Extreme events as a multi-point feature
-
Entropy production as a criterion for cascade process

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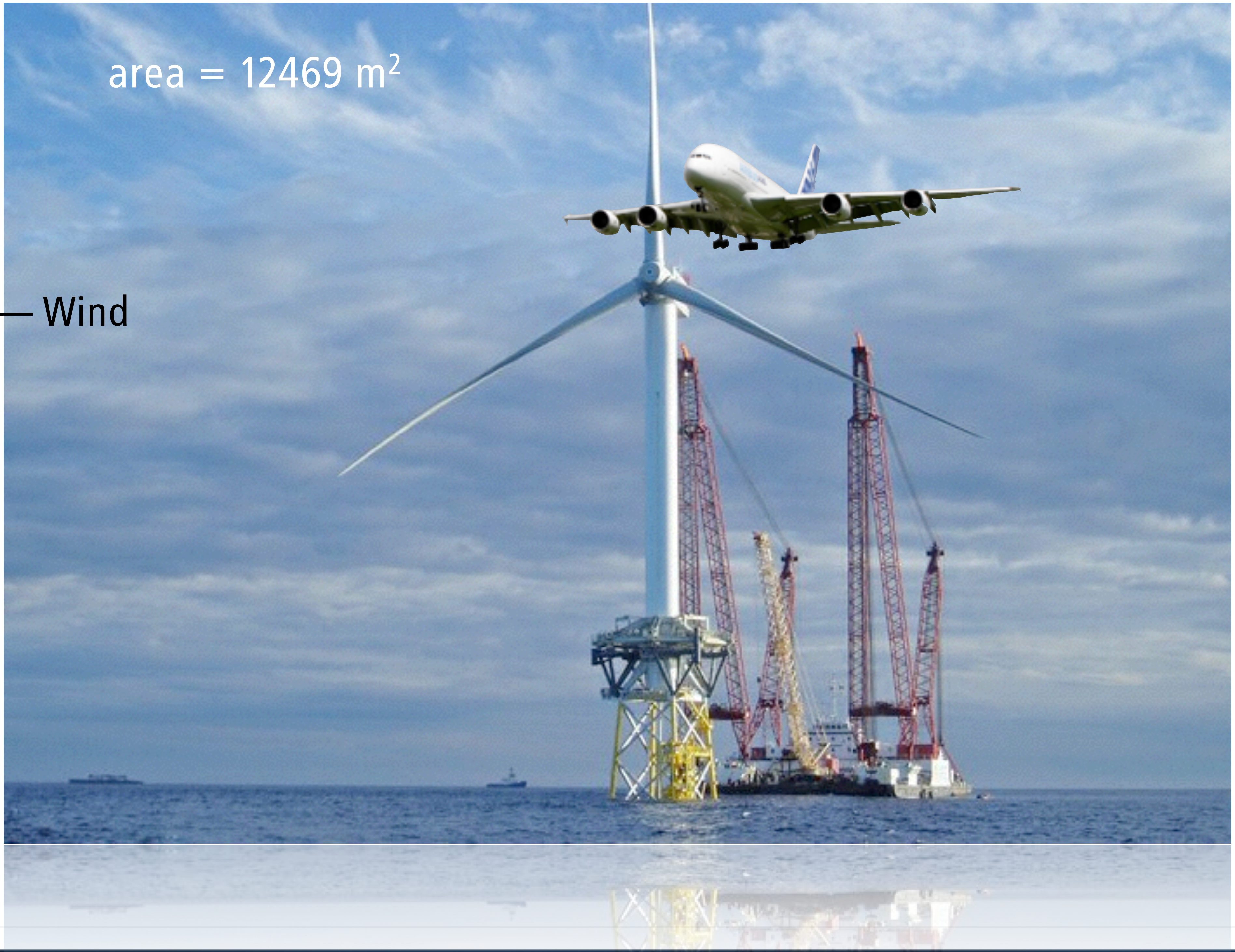
- motivation
  — wind energy
  — rogue waves
  — financial data
- n-point statistics of turbulence
- nonequilibrium thermodynamics and extrem

WEC >5MW

- energy resource — Wind



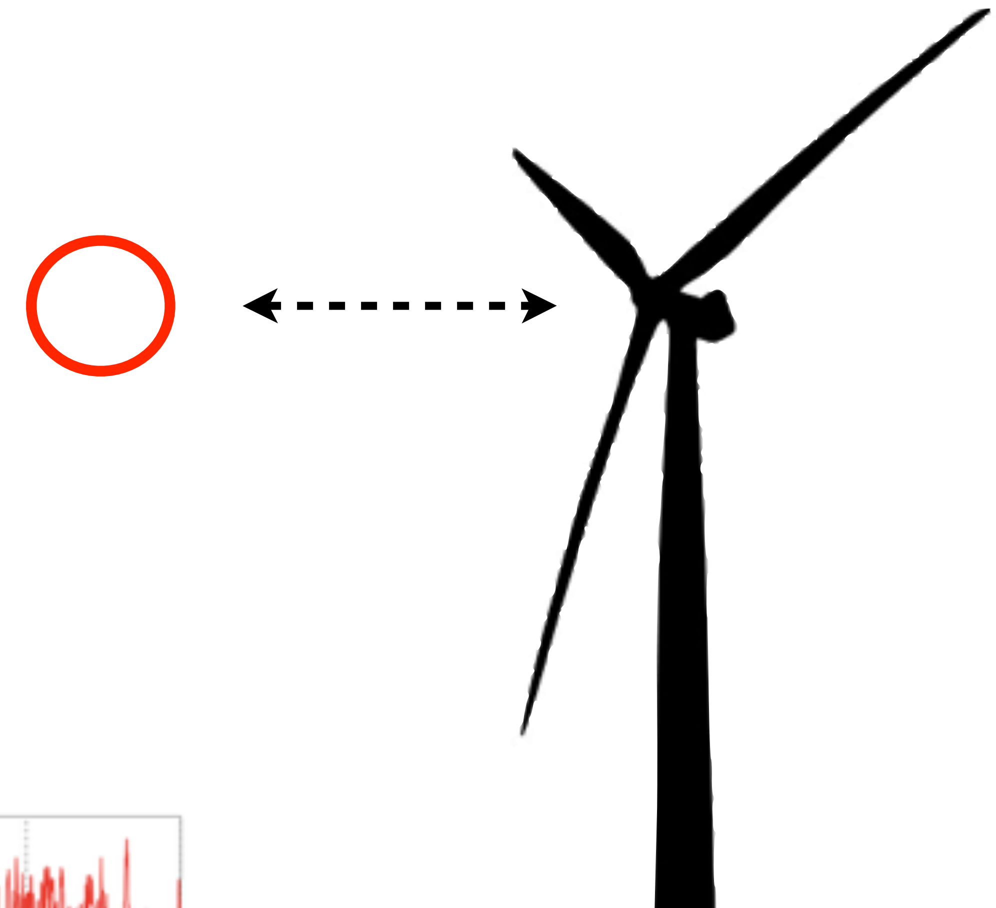
wind measurements and data analysis

INTERNATIONAL STANDARD

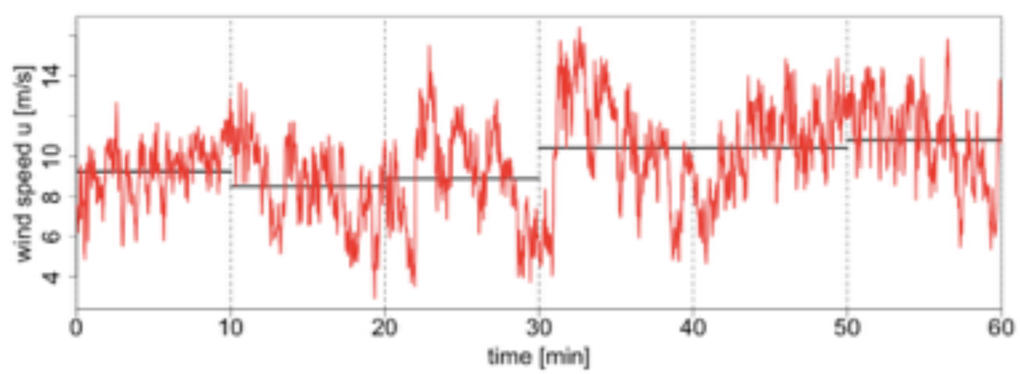
IEC 61400-1

Third edition 2005-08

- wind conditions after IEC
  - measurement at hub height in front of a turbine



measured time series



V_{ref}

- reference wind speed

Wind turbine class		I	II	III	S
V_{ref}	(m/s)	50	42,5	37,5	Values
A	I_{ref} (-)	0,16			specified
B	I_{ref} (-)	0,14			by the
C	I_{ref} (-)	0,12			designer

