AN EXPERIMENTAL STUDY OF TURBULENCE ON FRACTAL FOURIER SPACES

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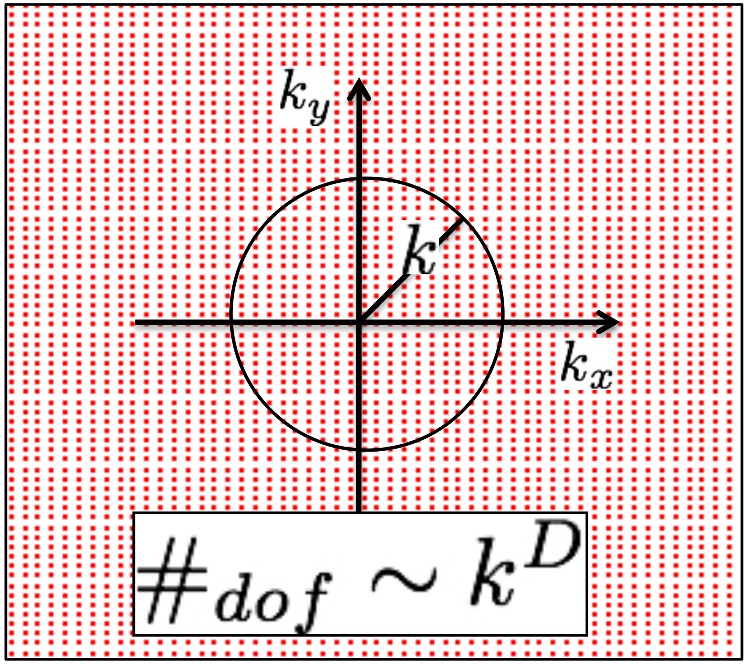
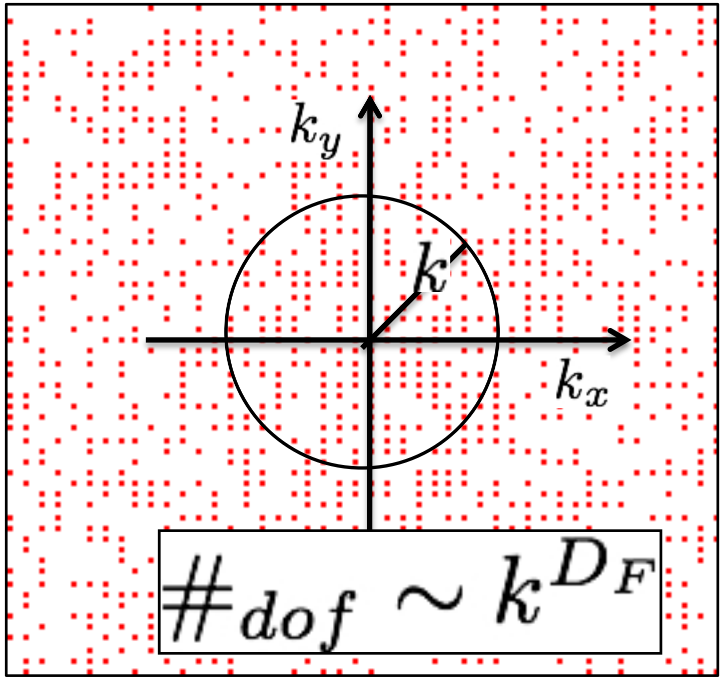
WPI MAY 2015





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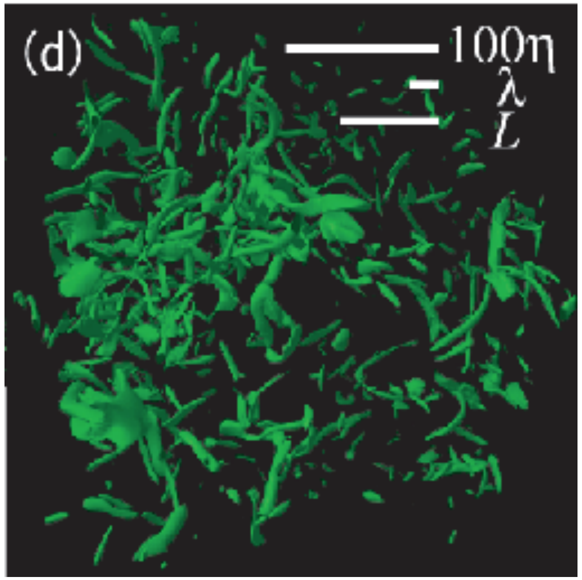
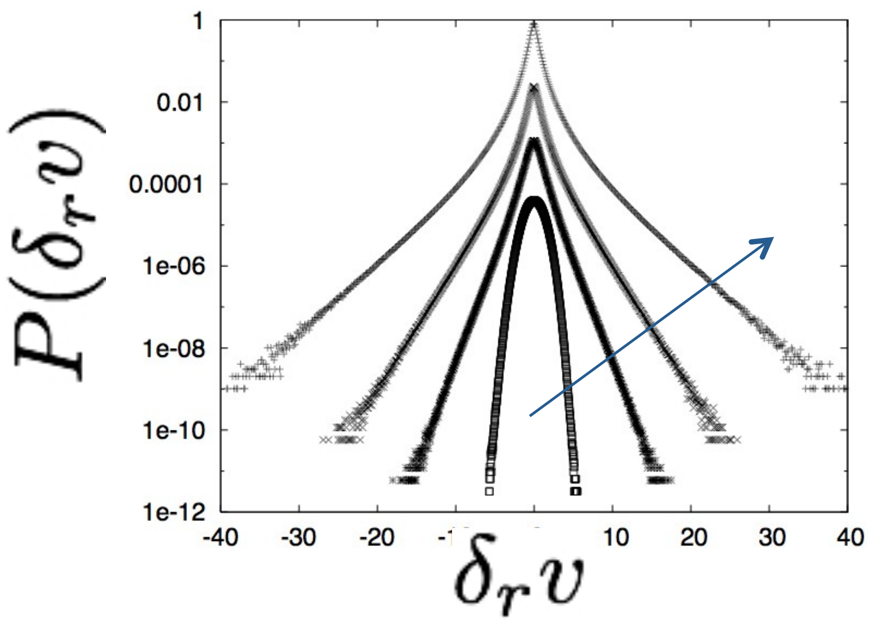
S. Malapaka (Tor Vergata Univ. Italy)
F. Toschi (TuE, The Netherlands)

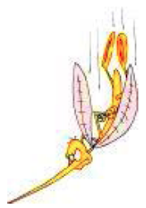
3D HOMOGENEOUS AND ISOTROPIC TURBULENCE

$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v} \cdot \partial)\mathbf{v} = -\partial P + \nu \Delta \mathbf{v} + \mathbf{F} \\ \partial \cdot \mathbf{v} = 0 \\ + \ Boundary \ Conditions \end{cases}$$

EXPERIMENTS IN-SILICO:
CAN WE ASK QUESTIONS ABOUT THE ENERGY TRANSFER BY DECIMATING INTERACTIONS IN THE NON LINEAR TERM?



