Lagrangian and Eulerian aspects of turbulent flows with dilute polymers - some representative results

Alex Liberzon and Arkady Tsinober

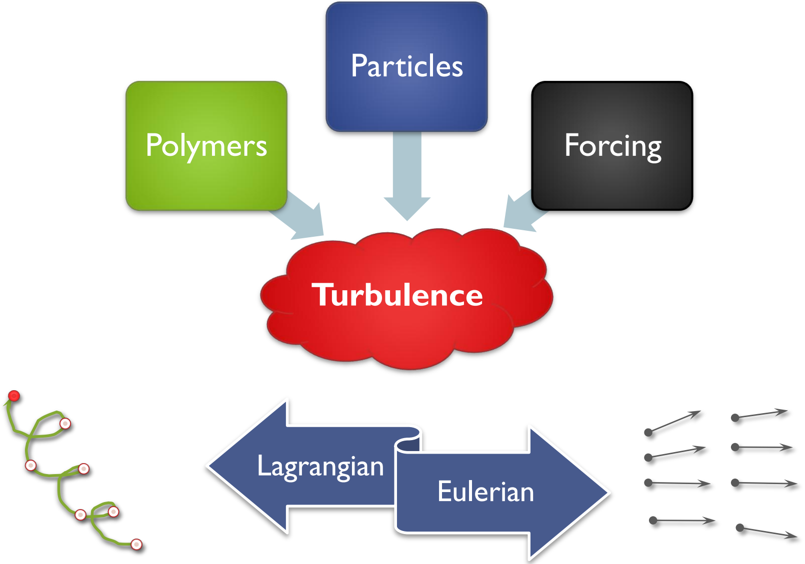
Turbulence Structure Laboratory, Tel Aviv University

WPI Workshop, May 7-10, 2012

Outline

- Background
- Motivation
- Experimental study - 3D-PTV
- Lagrangian/Eulerian results
- Discussion

Turbulence Structure Laboratory

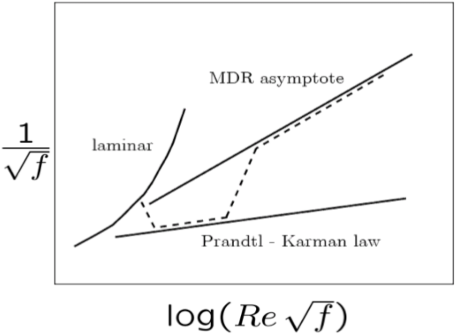


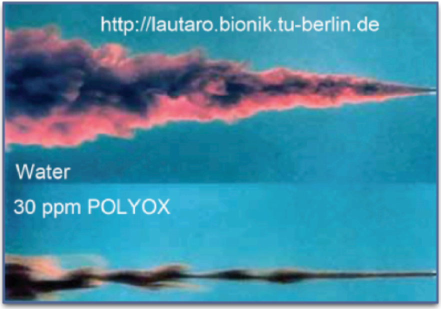
Motivation is both basic and practical

- Drag reduction has been studied since 1948 Toms effect
- Body of literature is huge, important contributions of the present in this room

*The **turbulence** which occurs in the presence of drag-reducing additives **is different** from the turbulence which occurs in the solvent alone. Indeed, in some cases of very dilute polymer solutions, the anomalous (i.e. less dissipative) turbulence is probably the only detectable non-Newtonian effect.* McComb 1990

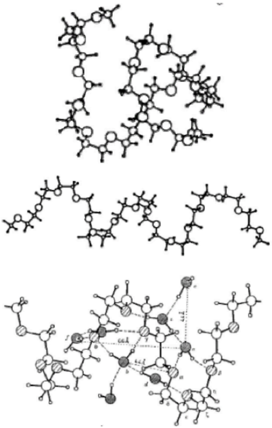
Not only drag reduction





Phenomenology of polymer effects

- Fluctuating and complex strain field is necessary to "turn the effect on"
- Reaction back changes the field of strain, e.g. resistance to large strain, suppression of strong events, bursts
- The flow could be considered intermittently rheological and not evenly distributed (networks)
- The polymer drag reduction is not necessarily associated with suppression of turbulence, but with qualitative changes of some of its structure and production. In other words, there exist turbulent flows with strongly reduced drag and consequently dissipation and strain.