INTERNATIONAL STANDARD

IEC 61400-1

Third edition 2005-08

- annual mean wind speed

$$V_{ave} = 0{,}2\, V_{ref}$$

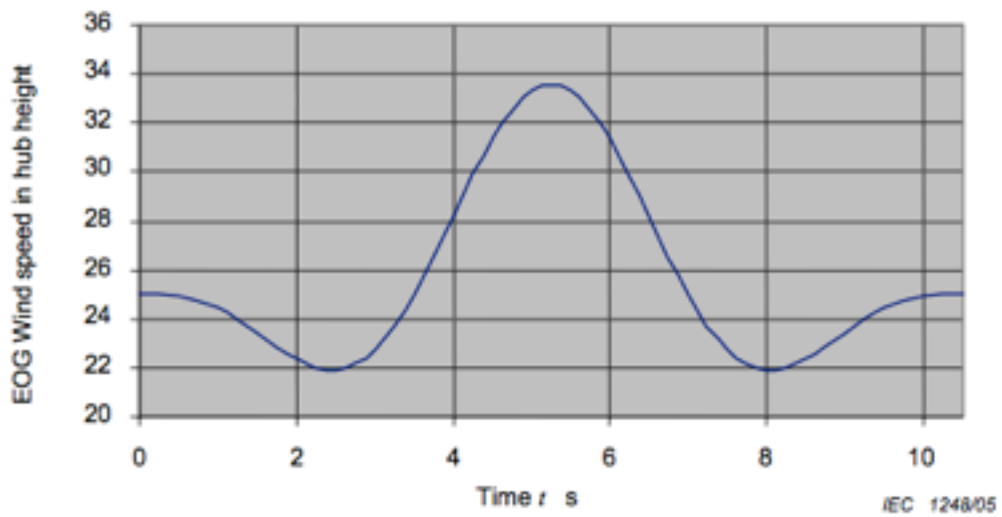
$$P_R(V_{hub}) = 1 - \exp\left[-\pi\left(V_{hub} / 2V_{ave}\right)^2\right]$$

- profile

$$V(z) = V_{hub}\left(z / z_{hub}\right)^{\alpha}$$

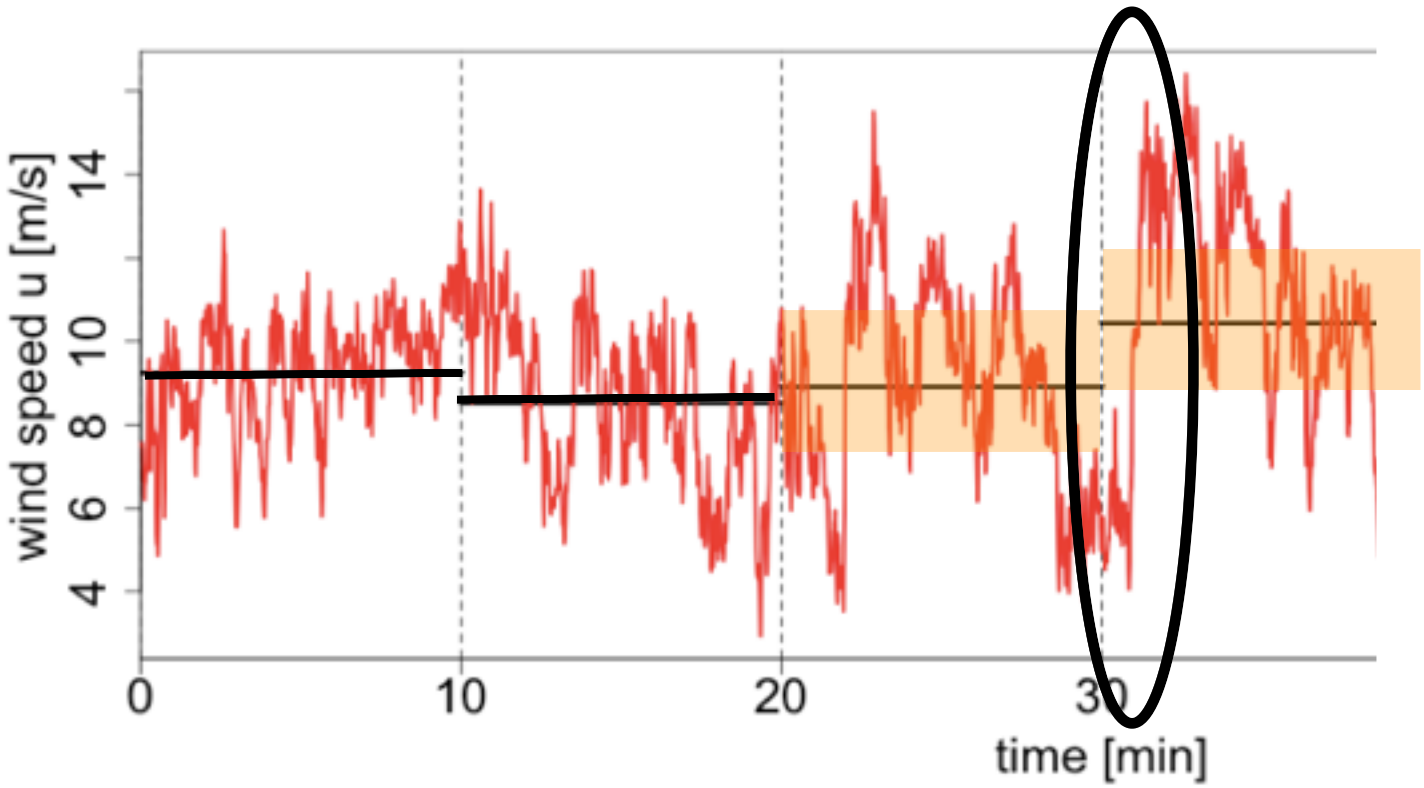
- extreme events -gusts

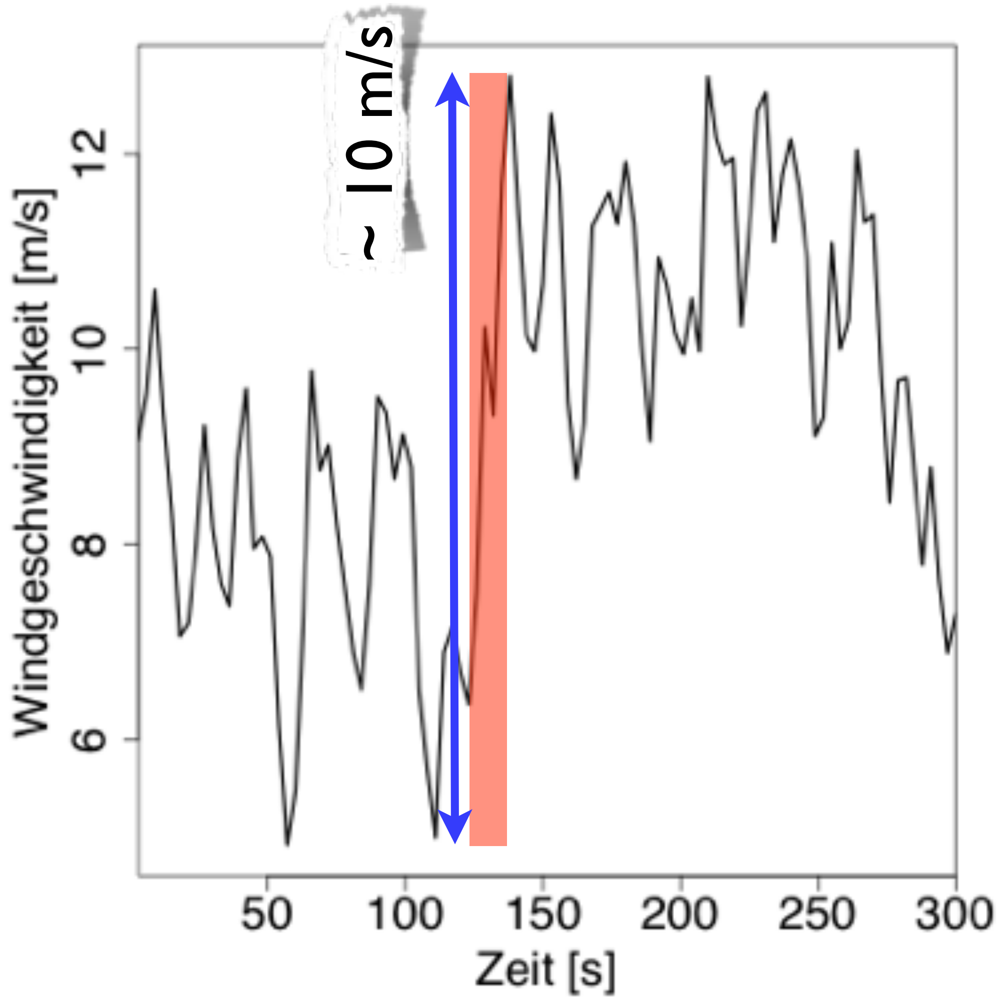
$$V(z,t) = \begin{cases} V(z) - 0{,}37\, V_{gust} \sin(3\pi t / T)\left(1 - \cos(2\pi t / T)\right) \\ V(z) \end{cases}$$