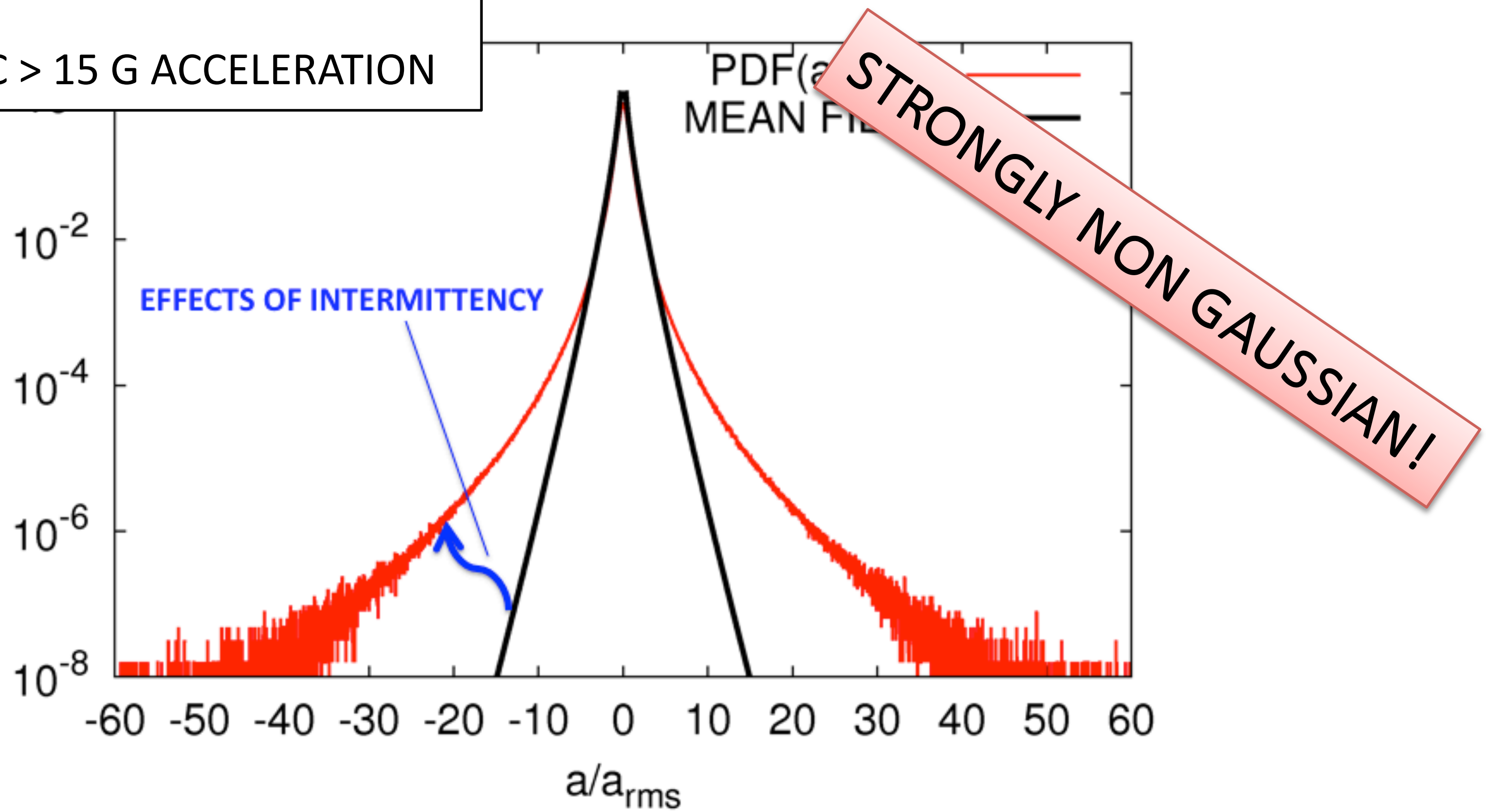




- wind speed 18km/h (5m/sec)
- height above ground 1m
- roughness height 0.05m
  (farmland with few trees in summer time)

↓ τ_η = 5 msec and η = 0.5 mm

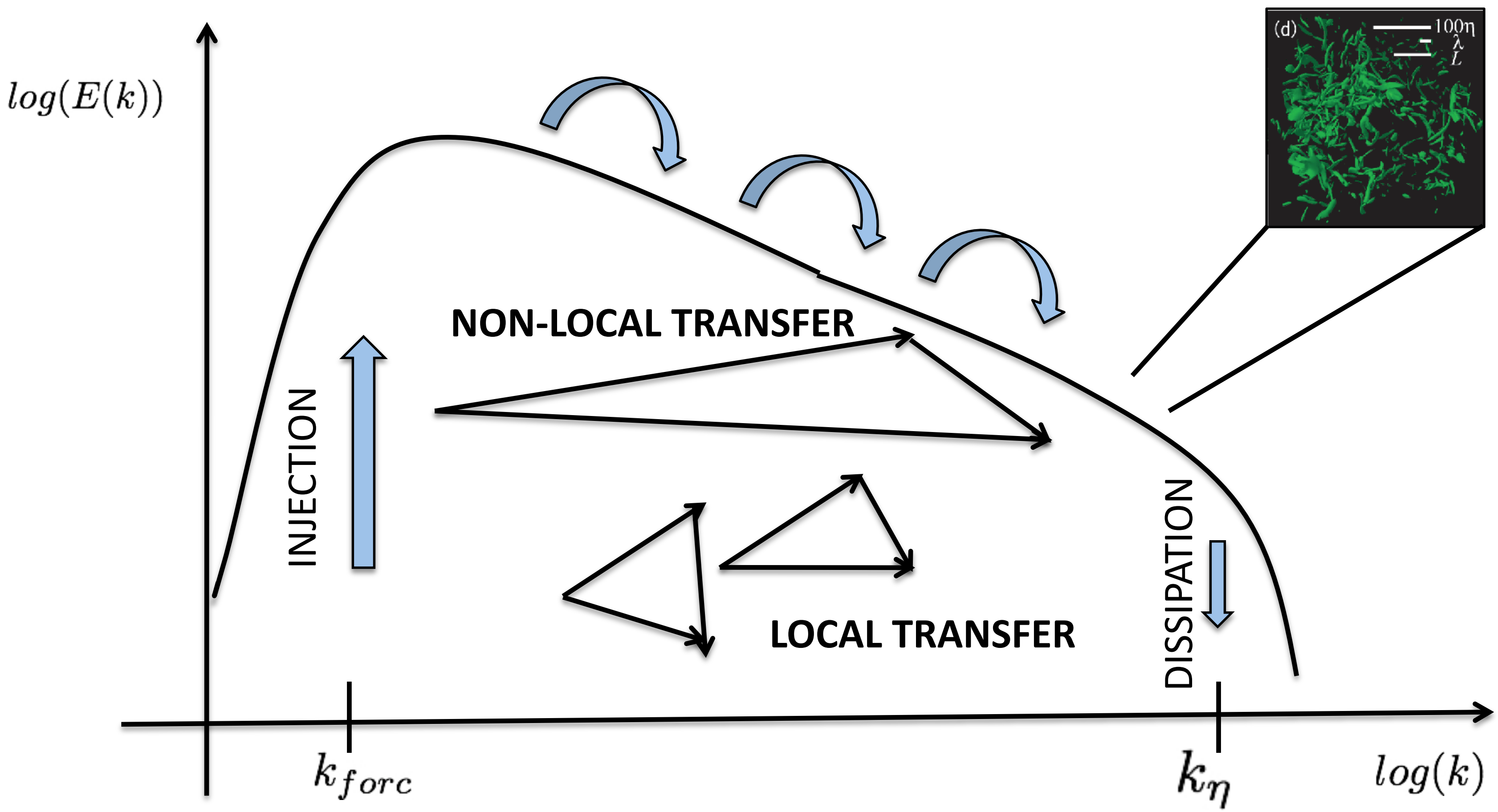
EVERY 15 SEC > 15 G ACCELERATION



ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ 10^5 [Bi04] COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)