Motivation

- **Turbulent flows with polymer solutions** - important example where the **Lagrangian approach is unavoidable**:
  1. The **material elements** (Lagrangian objects) are **not passive**;
  2. There are no equations reliably describing flows of polymer solutions (such as NSE for Newtonian fluids).

There is a need for Lagrangian experimentation with such turbulent flows (and any other active additives), but ....

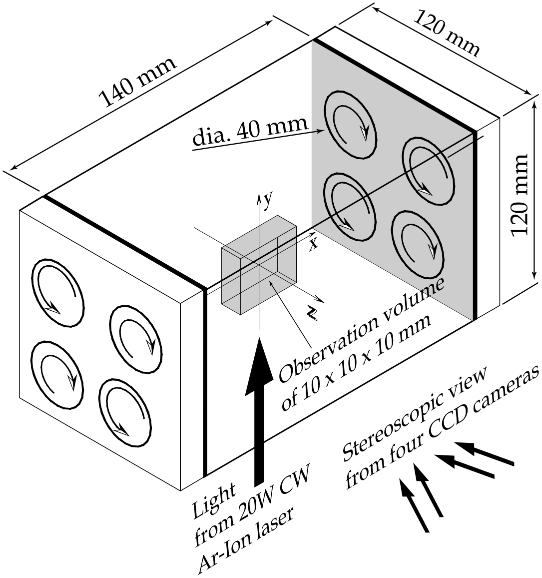
- **Lagrangian** methods alone are limited - there is a necessity of **Eulerian** approaches in parallel:
  1. The fluid particle acceleration $\boldsymbol{a} \equiv D\boldsymbol{u}/Dt$ (Lagrangian) and the Eulerian components.
  2. Evolution of small scales via Lagrangian approaches using strain and vorticity in Eulerian form.
  3. Dealing with the material elements one needs again quantities such as strain and vorticity in Eulerian form.
  4. Eulerian approaches are needed for large scale issues as Reynolds stresses and TKE production.
  5. Direct interaction of small and large scales may be exhibited by mixed quantities: $a_L = \omega \times u$

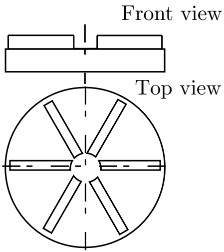
Representative results

The results presented will cover the following topics:

1. Accelerations
2. Velocity derivatives
3. Material elements
4. Large scale stuff (RS and TKE)
5. Direct interaction of SS and LS as may be exhibited by $a_L = \omega \times u$ and perhaps something else available $(\omega \cdot u)$ and (doubtfully) in the spirit of Brasseur.