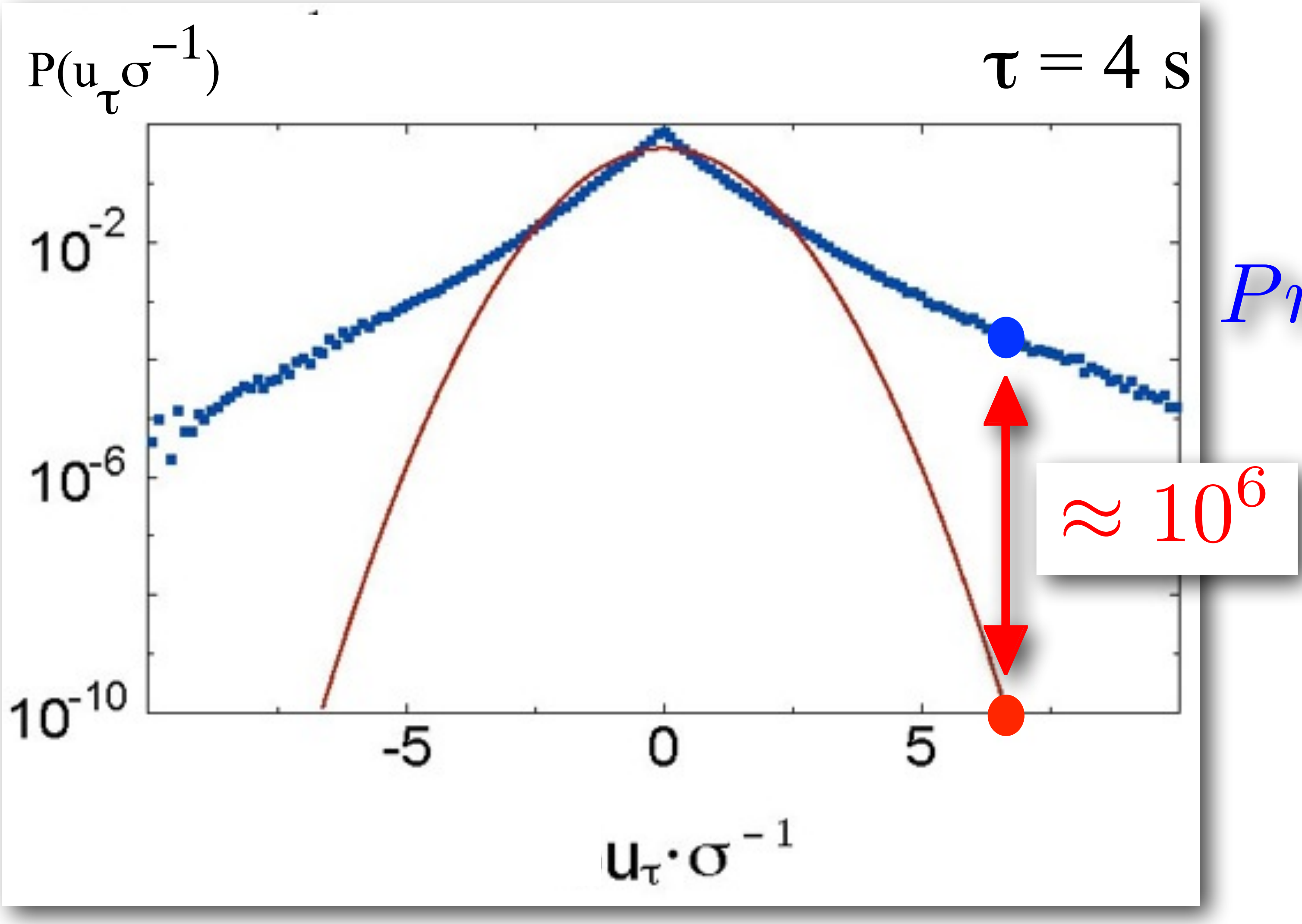
wind measurements and data analysis

- characterization after IEC norm - or what is a gust??





statistics of gusts



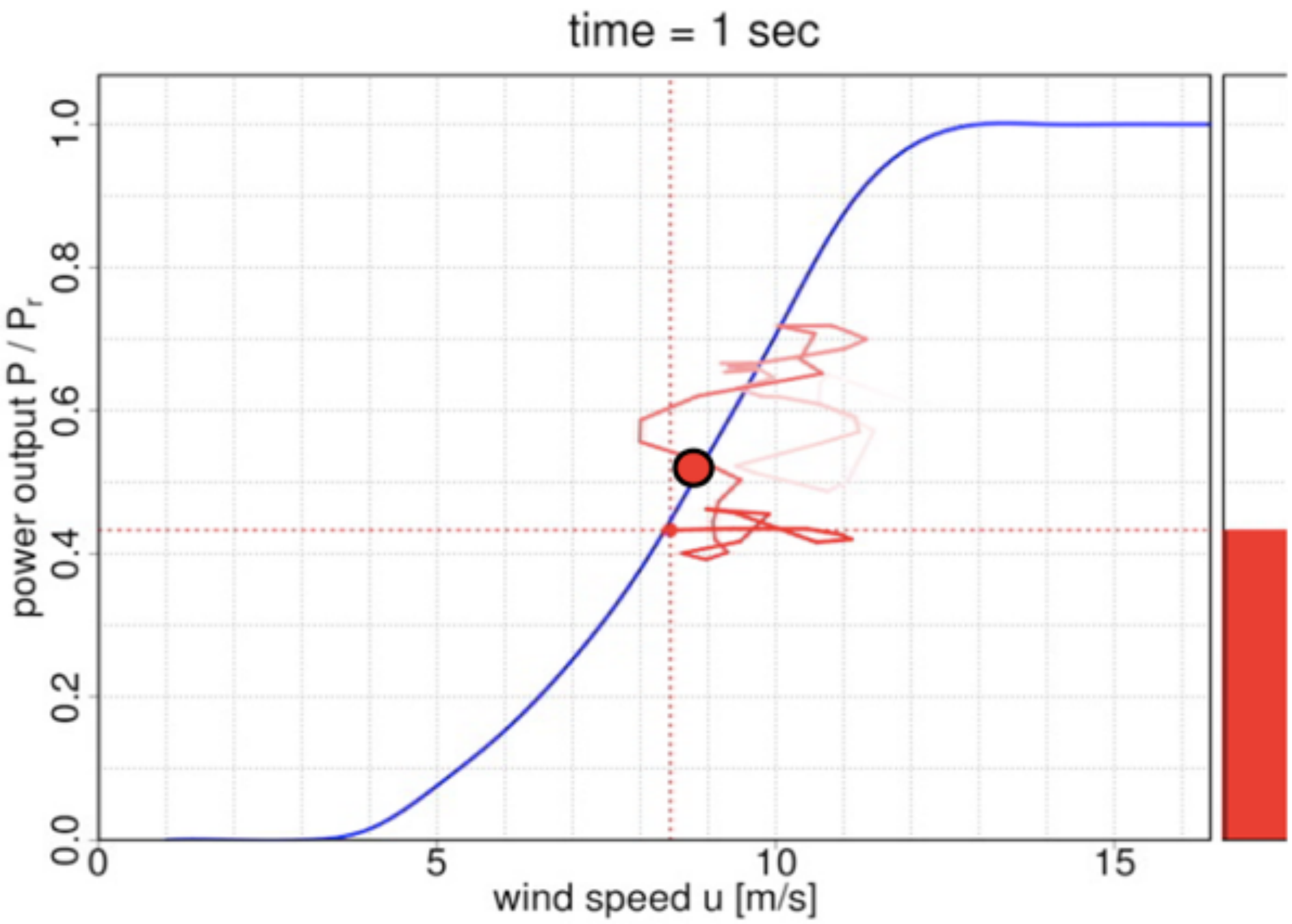
$Prob(u_\tau > 6\sigma) \approx 10^{-4}$ → 1/day