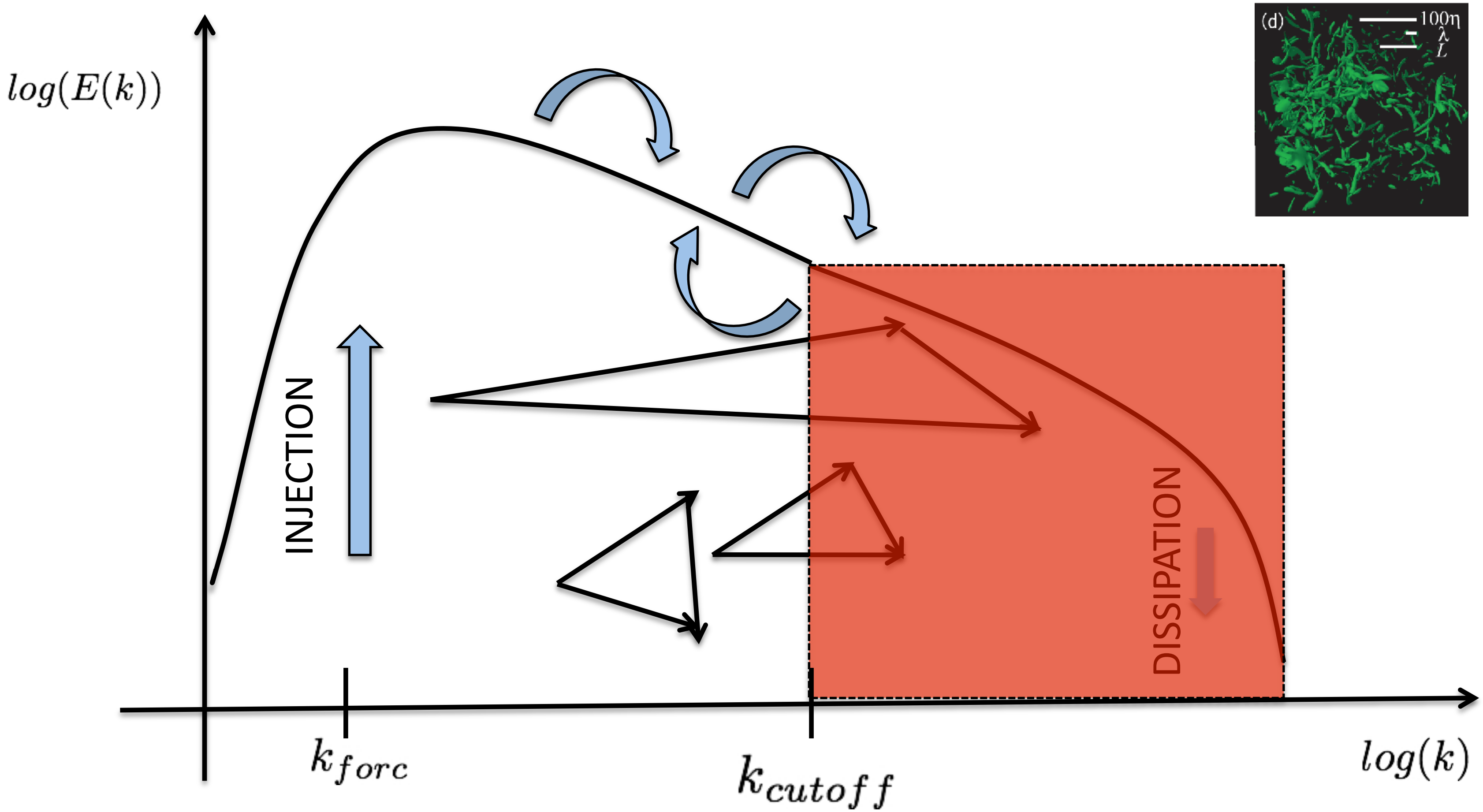
IF YOU WANT TO PREDICT/CONTROL EXTREME EVENTS YUO **CANNOT** NEGLECT INTERMITTENCY

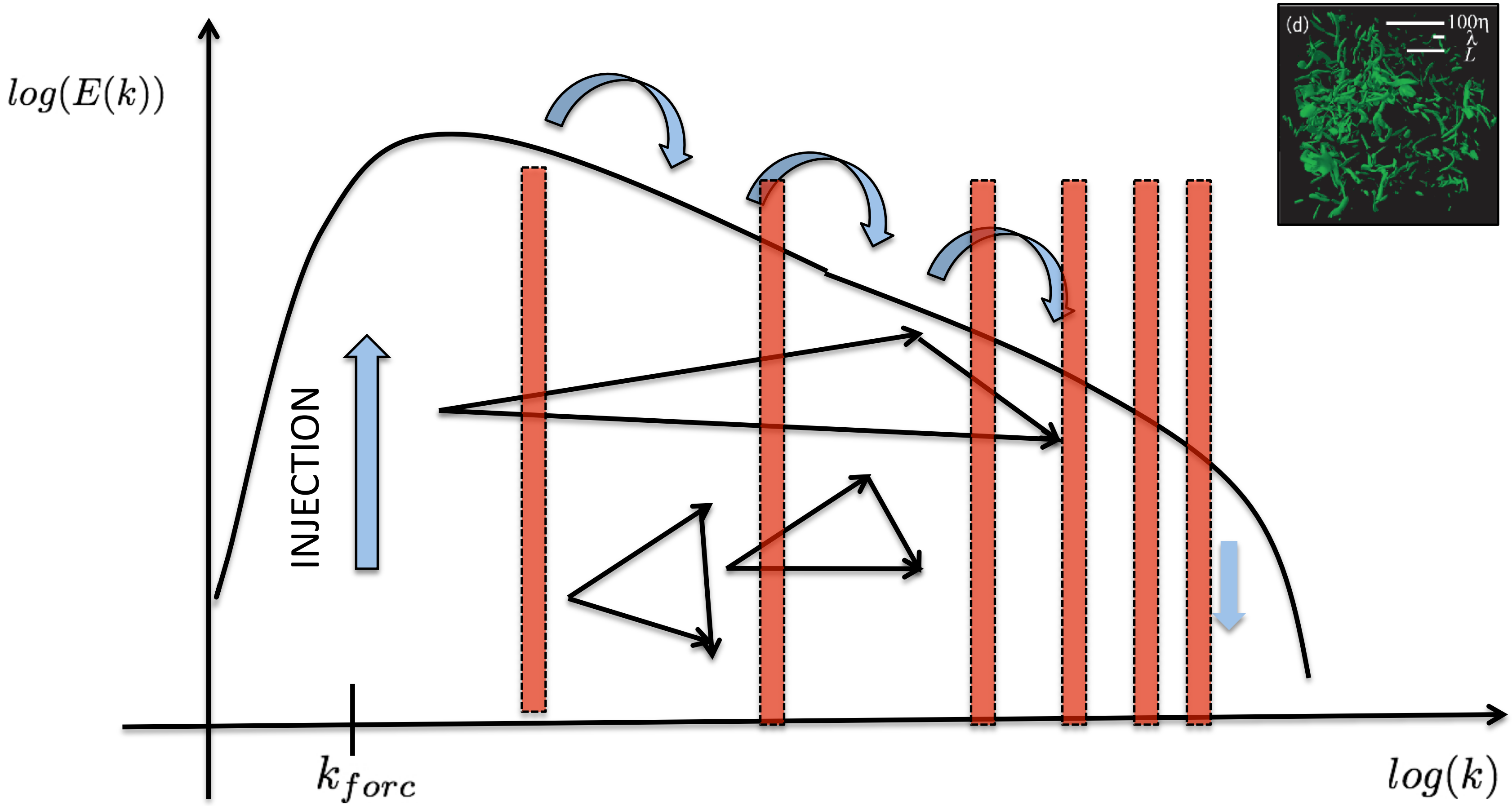
L. B., G. Boffetta, A. Celani, B. Devenish, A. Lanotte and F. Toschi *Phys. Rev. Lett.* **93**, 064502, 2004.



