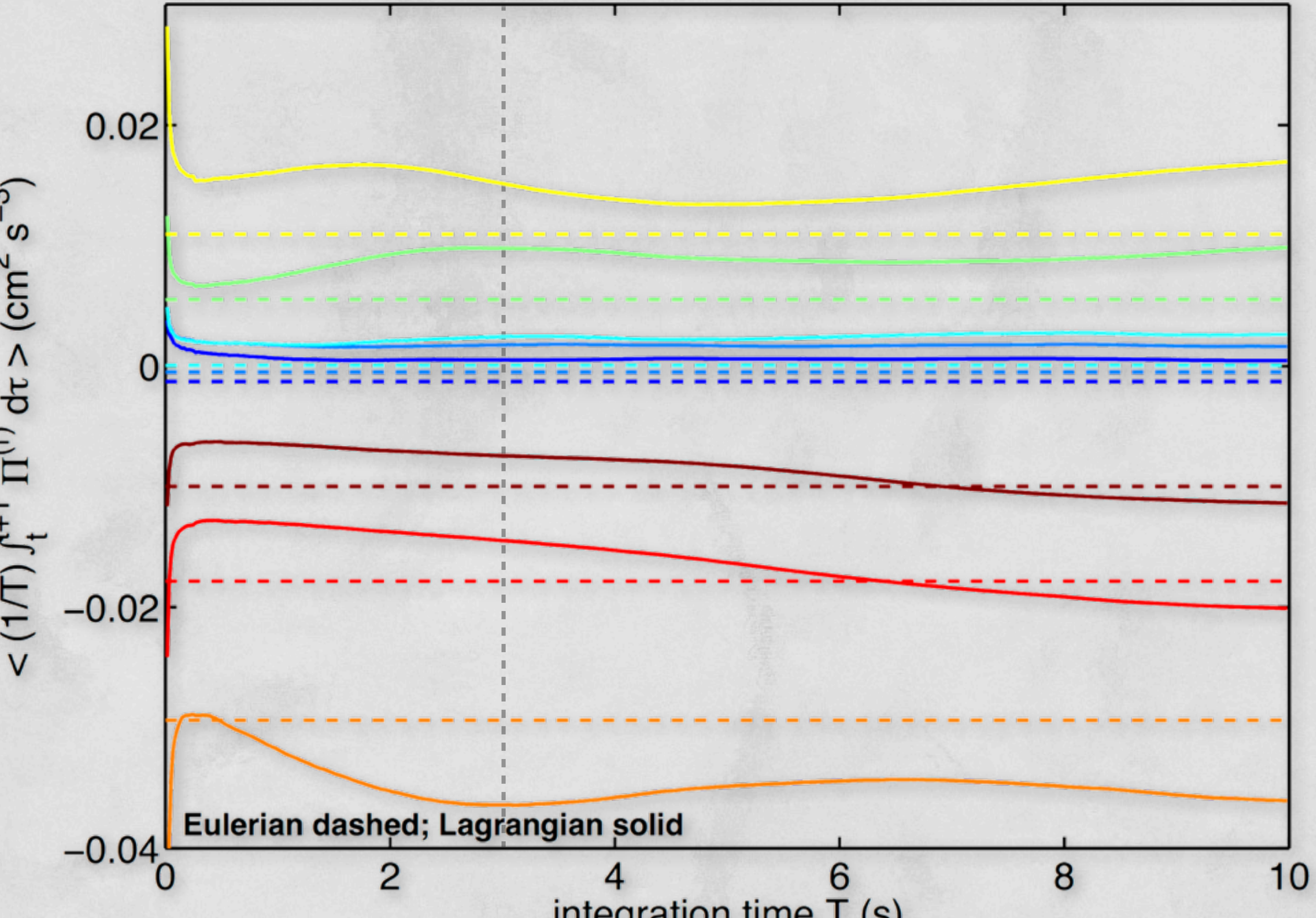


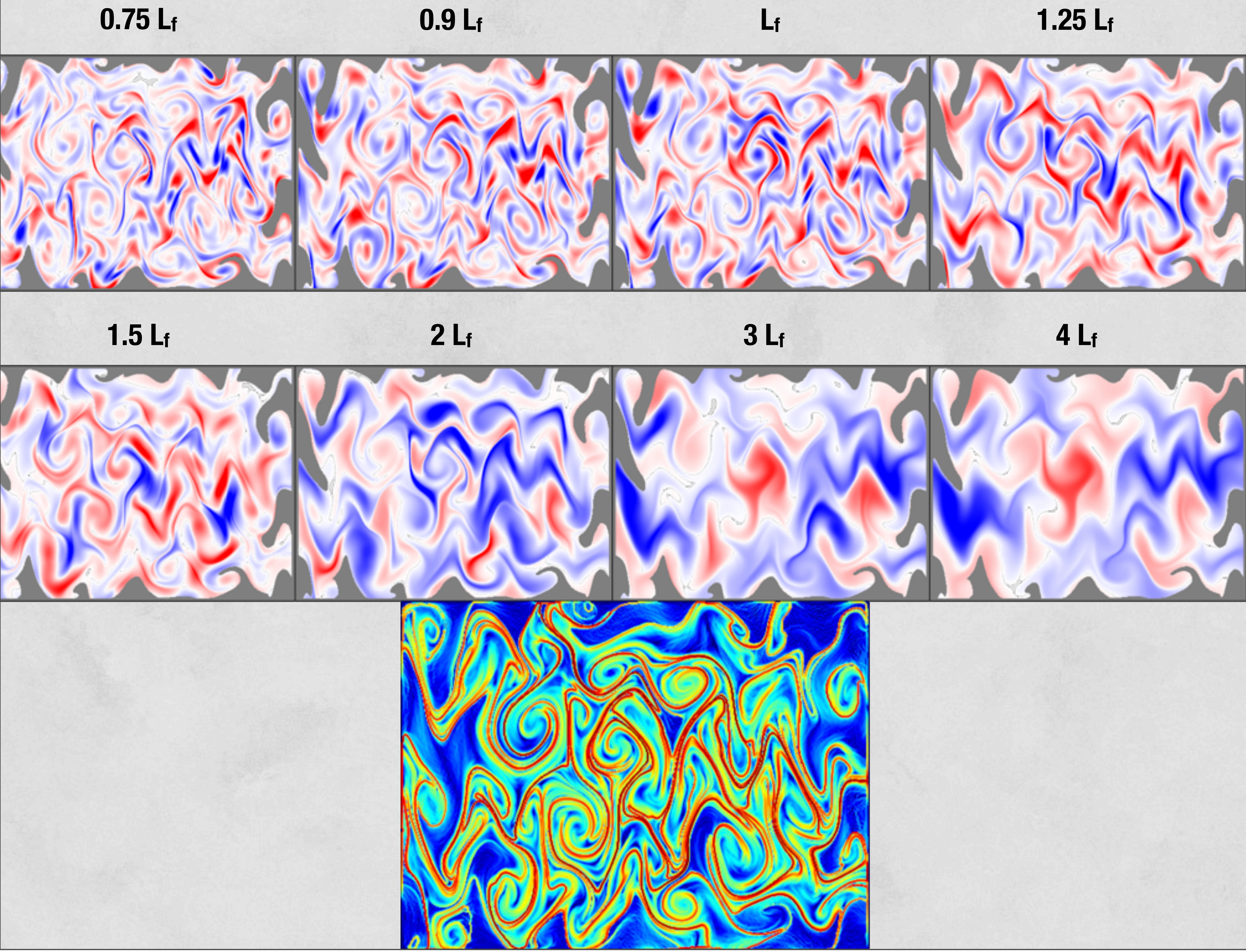

T = 0

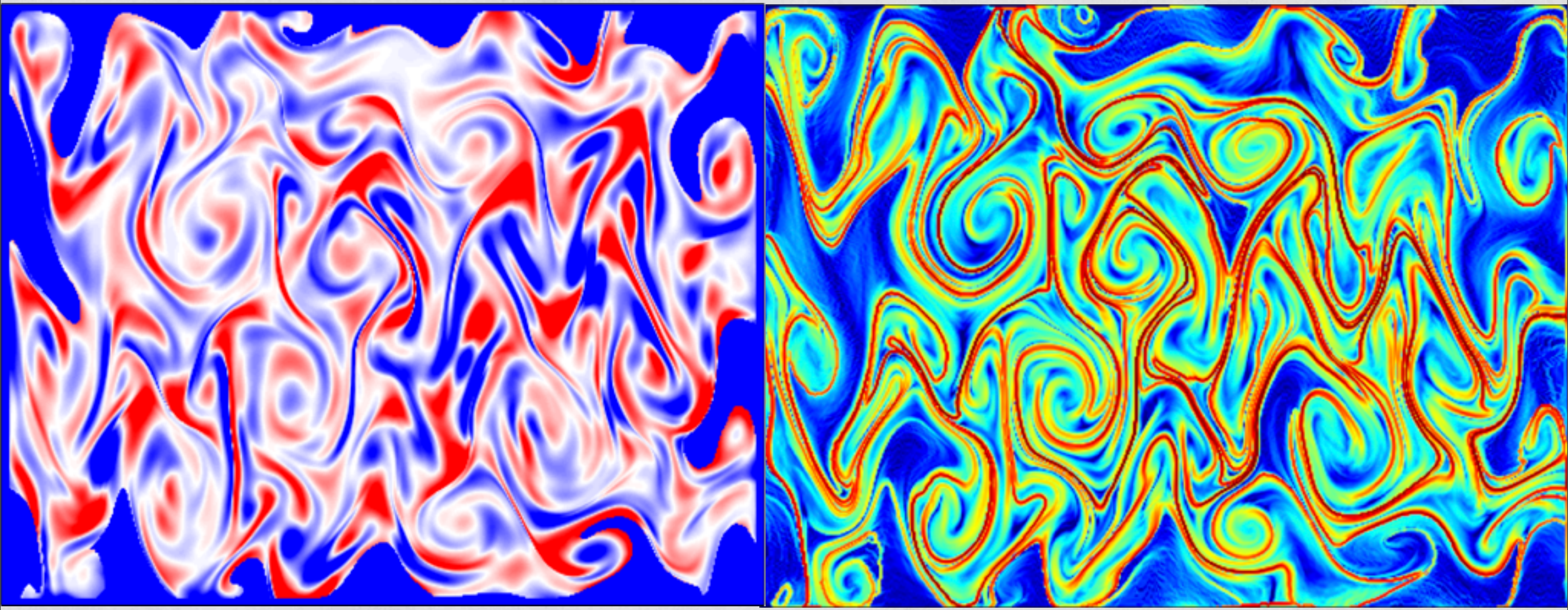