Boundary-Layer Meteorology **108** (2003)

$Prob(u_\tau > 6\sigma) \approx 10^{-10}$ → 1/3000 years

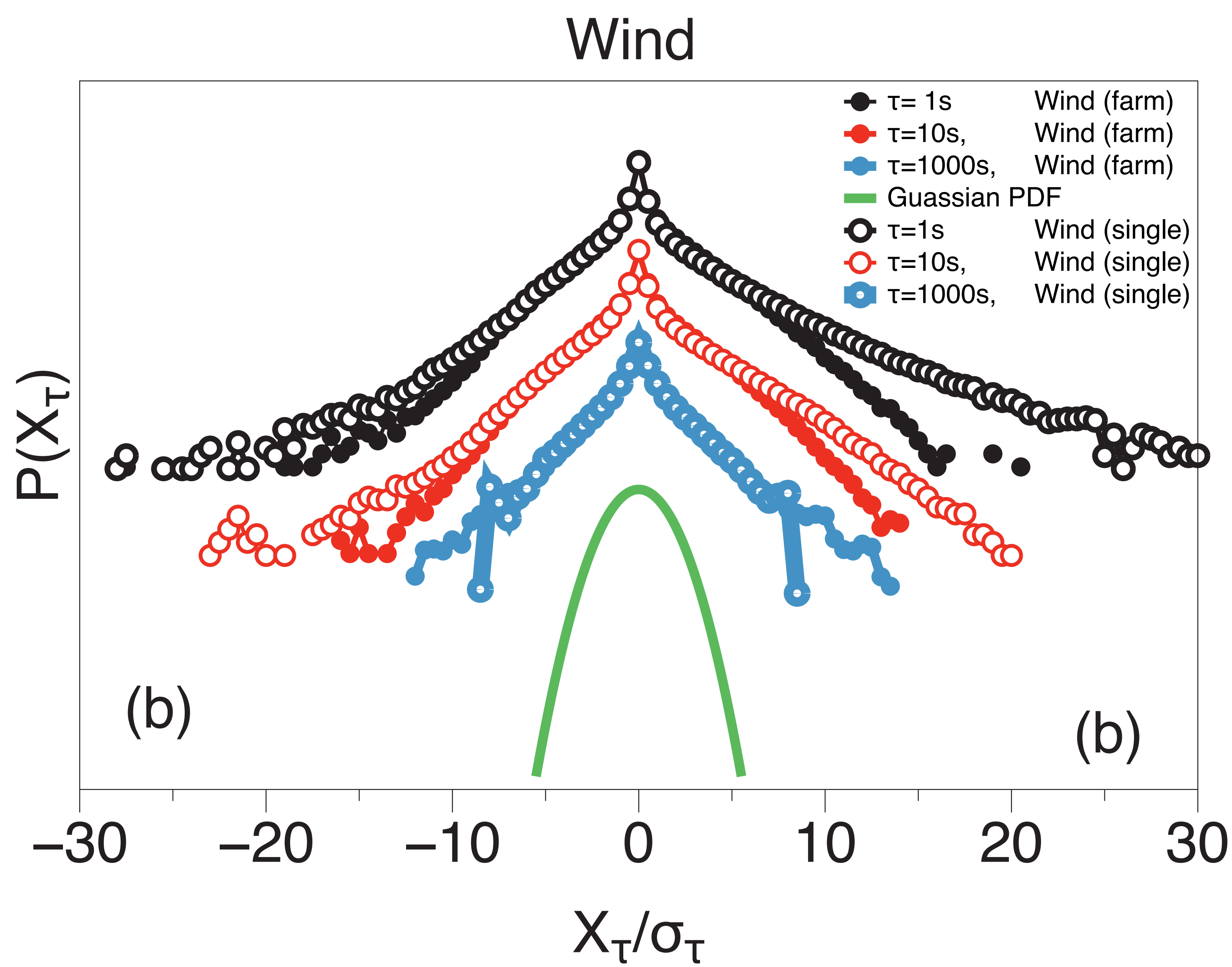
dynamics of power conversion

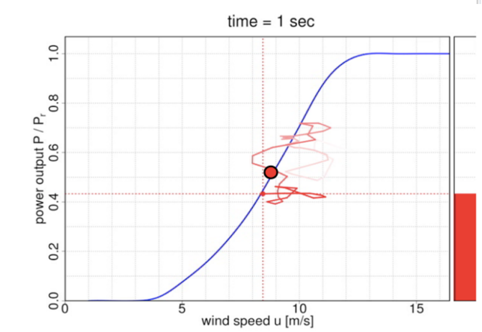
$$P_{WT} = \frac{1}{2} c_p(\lambda)\ \rho\ u_{wind}^3 \cdot A$$



increment statistics of power fluctuations

highly intermittent and turbulent power dynamics from wind turbines and wind farms





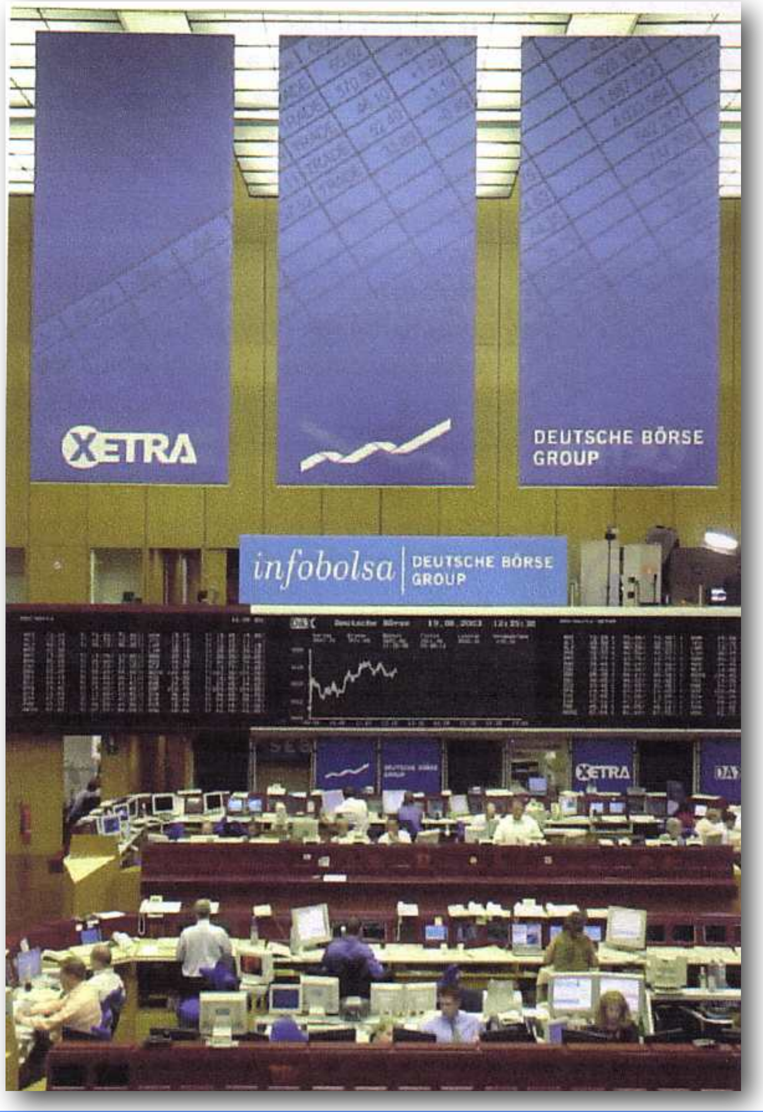
P. MIlan et.al.PRL 110, 138701 (2013)