$log(E(k))$
NON-LOCAL TRANSFER
INJECTION
LOCAL TRANSFER
DISSIPATION
k_{forc}
k_η
$log(k)$
(d)
100η
λ
L

LARGE EDDY SIMULATION





$$\mathbf{v}^D(\mathbf{x},t) = \mathcal{P}^D\mathbf{v}(\mathbf{x},t) = \sum_{\mathbf{k}\in\mathcal{Z}^3} e^{i\mathbf{k}\cdot\mathbf{x}}\,\gamma_{\mathbf{k}}\mathbf{u}(\mathbf{k},t)\,.$$

DECIMATED WITH PROBABILITY ~ $1 - k^{D_F-3}$

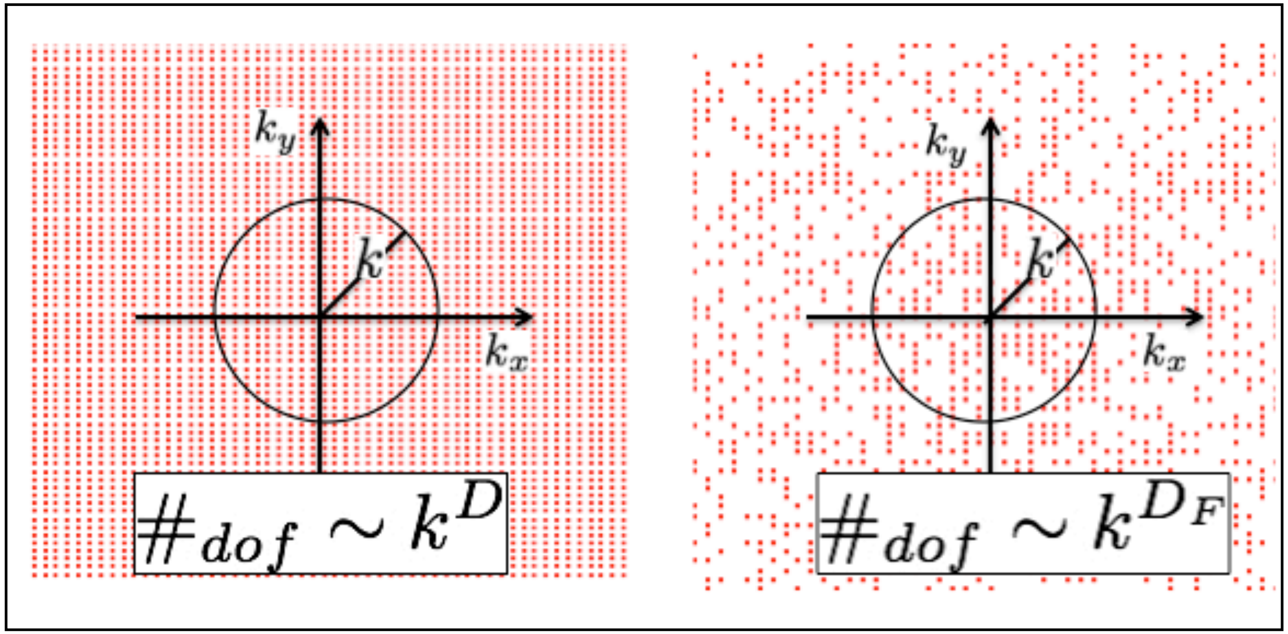
HOMOGENEOUS & ISOTROPIC & SELF-SIMILAR (NO EXTERNAL SCALES)

SELF- SIMILAR SURGERY OF NAVIER-STOKES INTERACTIONS

U. Frisch, A. Pomyalov, I. Procaccia and S. Ray PRL 2012
S. Grossmann, D. Lohse and A. Reeh, PRL 1996

$$\partial_t \mathbf{v}^{D_F} = P^{D_F} B(\mathbf{v}^{D_F}, \mathbf{v}^{D_F}) + \Delta \mathbf{v}^{D_F} + \mathbf{f}^{D_F}$$

SELF-SIMILAR GALERKIN TRUNCATION



Turbulence in non-integer dimensions by fractal Fourier decimation

Uriel Frisch,[1] Anna Pomyalov,[2] Itamar Procaccia,[2] and Samriddhi Sankar Ray[1]

[1] *UNS, CNRS, OCA, Laboratoire Cassiopée, B.P. 4229, 06304 Nice Cedex 4, France*

[2] *Department of Chemical Physics, The Weizmann Institute of Science, Rehovot 76100, Israel*

(Dated: August 8, 2011)

Fractal decimation reduces the effective dimensionality of a flow by keeping only a (randomly chosen) set of Fourier modes whose number in a ball of radius k is proportional to k^D for large k. At the critical dimension $D = 4/3$ there is an equilibrium Gibbs state with a $k^{-5/3}$ spectrum, as in [V. L'vov *et al.*, Phys. Rev. Lett. **89**, 064501 (2002)]. Spectral simulations of fractally decimated two-dimensional turbulence show that the inverse cascade persists below $D = 2$ with a rapidly rising Kolmogorov constant, likely to diverge as $(D - 4/3)^{-2/3}$.

$$E(k) = \frac{k^{D-1}}{\alpha + \beta k^2}; \quad \beta > 0, \quad \alpha > -\beta,$$

$$D = 4/3$$

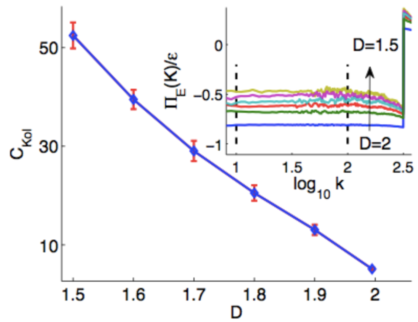


FIG. 3. (Color online) Dependence of the Kolmogorov constant on D. The lowest value, at $D = 2$, is about 5. The inset shows the energy flux normalized by the energy injection ε for the same values of D as in Fig. 2.