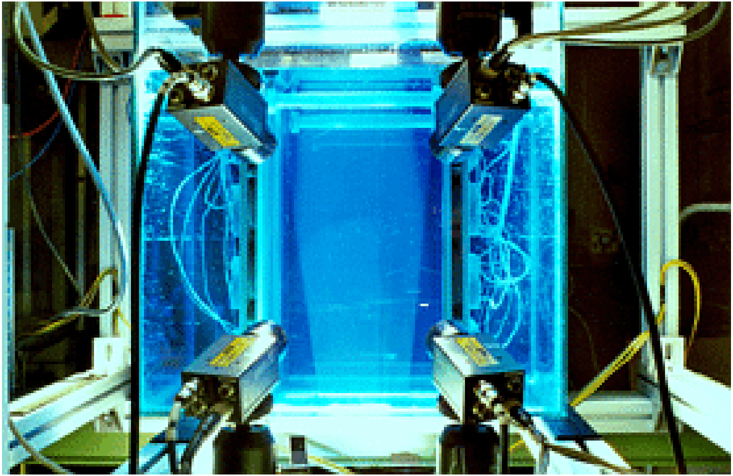
Experimental setup



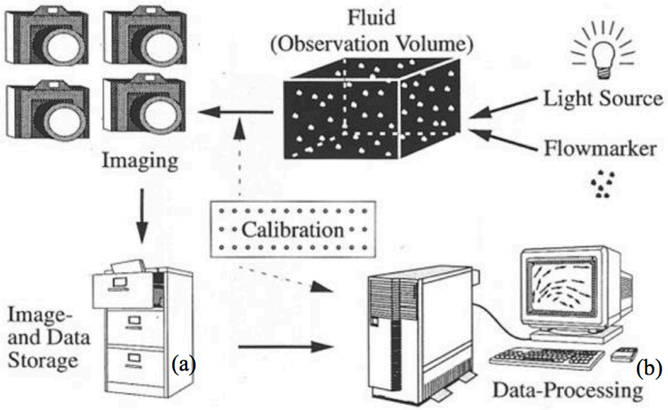


Schematic drawing
of a disk with 6 baffles

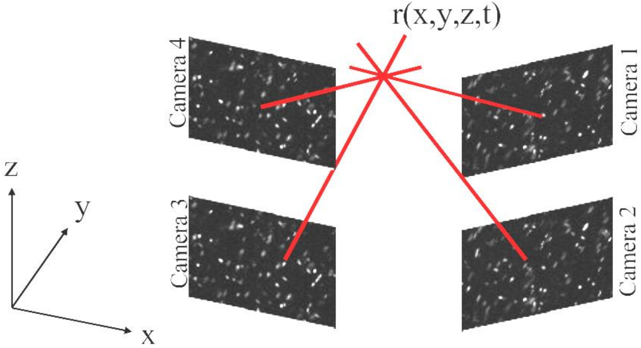
Experimental method

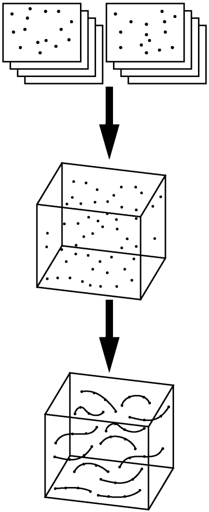


Experimental principles



PTV algorithm





The important thing is that we measure directly the full gradient tensor along the particle trajectories: $\partial u_i/\partial x_j$ and its evolution in time.

Quality checks: Lagrange vs Euler

Lagrangian acceleration, the material derivative of velocity vector, $\boldsymbol{a}$,

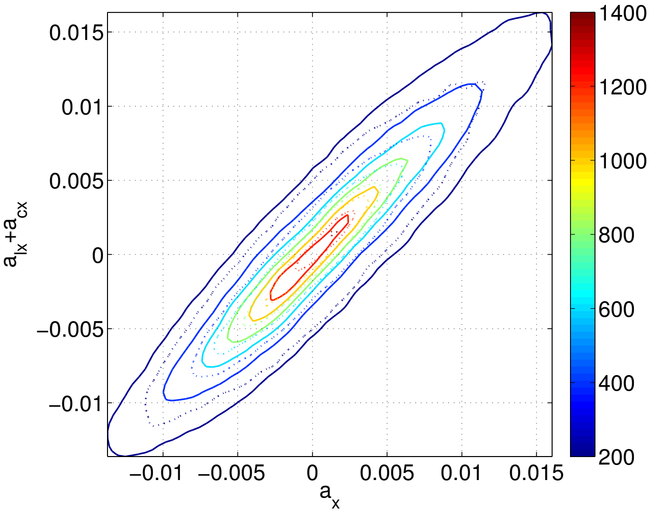
$$\mathbf{a} \equiv \frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u}$$

is studied in conjunction with its physically important Eulerian decompositions:

$$\boldsymbol{a} = \boldsymbol{a_l} + \boldsymbol{a_c} = \boldsymbol{a}_{||} + \boldsymbol{a}_{\perp} = \boldsymbol{a}_L + \boldsymbol{a}_B$$

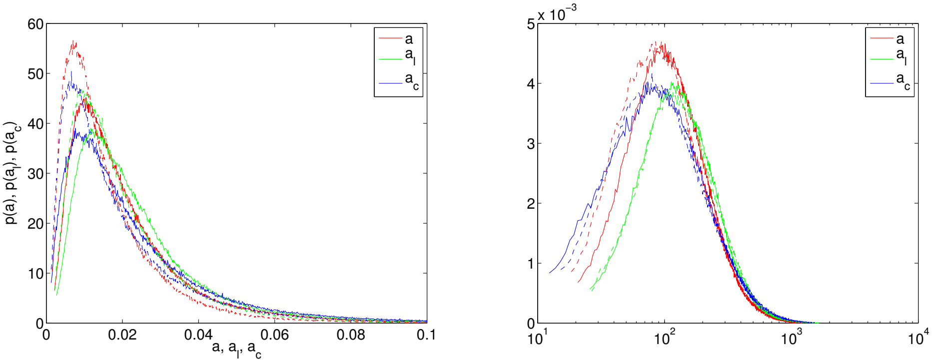
where $\boldsymbol{a}_l = \partial \boldsymbol{u}/\partial t$ is the local acceleration, $\boldsymbol{a}_c = (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}$ is the convective acceleration, $\boldsymbol{a}_{||} = (\boldsymbol{a} \cdot \boldsymbol{u})\boldsymbol{u}$ is the acceleration component parallel to the velocity vector, $\boldsymbol{a}_{\perp} = \boldsymbol{a} - \boldsymbol{a}_{||}$ is the acceleration component normal to the velocity vector, $\boldsymbol{a}_L = \boldsymbol{\omega} \times \boldsymbol{u}$ is the Lamb vector and $\mathbf{a}_B = \nabla(u^2/2)$;