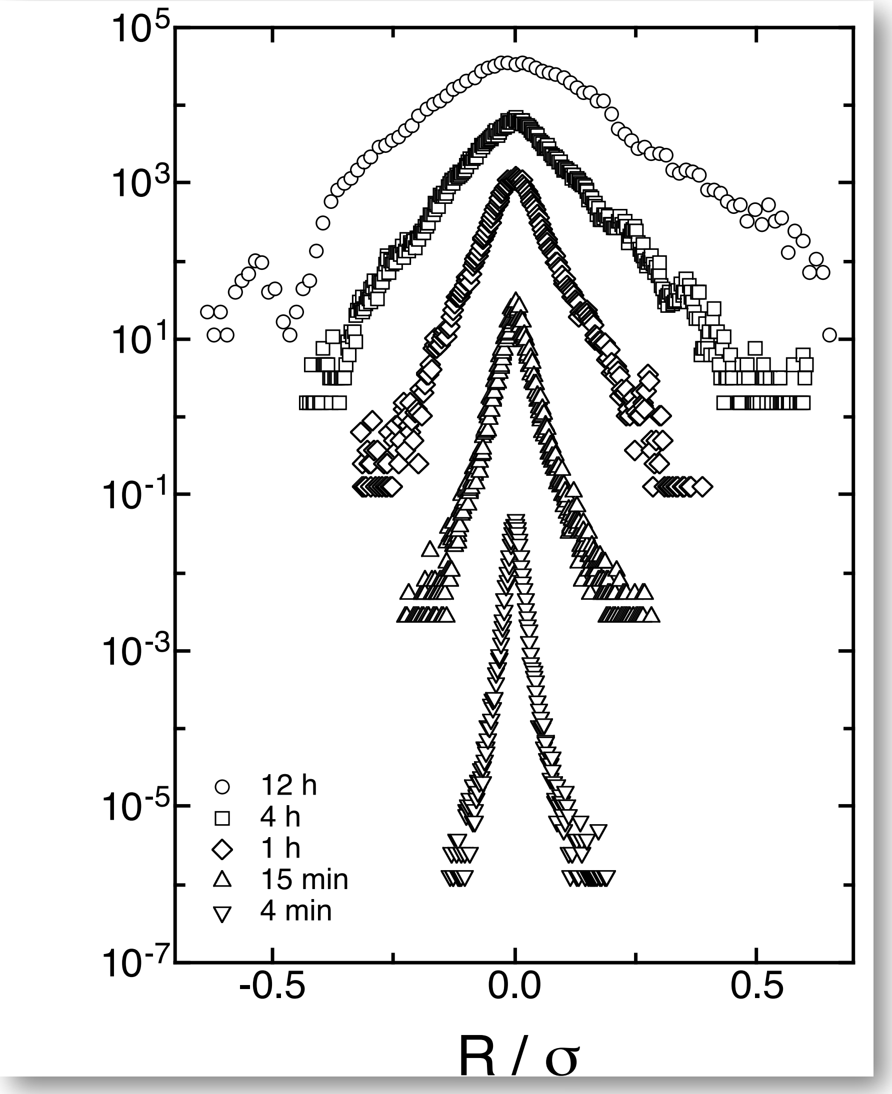
finance

scale dependent quantity for measuring the disorder

$$Q(x,r) \Rightarrow r(t,\tau) = \frac{x(t+\tau)}{x(t)} \quad \text{or } R(t,\tau) = \log r(t,\tau)$$

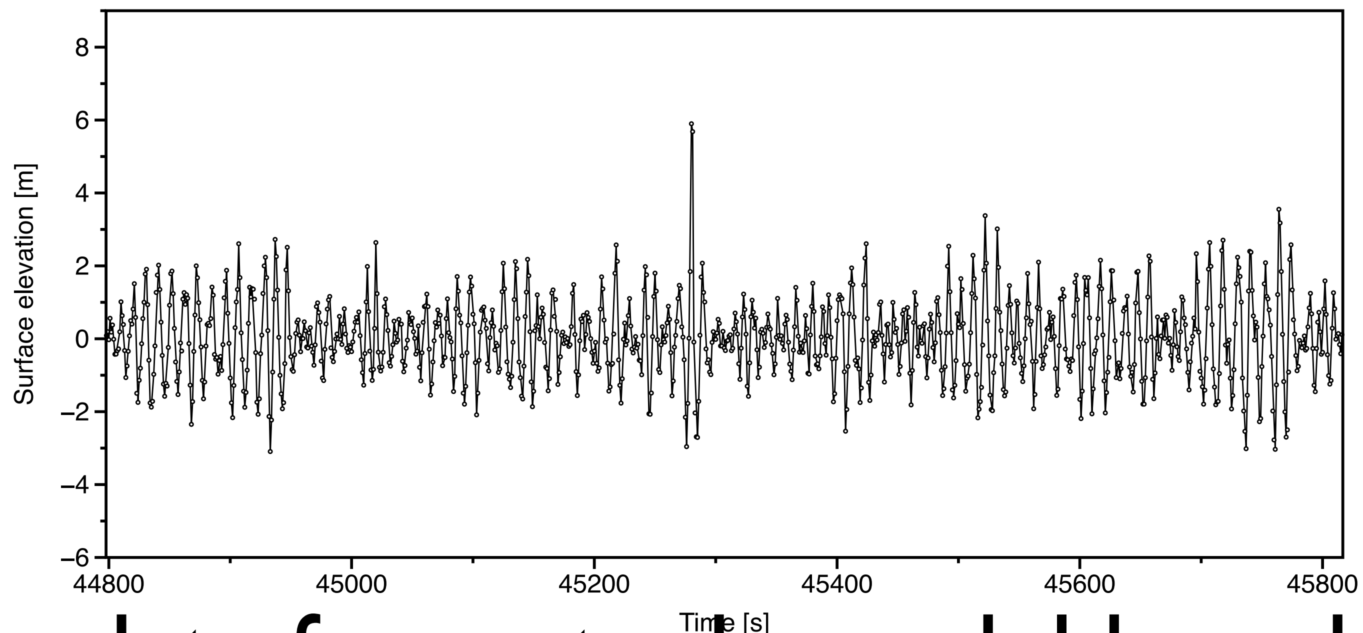
return or log return for different time scales