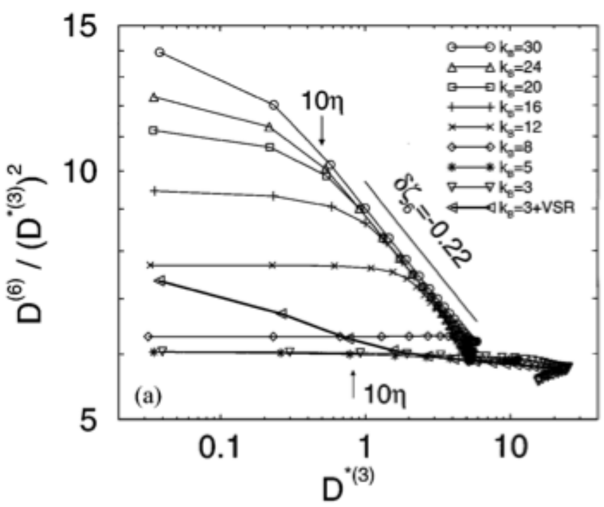
Enstrophy equipartition <-> 5/3 Kolmogorov spectrum

Developed Turbulence: From Full Simulations to Full Mode Reductions

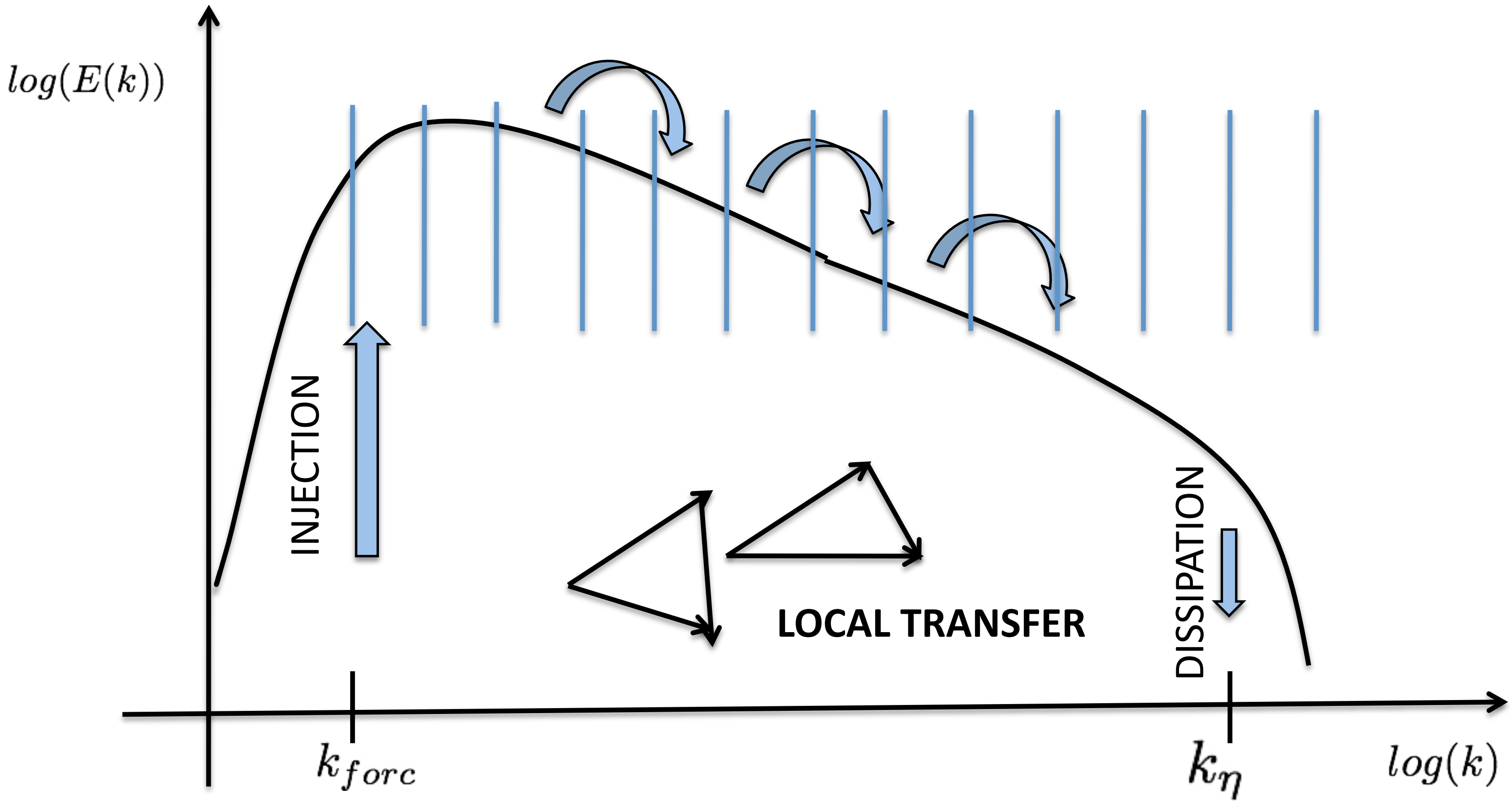
Siegfried Grossmann,* Detlef Lohse,† and Achim Reeh‡
Fachbereich Physik der Universität Marburg, Renthof 6, D-35032 Marburg, Germany
(Received 5 August 1996)

Developed Navier-Stokes turbulence is simulated with varying wave-vector mode reductions. The flatness and the skewness of the velocity derivative depend on the degree of mode reduction. They show a crossover towards the value of the full numerical simulation when the viscous subrange starts to be resolved. The intermittency corrections of the scaling exponents ζ_p of the pth order velocity structure functions seem to depend mainly on the proper resolution of the inertial subrange. *Universal* scaling properties (i.e., independent of the degree of mode reduction) are found for the relative scaling exponents $\rho_{p,q} = (\zeta_p - \zeta_3 p/3)/(\zeta_q - \zeta_3 q/3)$. [S0031-9007(96)01942-4]