Joint PDF of $\boldsymbol{a}$ and $\boldsymbol{a}_l + \boldsymbol{a}_c$



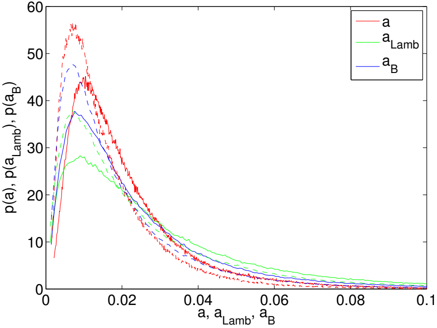
Solid line - water, dashed line - polymers

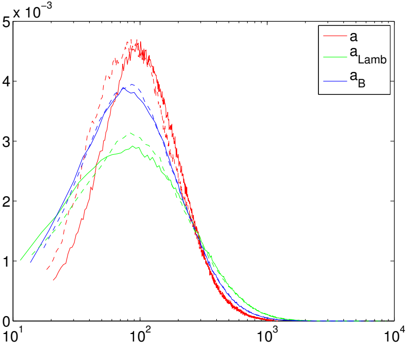
PDFs of acceleration components



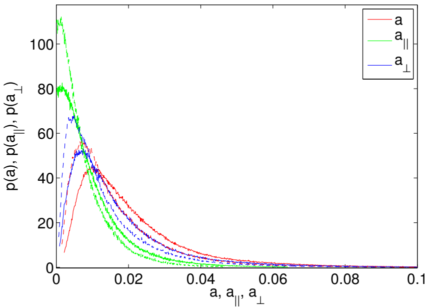
PDFs of the magnitudes of the acceleration vector ($|\boldsymbol{a}|$) and of its components for water (solid lines) and polymer (dashed lines). (left) dimensional form (right) dimensionless form, normalized with $\varepsilon^{3/2}\nu^{-1/2}$

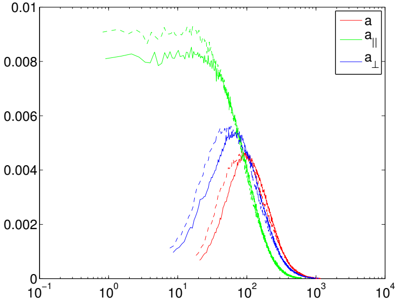
PDFs of acceleration components (cont.)



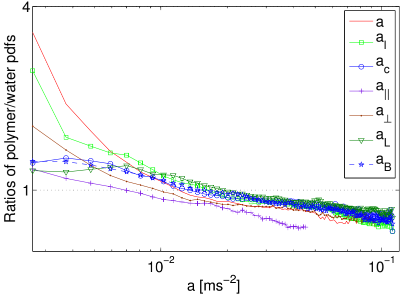


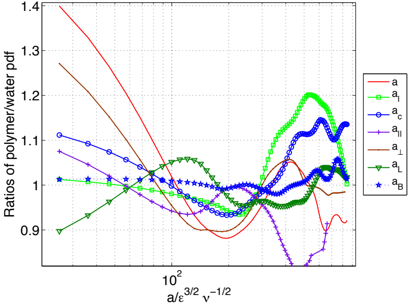
PDFs of acceleration components (cont.)