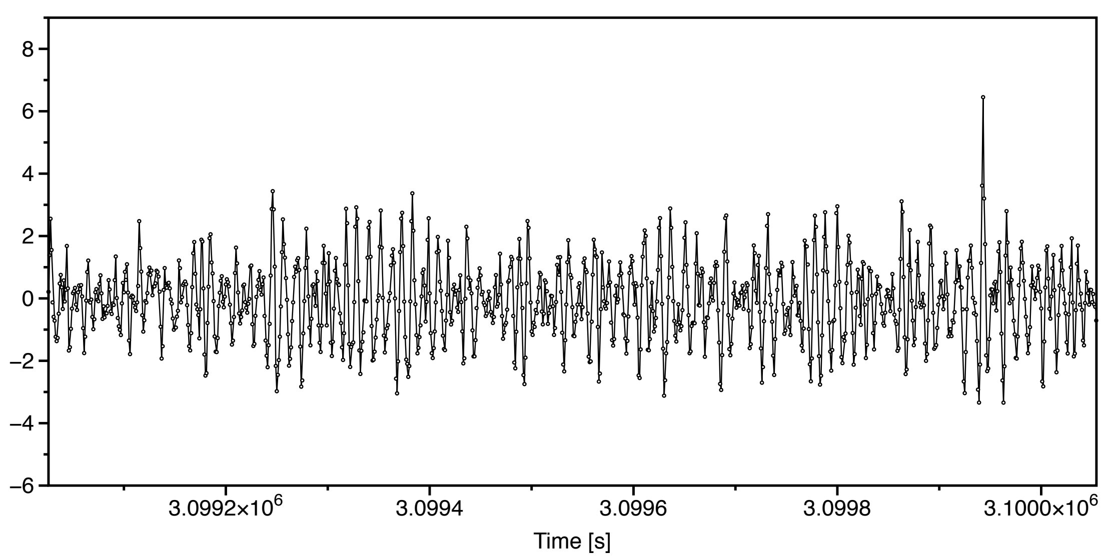


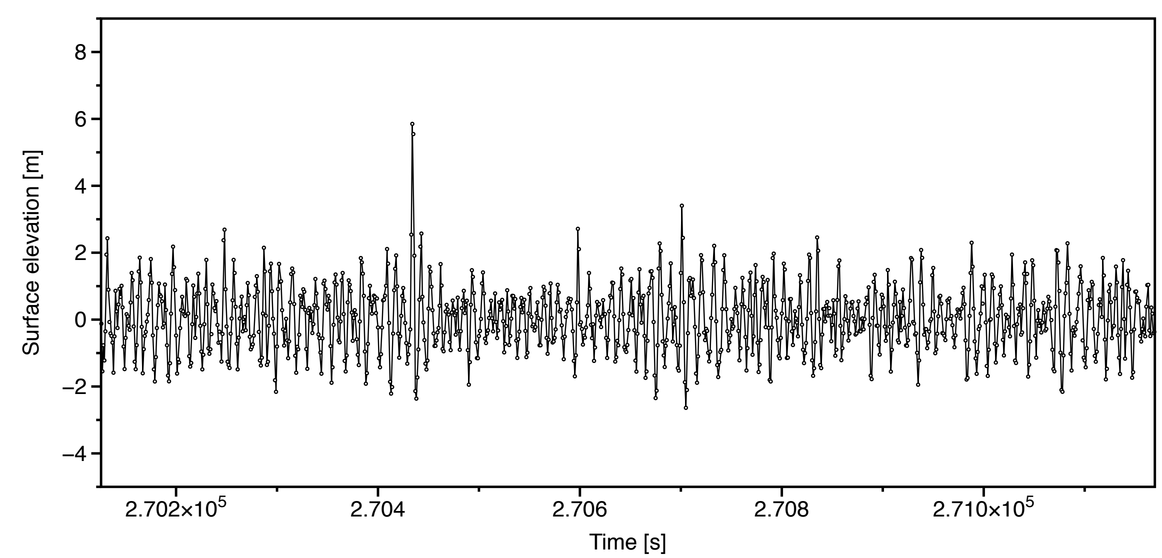
rogue waves

measurement Japan



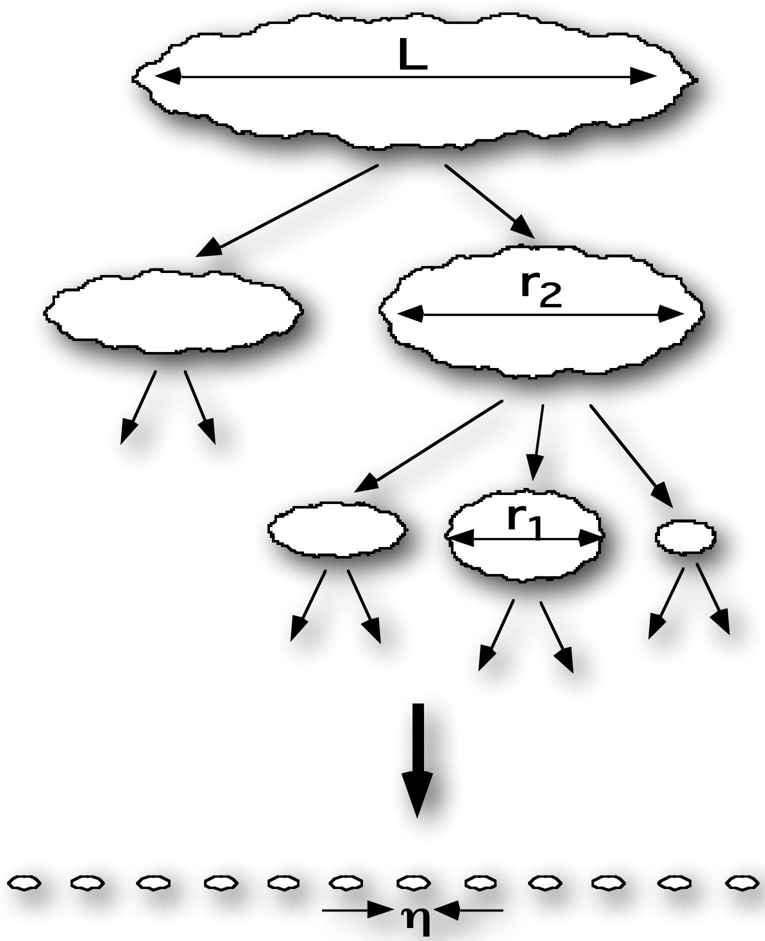
data from stoch. model based in multi point statistics