Spatial Averages



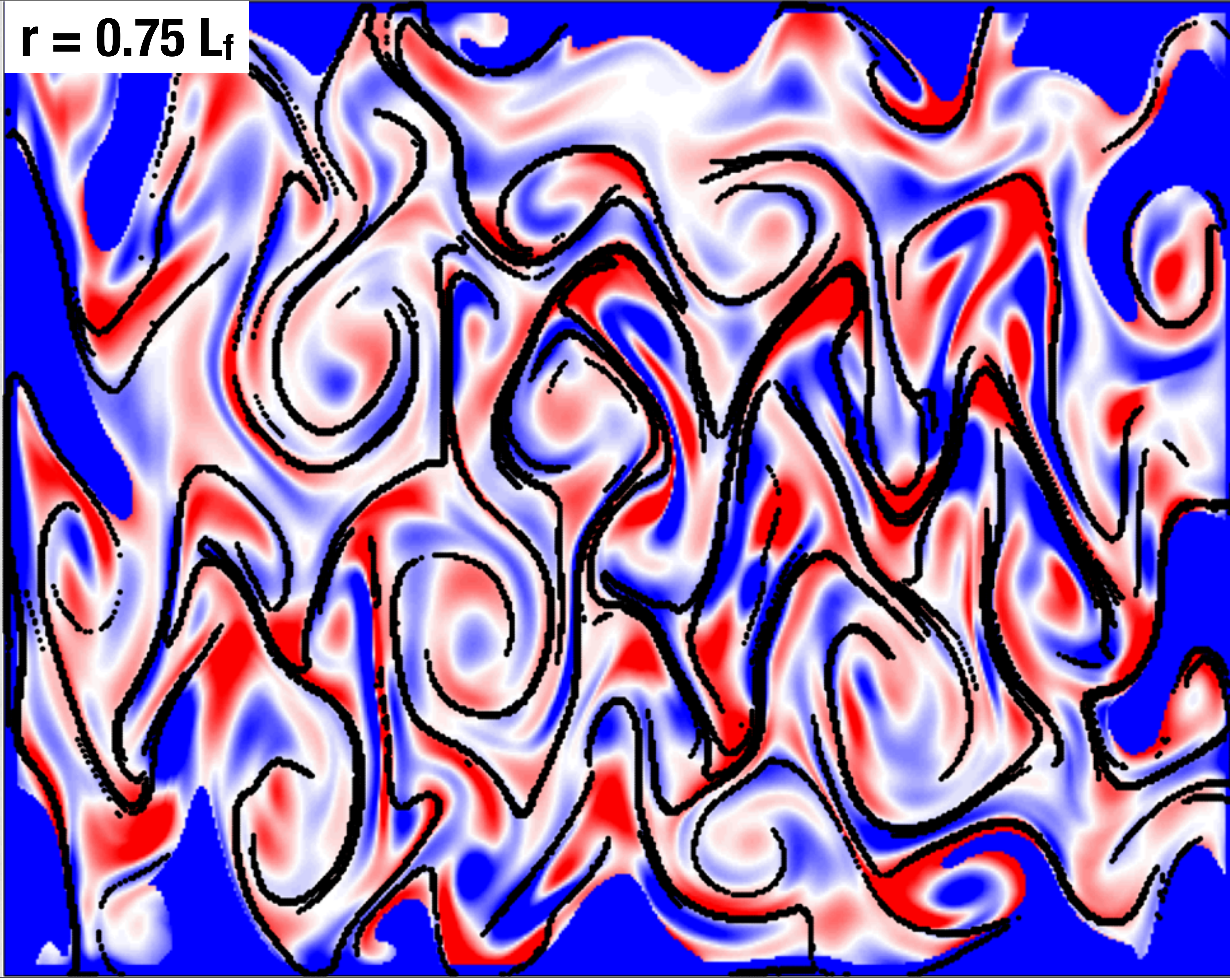
Spatiotemporal Averages