PACS numbers: 47.27.Eq, 47.11.+j

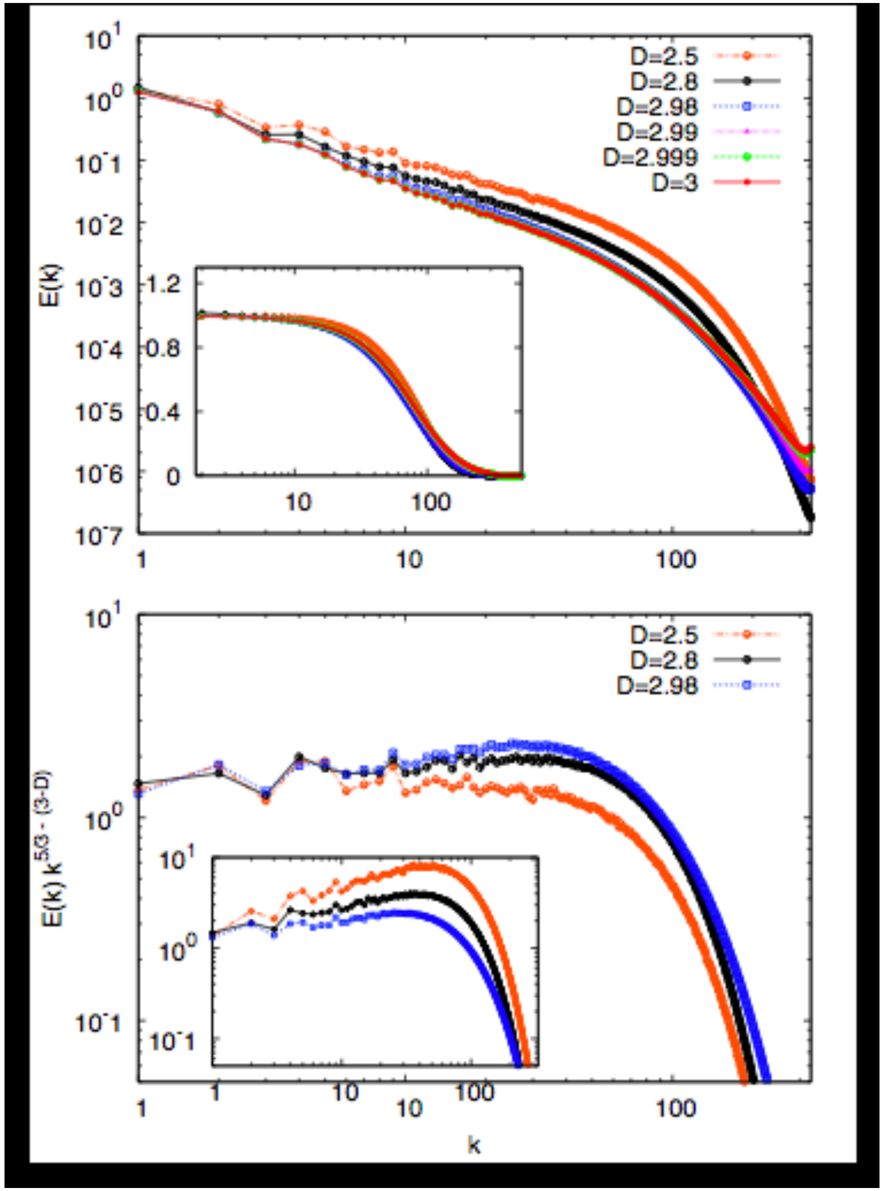


In summary, we repeat that in 3D Navier-Stokes turbulence the main origin of intermittency corrections seems to be the proper resolution of the phase space at the scale of interest. Reflections from the VSR seem to be of minor importance. Why does the energy flux reflected from the VSR [10,11] reach so far in the ISR for the GOY model but apparently not for 3D Navier-Stokes turbulence? We speculate that in 3D the phases of the modes are subjected to far more fluctuations than in the 1D GOY model. Therefore, coherences get destroyed easier. Some coherence, however, must remain, otherwise no energy could be transported downscale.



$$\frac{d}{dt}u(k_n) = k_n[a\,u(k_{n+2})u(k_{n+1}) + b\,u(k_{n+1})u(k_{n-1}) + c\,u(k_{n-2})u(k_{n-1})] - \nu k_n^2 u(k_n)$$

Bohr T., Jensen M. H., Paladin G. and Vulpiani A., *Dynamical Systems Approach to Turbulence*, Cambridge, in press (1998)



$$E^D(k) = \int_{|\mathbf{k}_1|=k} d^3k_1\, \gamma_{\mathbf{k}_1} \int d^3k_2\, \gamma_{\mathbf{k}_2} \langle\, \mathbf{u}(\mathbf{k}_1)\, \mathbf{u}(\mathbf{k}_2)\, \rangle\,.$$

$$\Pi^D(k) = \int_{|\mathbf{k}_1|<k} d^3k_1 \gamma_{\mathbf{k}_1} \int d^3k_2\, d^3k_3\, \gamma_{\mathbf{k}_2}\gamma_{\mathbf{k}_3} S(\mathbf{k}_1|\mathbf{k}_2,\mathbf{k}_3)\,,$$