turbulent cascade

turbulent cascade: - large vortices are generating smaller

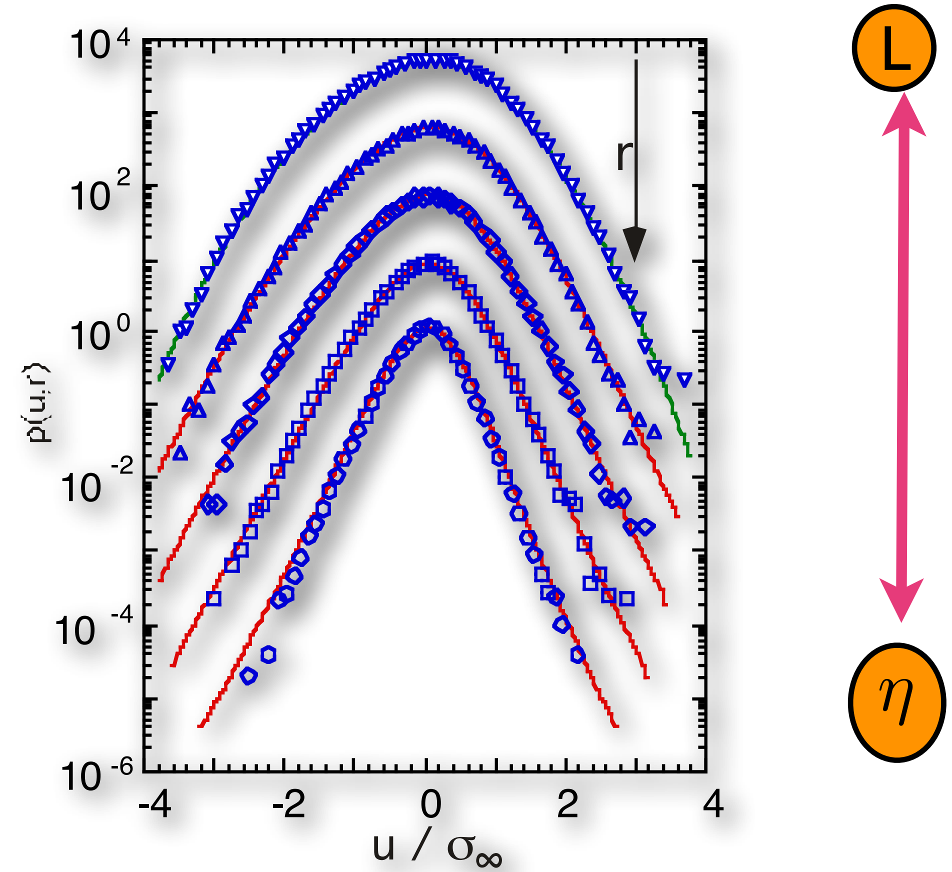


multi scale analysis

$$\mathbf{u_r} := \mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})$$

$$< u_r^n > \quad \propto \quad r^{\xi_n}$$

$$< u_r^n > = \int u_r^n \, p(u_r) \, du_r$$

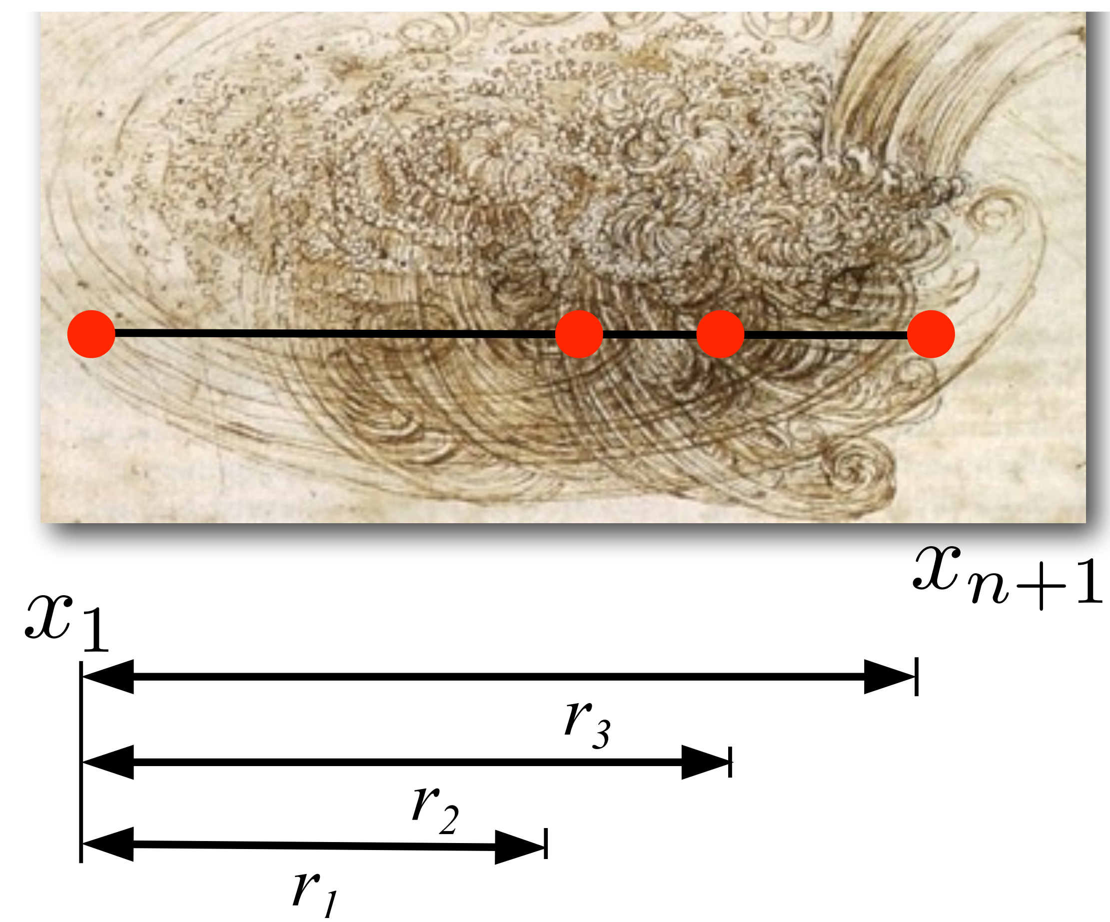


n-point statistics

n- point statistics expressed by increment statistics

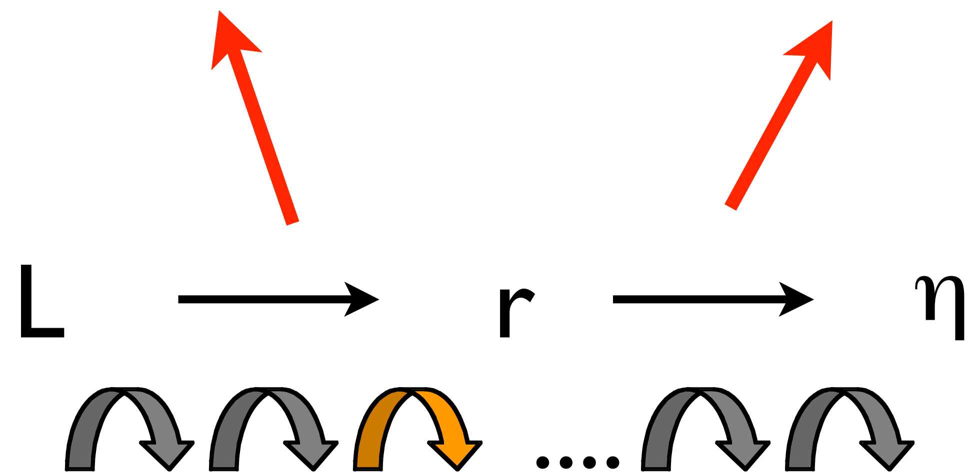
$$p(u(x_1), ..., u(x_{n+1})) \quad = \quad p(u_{r_1}, ..., u_{r_n}, u(x_1))$$

$$u_{r_i} = u(x + r_i) - u(x_i)$$



n-point statistics

$$p(u(x_1), ..., u(x_{n+1}))$$
$$= p(u_{r_1}|u_{r_2}, u(x_1)) \ldots p(u_{r_{n-1}}|u_{r_n}, u(x_1)) \cdot p(u(x_1))$$



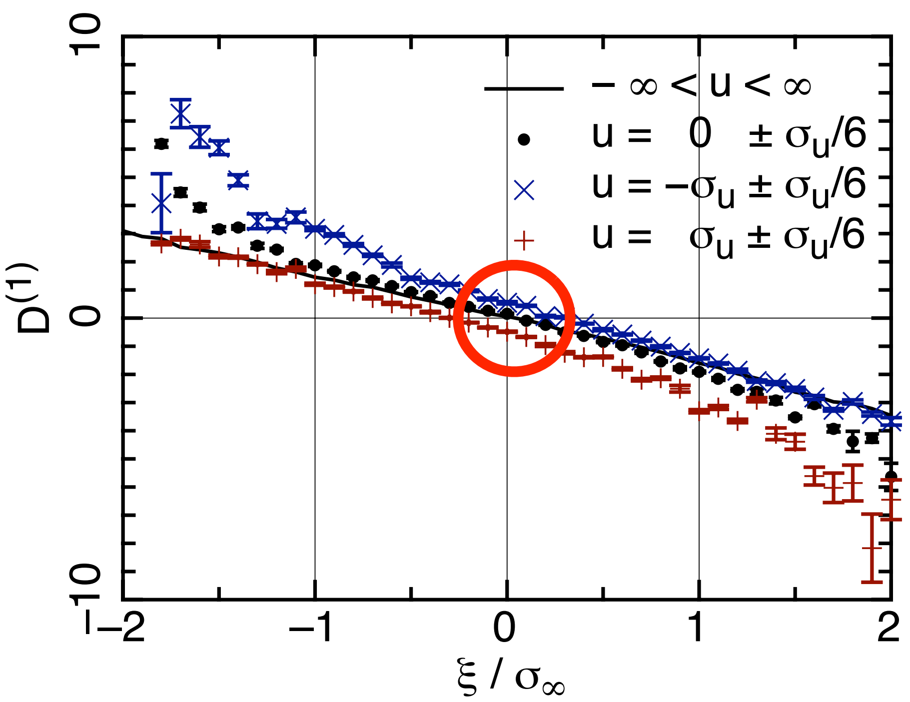
Markow prop & cascade with Fokker-Planck Equation

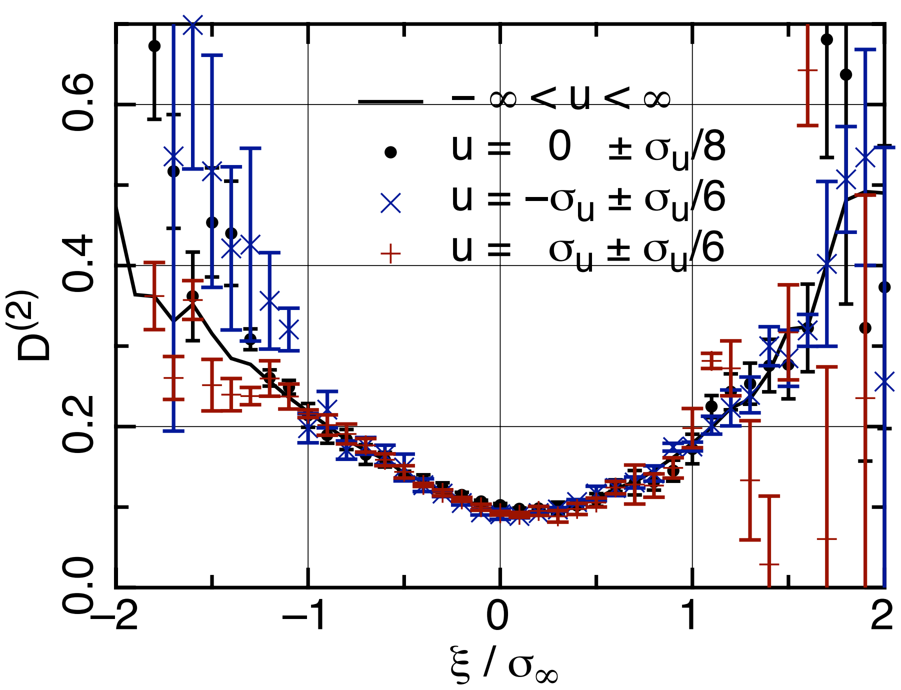
$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial \xi_j^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\}\ p(u_{r_j}|u_{r_k}, u(x_1))$$

n-point statistics

Markow prop & cascade with Fokker-Planck Equ

$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j}|u_{r_k}, u(x_1)) = \{-\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial \xi_j^2} D^{(2)}(u_{r_j}, r_j, u(x_1))\} \, p(u_{r_j}|u_{r_k}, u(x_1))$$