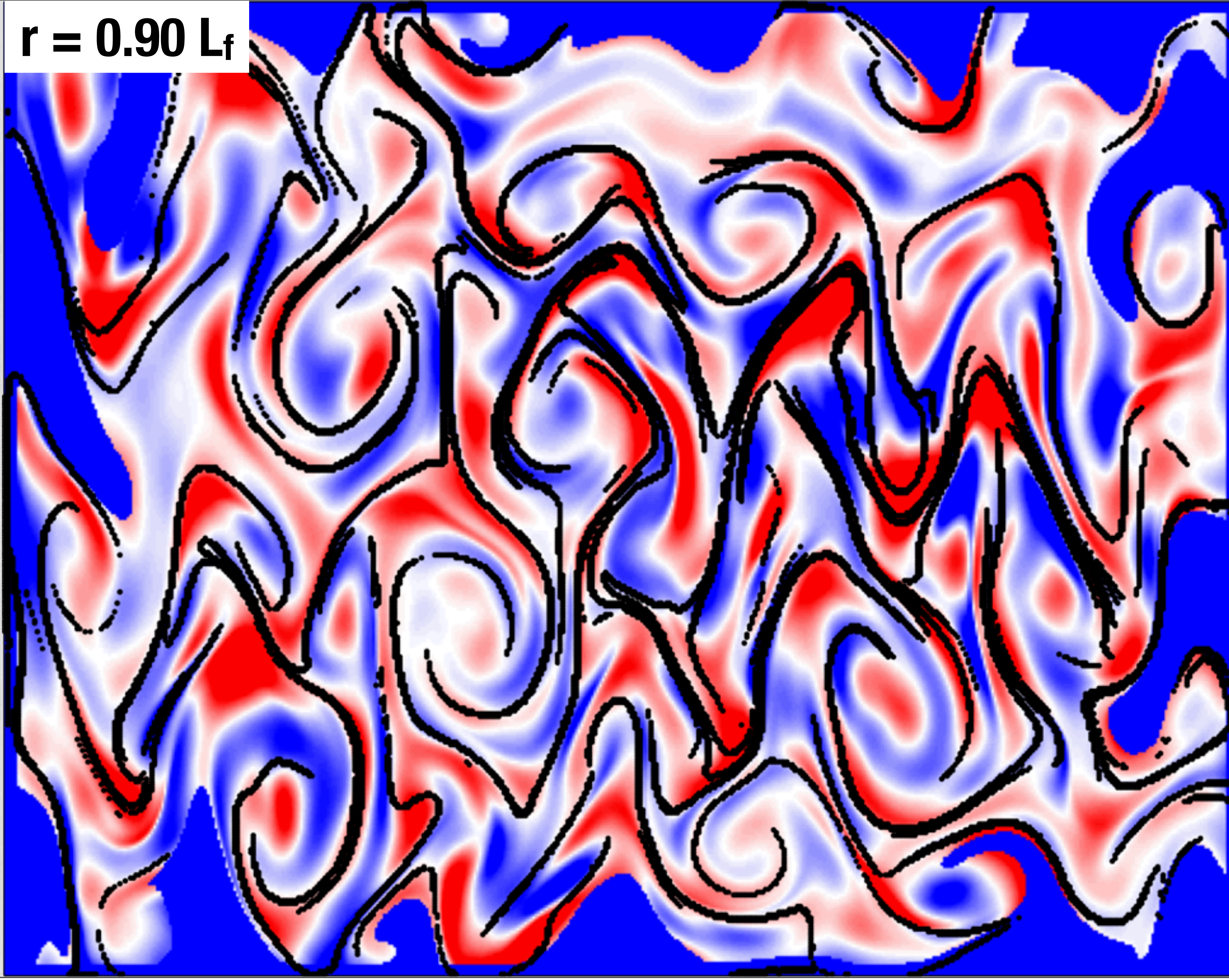


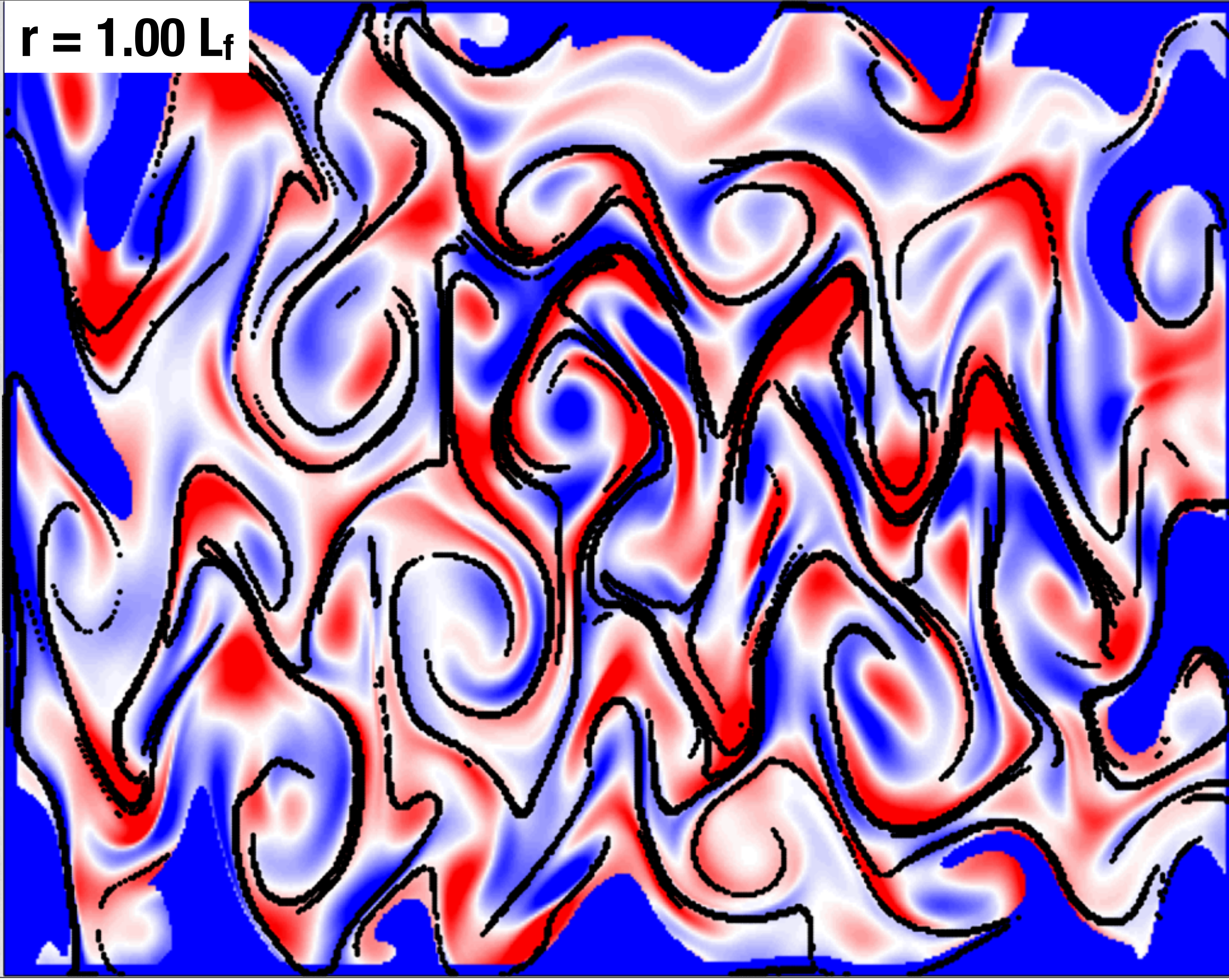

0.75 L_f
0.9 L_f
L_f
1.25 L_f
1.5 L_f
2 L_f
3 L_f
4 L_f

