

# Bioremediation Modeling and Traveling Wave Analysis

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## ABSTRACT

Bioremediation, i.e. the degradation of contaminants by microorganisms, is a promising technology for restoring contaminated groundwater and soil. In order to predict the merits of this method, mathematical models are of special interest. In this presentation a particular bioremediation model is derived and analyzed.

We develop a mathematical model that describes the degradation of an arbitrary number of substrates and the related growth of specific bacteria. Due to natural correlations, the modeling process involves the analysis of enzymatic reactions as well as bacterial growth depending on these reactions. The resulting bioremediation model consists of advection-reaction equations for the substrate concentrations and a rate equation related to the biomass concentration.

For the special cases of one and two substrates involved in the bioremediation process, we analyze our model with respect to traveling wave solutions, which form an important class of solutions occurring in various problems in the natural sciences.

By phase plane arguments we show the existence of traveling wave fronts in both models of interest, and we specify the solutions in terms of the wave profile and occurring wave speeds.

Furthermore, we study the wave fronts with respect to their stability in  $L^2$  as well as in exponentially weighted  $L^2$  spaces. We deduce linear stability results from the spectral analysis of differential operators involved in the problem, where their Fredholm properties and specific asymptotic properties are of particular importance.