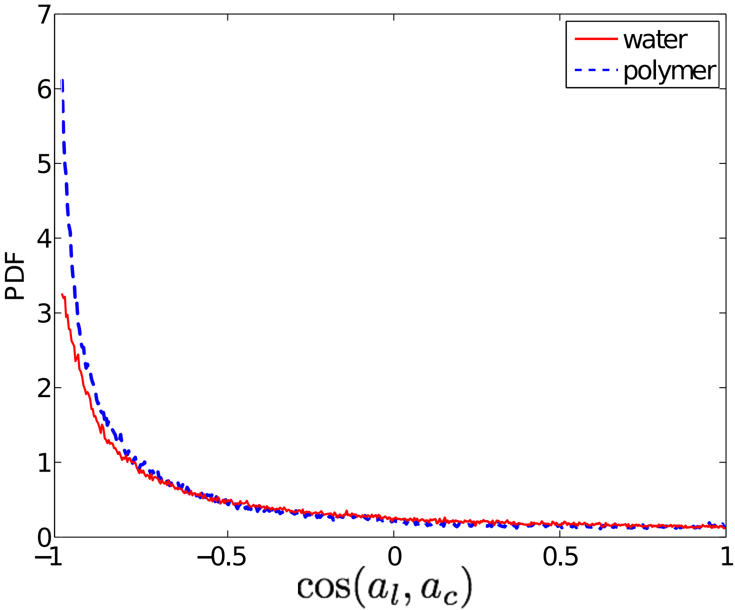


Ratios of PDFs of polymer to water





Alignment of $\boldsymbol{a}_l$ and $\boldsymbol{a}_c$



Lagrangian information on the evolution of material elements

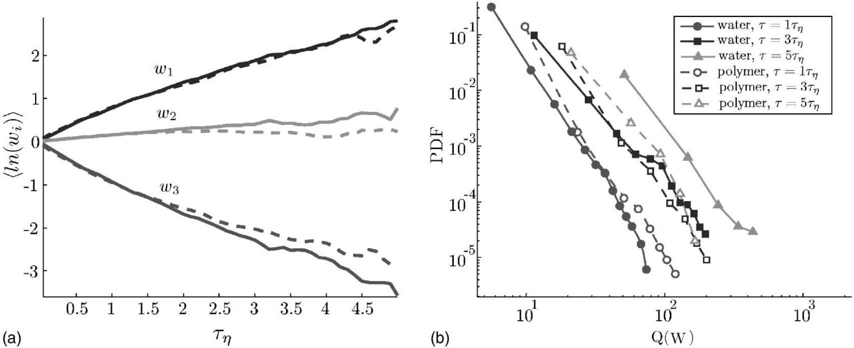
Infinitesimal material lines, l_i evolve according to a purely kinematic equation :

$$\frac{Dl_i}{Dt} = W_i^l$$

$$W_i^l = l_j s_{ij} + (1/2)\varepsilon_{ijkj} l_k \equiv (\boldsymbol{s} \cdot \boldsymbol{l})_i + (1/2)(\boldsymbol{\omega} \times \boldsymbol{l})_i$$

Term 1) Change of magnitude of $\boldsymbol{l}$, and Term 2) the tilting of $\boldsymbol{l}$. *More details in Liberzon et al. PoF (2005)*

Stretching related quantities - Cauchy-Green tensor eigenvalues



$$\ell_i(t) = B_{ij}(t)\ell(0),\ dB_{ij}/dt = (\partial u_i/\partial x_k)\, B_{kj},\ B_{ij}(0) = \delta_{ij},\ W_{ij} = B_{ik}B_{kj}$$

Stretching dynamics of infinitesimal material lines through a *single* tensor

$$\ell_i(t) = B_{ij}(t)\,\ell_j(0), \quad dB_{ij}/dt = (\partial u_i/\partial x_k)\,B_{kj} \quad B_{ij}(0) = \delta_{ij}$$

$$\ell_i \ell_j s_{ij} = B_{ik} B_{jm} s_{ij} \ell_k(0) \ell_m(0) \equiv T_{km}(t) \ell_k(0) \ell_m(0)$$

$$T_{km}(t) \ell(0) \ell_m(0) = \ell^2(0) \left[\mathcal{T}_i \cos^2(\ell(0), \tau_i) \right]$$

$$\langle \ell_i \ell_j s_{ij} \rangle = \langle \mathcal{T}_i \rangle \times \langle \cos^2(\ell^0, \tau_i) \rangle = \frac{1}{3} \langle \ell^2(0) \rangle \langle \mathcal{T}_1 + \mathcal{T}_2 + \mathcal{T}_3 \rangle$$