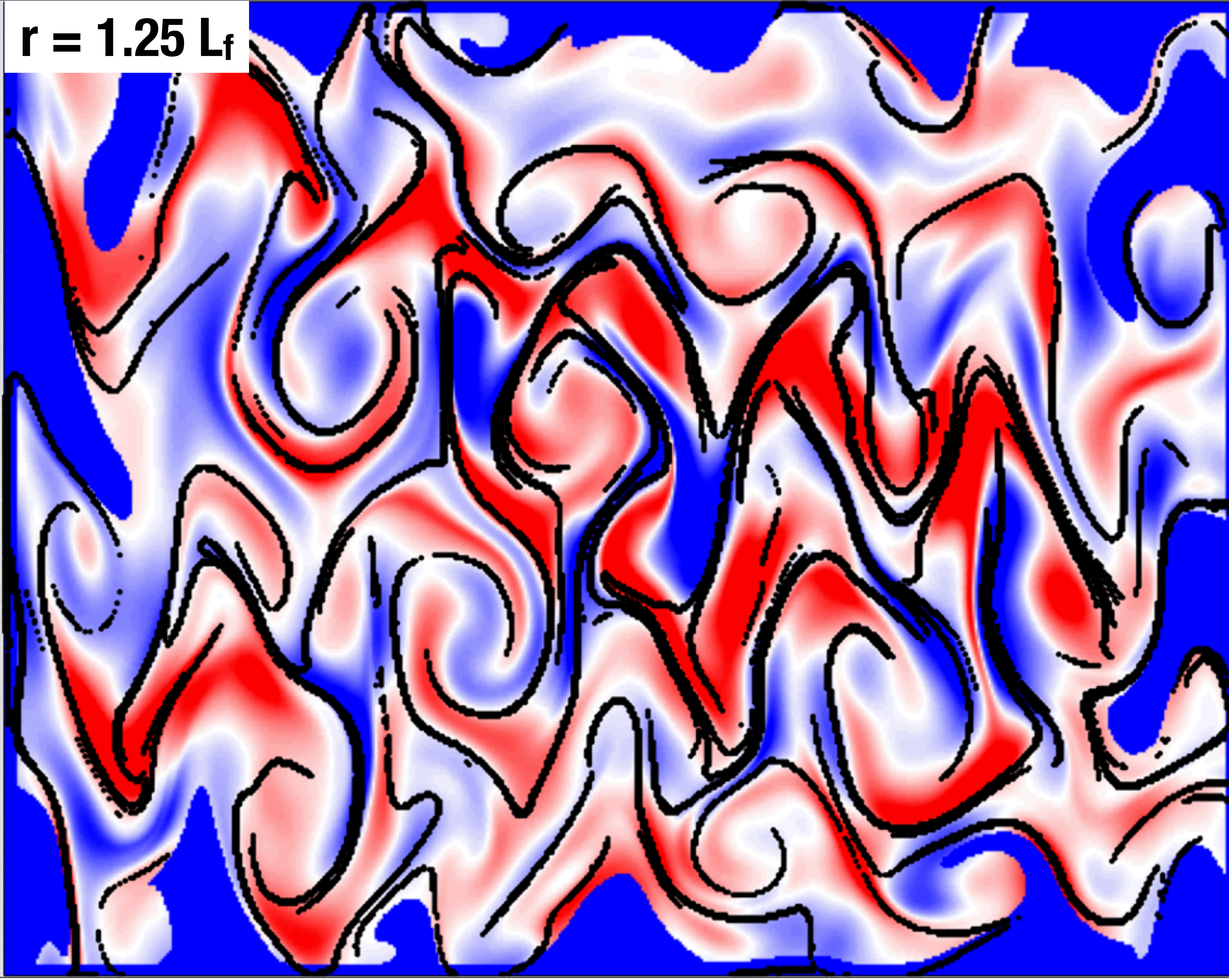


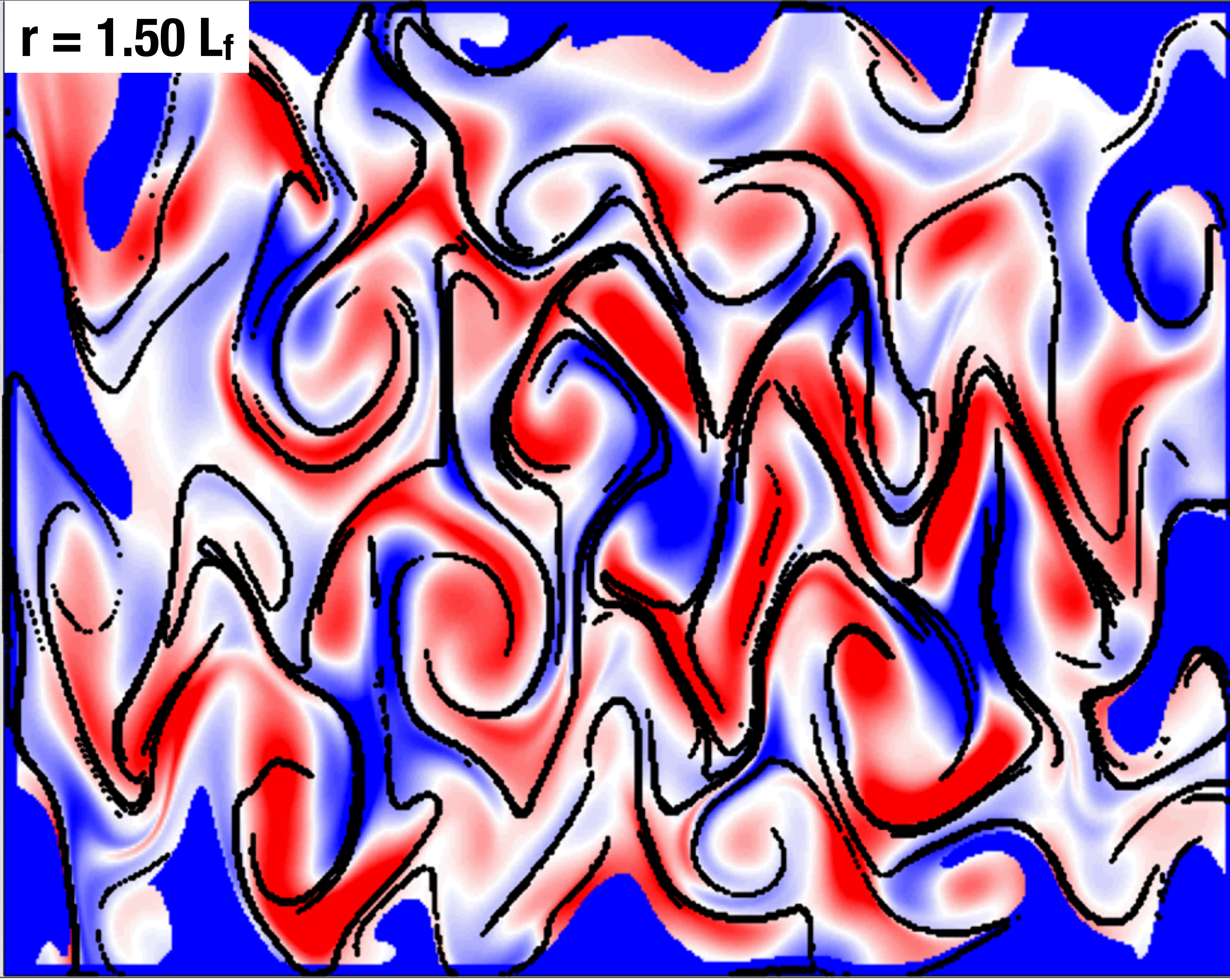
$r = 0.75\ L_f$