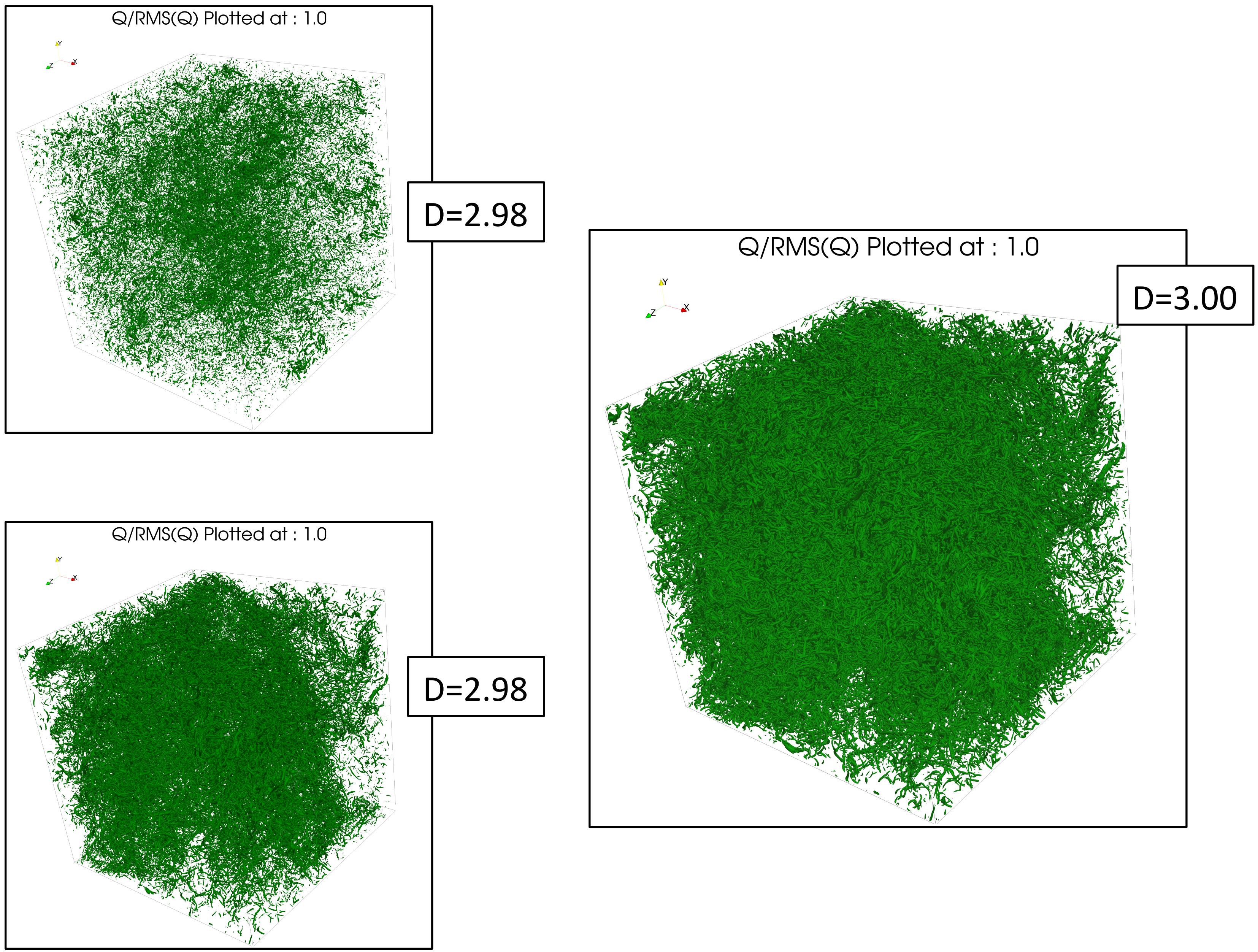
$$\mathbf{u}(\mathbf{k}) \sim k^{-a}$$

$$\Pi^D(\lambda k) \;\sim\; \lambda^{3D+1-3a}\Pi^D(k).$$

$$a = D + 1/3 \rightarrow E^D(k) \sim E^{K41}(k) k^{3-D}$$

DF	2.5	2.8	2.98	2.99	2.999	3.0
1024^3	X	X	X	X	X	X
2048^3			X	X		

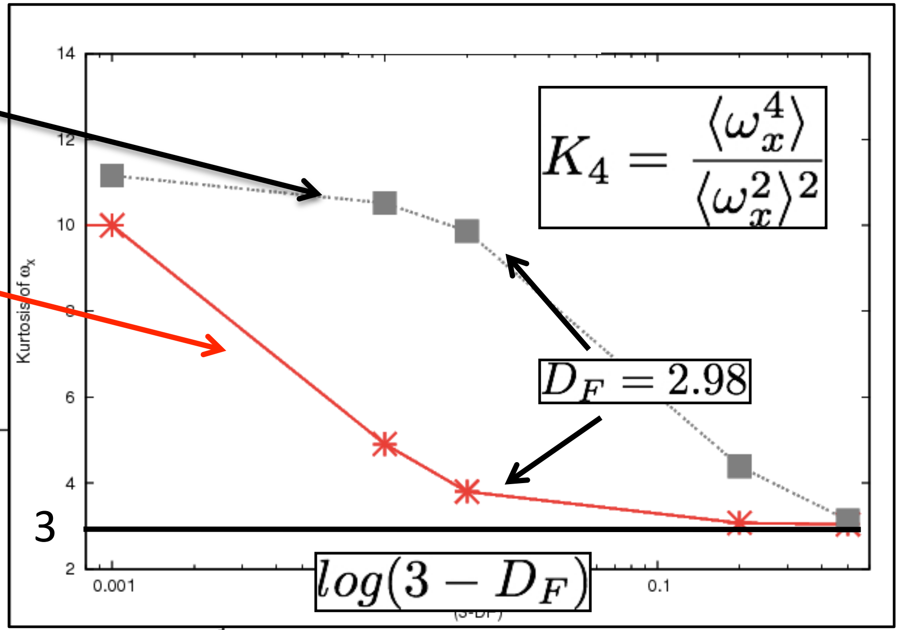
DF	2.5	2.8	2.98	2.99	2.999	3.0
1024^3	3%	25%	87%	93%	99%	100%

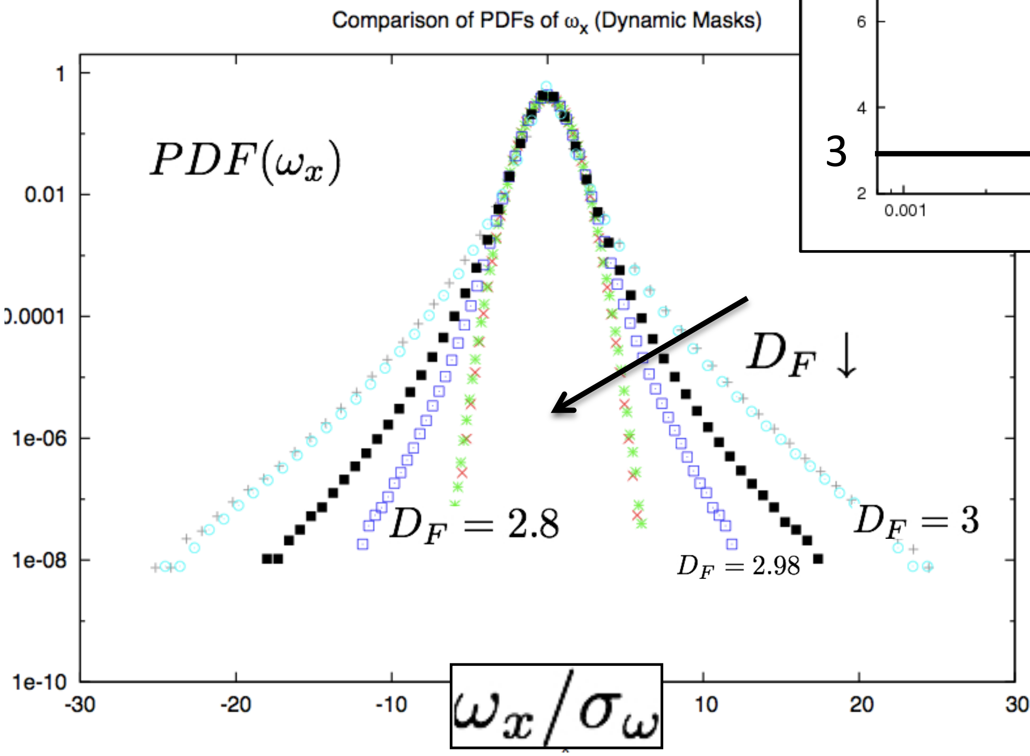

Q/RMS(Q) Plotted at : 1.0
D=2.98
Q/RMS(Q) Plotted at : 1.0
D=2.98
Q/RMS(Q) Plotted at : 1.0
D=3.00