1. trace $tr(\mathcal{T})$ is positive on average
2. empirically found that one eigenvalue is three orders of magnitude larger than others
3. it was shown to be strongly reduced in dilute polymers flow

Strong reduction of the "stretching eigenvalue" in polymers

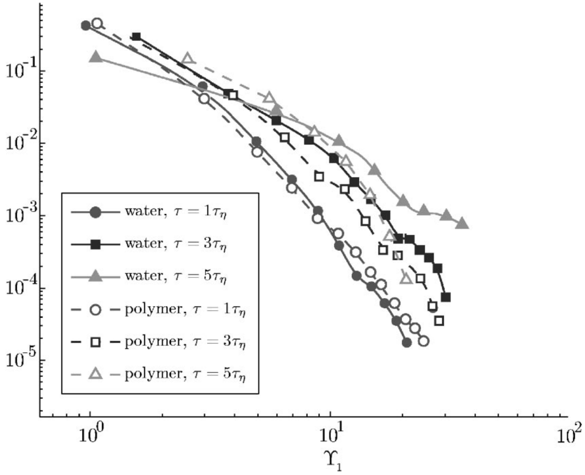
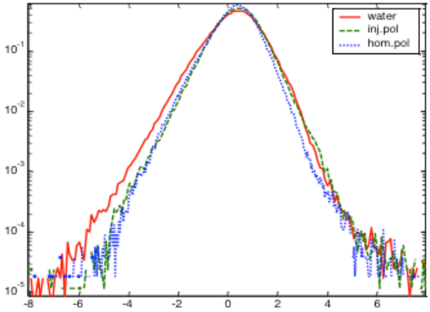
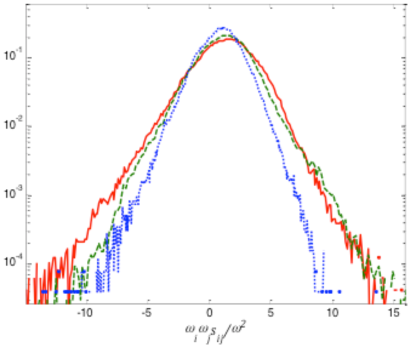


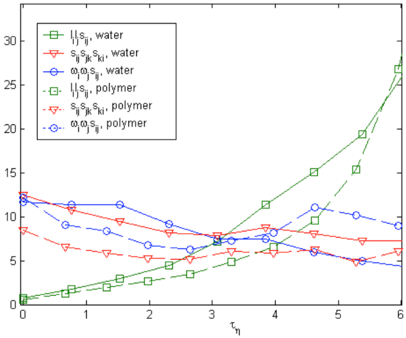
FIG. 5. PDF of the first eigenvalue Υ_1 of the T matrix for water (solid lines) and polymer solution (dashed lines) for different time moments.