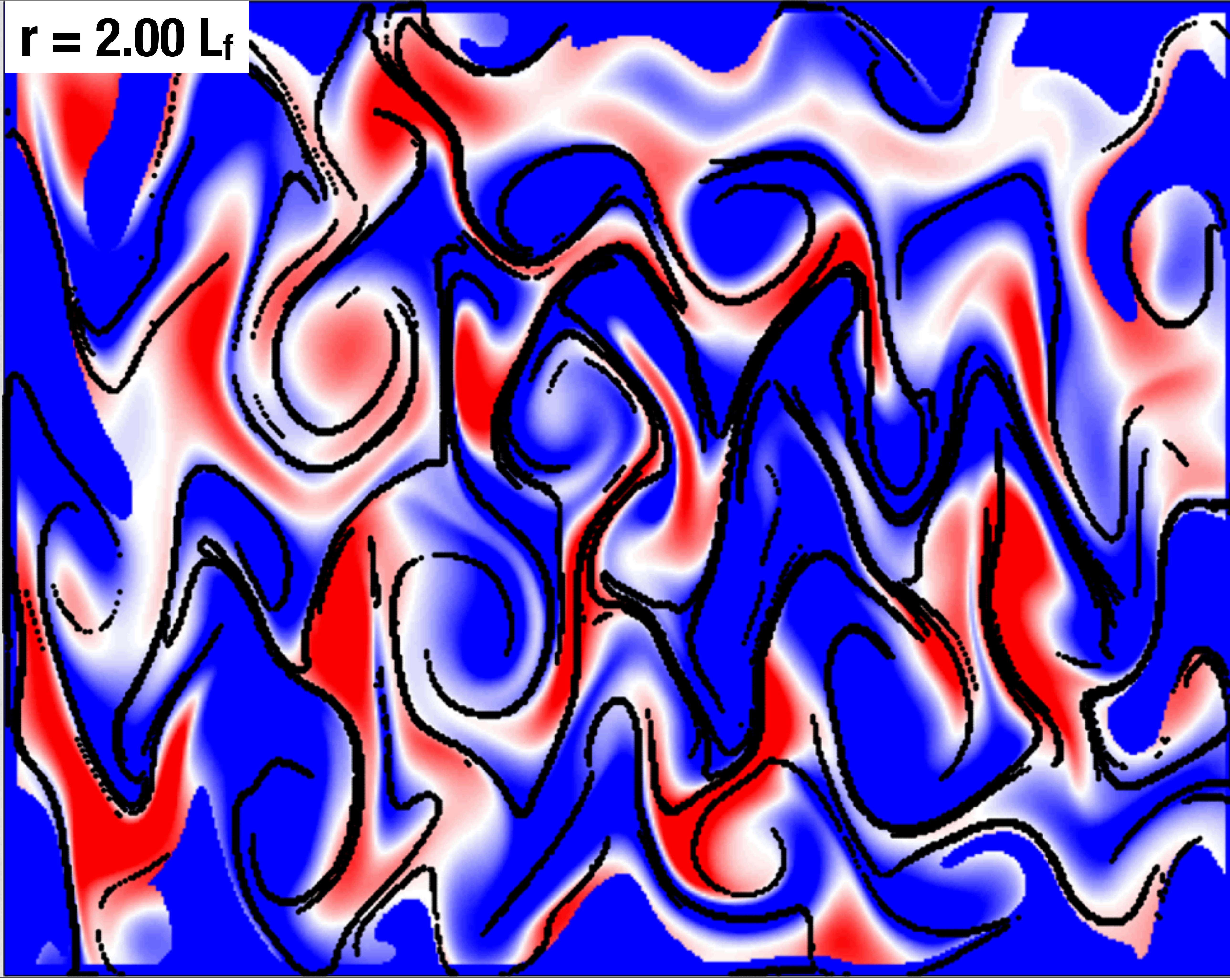

r = 0.75 L_f

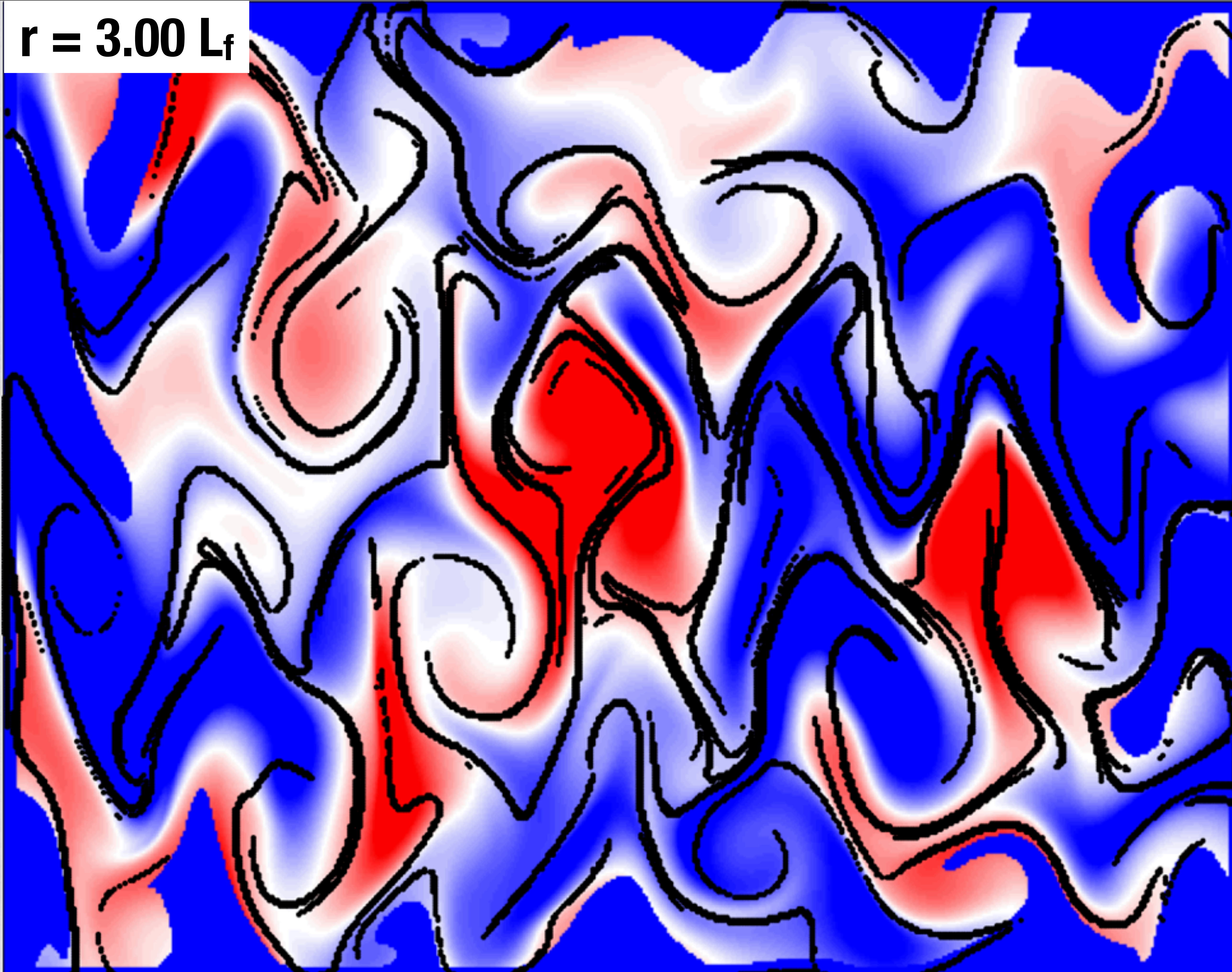

r = 0.90 L_f

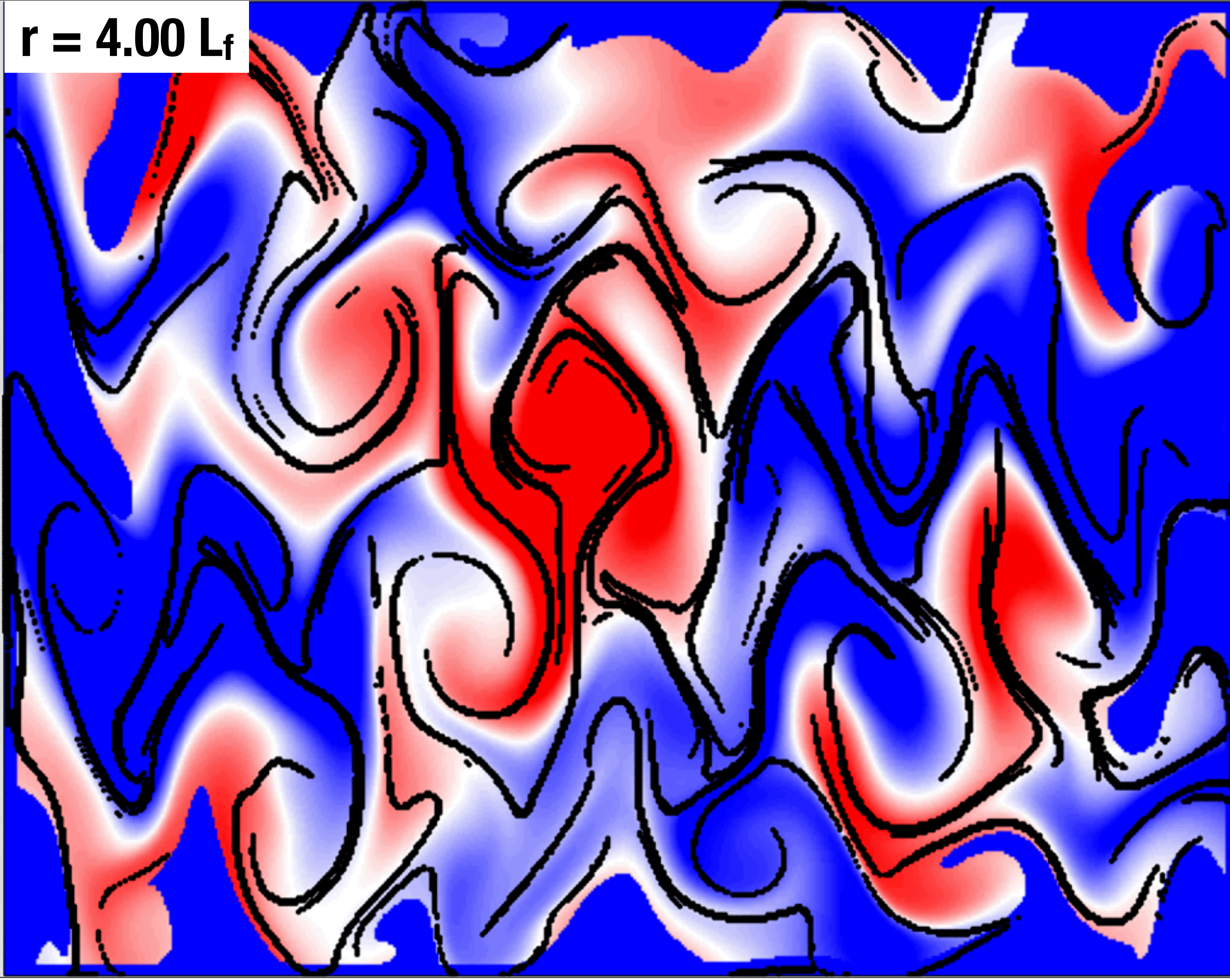

r = 1.00 L_f


r = 1.25 L_f


r = 1.50 L_f


r = 2.00 L_f


r = 3.00 L_f



r = 4.00 L_f

Summary

Spectral fluxes have nontrivial spatiotemporal structure

Spectral transport couples to spatial transport

Appropriate Lagrangian averages reveal coherent dynamics

LCS may separate dynamically distinct regions

http://leviathan.eng.yale.edu