PDF OF VORTICITY AT CHANGING FRACTAL DIMENSION

$$\begin{cases} \partial_t \mathbf{v} = B(\mathbf{v},\mathbf{v}) + \Delta \mathbf{v} + \mathbf{f} \\ \mathbf{v} \to P^{D_F} \mathbf{v} \end{cases}$$

$$\partial_t \mathbf{v}^{D_F} = P^{D_F} B(\mathbf{v}^{D_F}, \mathbf{v}^{D_F}) + \Delta \mathbf{v}^{D_F} + \mathbf{f}^{D_F}$$





$$S_p(r) = \langle (u(x+r) - u(x))^p \rangle \sim r^{\zeta_E(p)}$$

$$S_p(\tau) = \langle (v(t+\tau) - v(t))^p \rangle \sim \tau^{\zeta_L(p)}$$

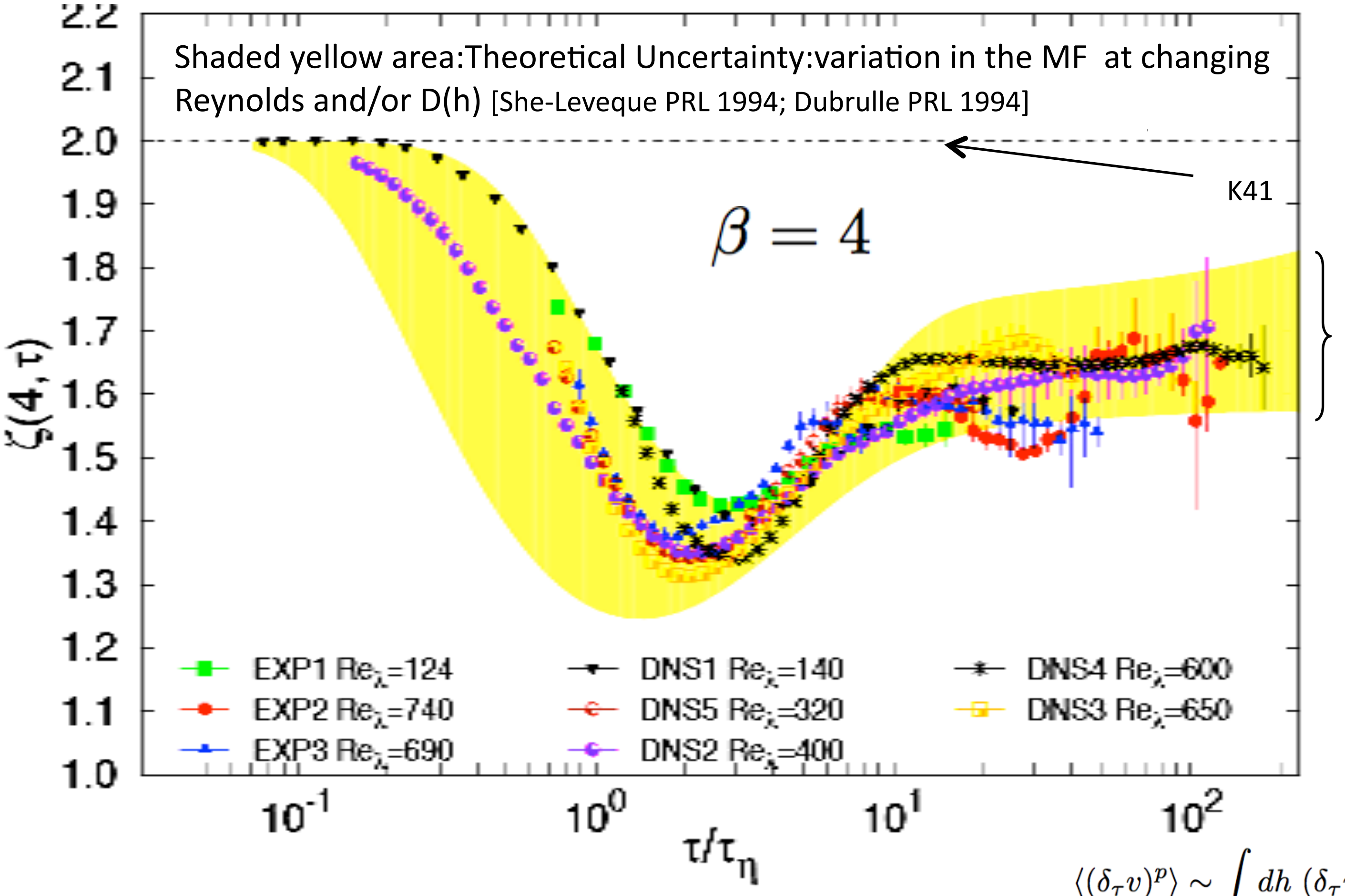
Bridge relation between Lagrangian and Eulerian increments

$$\delta_\tau v \sim \delta_r u$$

$$\tau_r \sim r / \delta_r u$$

$$\tau \sim \frac{L_0^h}{v_0} r^{1-h}$$

[Borgas (1993); Boffetta et al (2002); Chevillard et al. (2003)]



$$\langle (\delta_\tau v)^p \rangle \sim \int dh\ (\delta_\tau v)^p\ \mathcal{P}_h(\frac{\tau}{T_L}, \frac{\tau_\eta}{T_L})$$

International Collaboration for Turbulence Research, A. Arneodo,[1] J. Berg,[2] R. Benzi,[3] L. Biferale,[3] E. Bodenschatz,[4] A. Busse,[5] E. Calzavarini,[6] B. Castaing,[1] M. Cencini,[7] L. Chevillard,[1] R. Fisher,[8] R. Grauer,[9] H. Homann,[9] D. Lamb,[8] A.S. Lanotte,[10] E. Leveque,[1] B. Lüthi,[11] J. Mann,[2] N. Mordant,[12] W.-C. Müller,[5] S. Ott,[2] N. Oullette,[13] J.-F. Pinton,[1] S.B. Pope,[14] S.G. Roux,[1] F. Toschi,[15,16] H. Xu,[4] and P.K. Yeung[17]

WE LEARN ABOUT:
(i) INTERMITTENCY; (ii) UNIVERSALITY; (iii) ANISOTROPY

CONCLUSIONS

FRACTAL DECIMATION: MILDEST REMOVAL OF DEGREE OF FREEDOM
HOMOGENEOUS & ISOTROPIC & SELF SIMILAR

+ QUANTIFY IMPORTANCE OF LOCAL VS NON-LOCAL TRIADIC INTERACTIONS

+/- QUANTIFY IMPORTANCE OF $\#_{DOF}$ FOR VORTEX STRETCHING

+ CORRECTION IN THE MEAN RESPONSE (SPECTRUM) PROPORTIONAL TO 3-D_F: YOU CAN HAVE A LITTLE CHANGE IN THE SPECTRAL PROPERTIES AND STILL GAINING IN THE $\#_{DOF}$

+ CORRECTION IN FLUCTUATIONS: HUGE. SMALL SCALE VORTICITY IS STRONGLY SENSITIVE TO DECIMATION. “CHOERENT” SMALL-SCALE STRUCTURES FEEL NON-LOCAL CORRELATIONS ACROSS SCALES IN FOURIER

+ WE STILL MISS A CLEAR DEFINITION OF INTERMITTENCY IN FOURIER SPACE