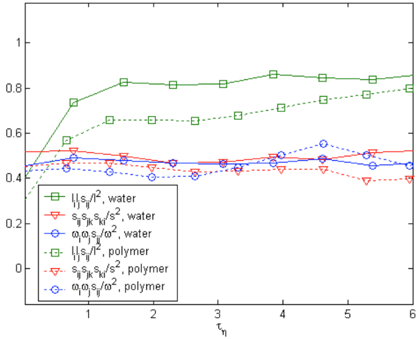
Stretching rates



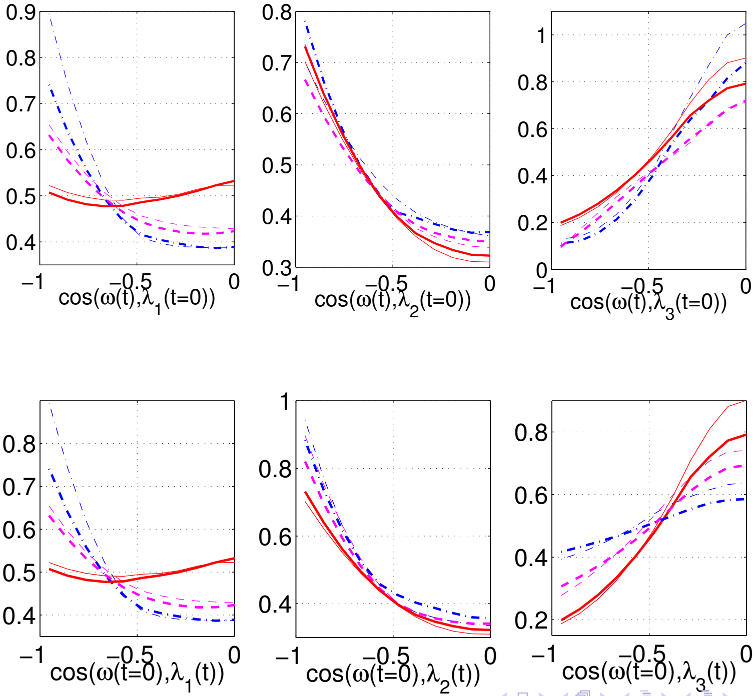


Stretching rates - time evolution



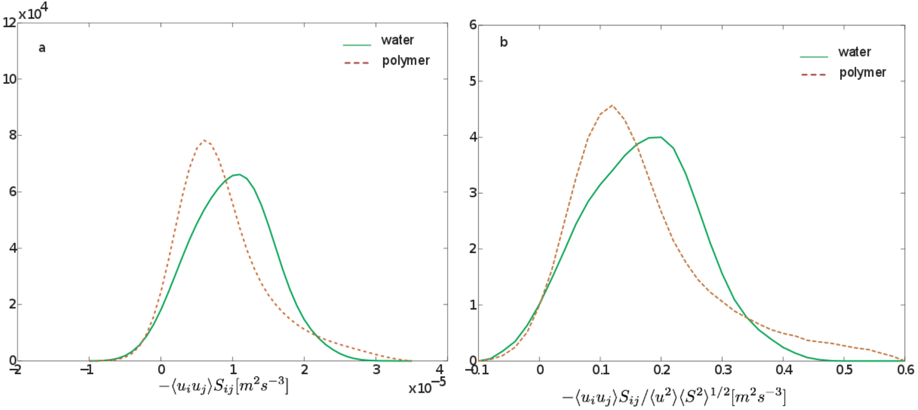


Notice the “delay” of polymer stretching rate - could explain the resistance to strong strain via mis-alignment or tilting.

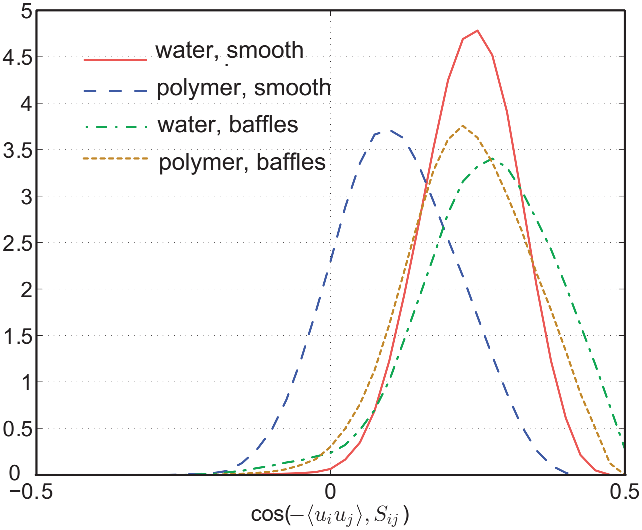


0.9
0.8
0.7
0.6
0.5
0.4
−1
−0.5
0
cos(ω(t),λ1(t=0))
0.8
0.7
0.6
0.5
0.4
0.3
cos(ω(t),λ2(t=0))
1
0.8
0.6
0.4
0.2
0
cos(ω(t),λ3(t=0))
cos(ω(t=0),λ1(t))
cos(ω(t=0),λ2(t))
cos(ω(t=0),λ3(t))

Large scale effects, TKE production



PDF of alignment



Discussion

1. Lagrangian information is crucial in the case of dilute polymers (and probably particles, bubbles, fibers, colloids, etc.)
2. Eulerian information is crucial, maybe because our Lagrangian formulation is very limited and we need dynamics explained by strain, vorticity, etc.
3. Mixing Lagrangian and Eulerian information could help to get some new ideas.

Acknowledgments

1. Institute of Environmental Engineering (IfU), ETH Zurich
2. Turbulence Structure Laboratory team
3. Funding agencies: SNF, ISF, GIF, BSF