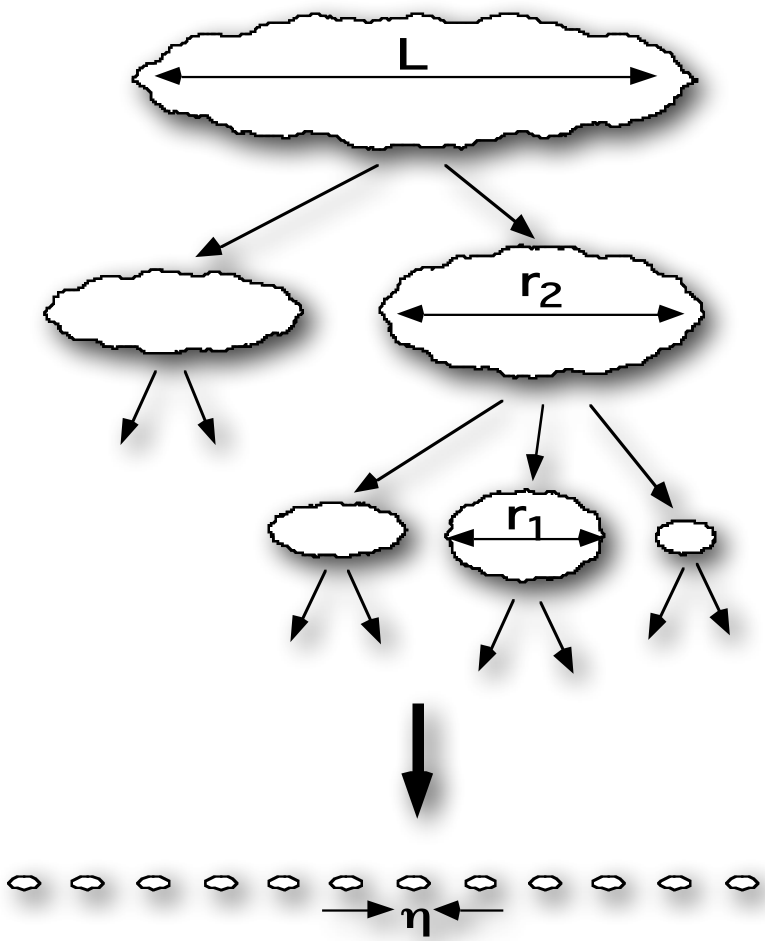




shift of drift function, no *u*-dependence of diffusion function

Stresing et.al. New Journal of Physics **12** (2010)

how are the scaling and the process approaches connected



multi scale analysis

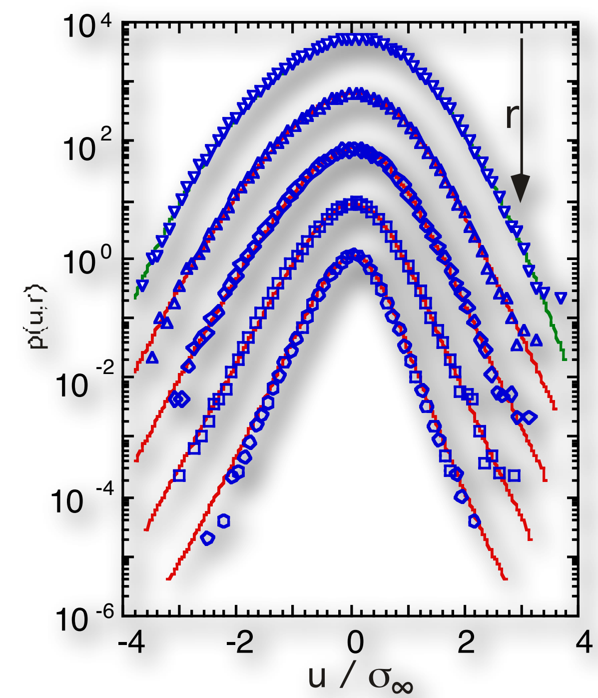
$$\mathbf{u_r} := \mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})$$

$$< u_r^n > \quad \propto \quad r^{\xi_n}$$

stochastic cascade process evolving in r - multi point

$$\partial_r u_r$$

$$\partial_r p_r(u_r)$$



cascade process as stochastic process -2-

Model: multifractal models [Frisch et al., 1978, 1995]

[Nickelsen, 2014]

$$\langle u(r)^n \rangle = c_n^{(1)} r^{\zeta_1} + c_n^{(2)} r^{\zeta_2} + c_n^{(3)} r^{\zeta_3} + \dots$$

stochastic process - stationary

$$D^{(1)}(u,r) = -\sum_{k=0}^{\infty} \frac{a_k}{r} u^k \quad , \quad D^{(2)}(u,r) = \sum_{k=0}^{\infty} \frac{b_k}{r} u^k$$

Model: log-Poisson random cascade model [She-Leveque, 1994]

[Nickelsen, 2014]

$$\langle u(r)^n \rangle = c_n r^{\zeta_n} \, , \quad \zeta_n = \frac{n}{9} + 2\left[1 - \left(\frac{2}{3}\right)^{\frac{n}{3}}\right]$$

stochastic process - jump process

$$D^{(1)}(u,r) = \left[\left(\frac{16}{3}\right)^{\frac{1}{3}} - \frac{19}{9}\right] \frac{1}{r} u \quad , \quad D^{(k)}(u,r) = \frac{1}{k!}\left[\left(\frac{16}{3}\right)^{\frac{1}{3}} - 2\right]^k \frac{1}{r} u^k$$

Model: field theoretic approach [Yakhot, 1998]

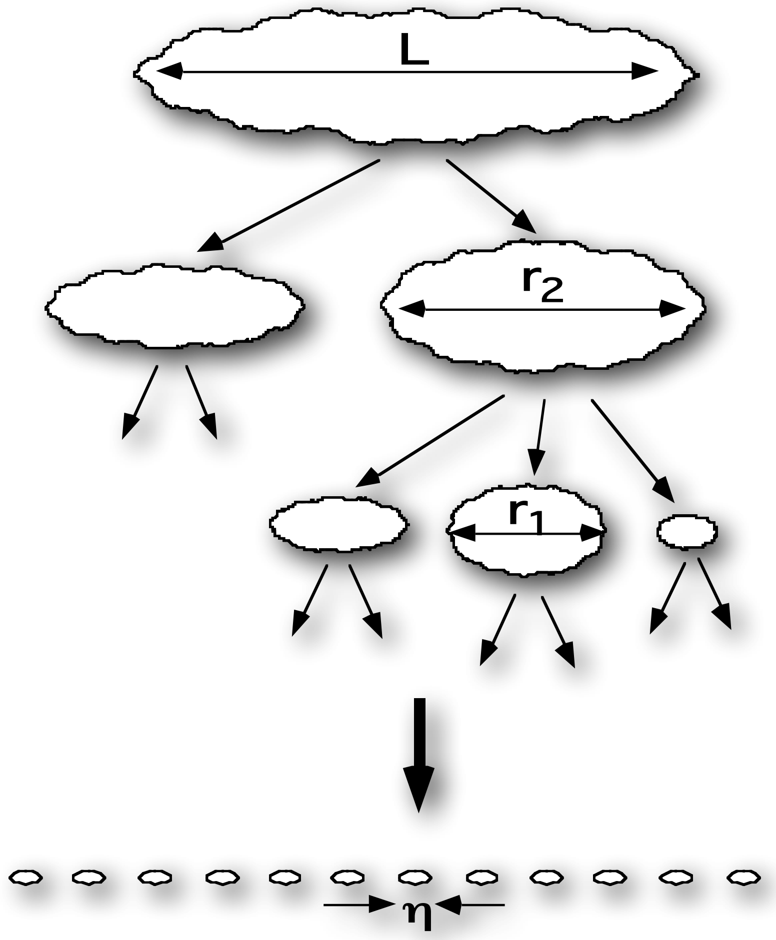
PDE for $p(u,r)$ $\Rightarrow$ $\zeta_n = \frac{n}{3}\frac{B+3}{B+n}$

[Davoudi-Tabar, 1999]

stochastic process - jump process

$$D^{(1)}(u,r) = -\frac{a_1}{r} u \quad , \quad D^{(k)}(u,r) = \frac{b_2^{(k)}}{r} u^k$$

thermodynamics of cascade



- new approach to non- equilibrium thermodynamics by Seifert PRL 95 (2005)
- the evolution along cascade as thermodynamical process
- entropy production along a single trajectory
- trajectory obeys an integral fluctuation theorem - Jarzynski relation PRL 78 (1997)

thermodynamics of cascade

1st law : energy balance of a single trajectory

$$\Delta U = W(u_r) - Q(u_r)$$

ΔU equilibrium energy difference

$W(u_r)$ work done on trajectory

$Q(u_r)$ heat transferred to system

entropy production for different paths u_r (Seifert 2005)

$$S(u_r) = \frac{Q(u_r)}{k_B T}$$

thermodynamics of cascade : entropy balance

2nd law : entropy balance

entropy production in the system (Seifert 2005)

contribution of fluctuations relaxing to the steady state

$$S_m(u_r) = -\int_L^\lambda \partial_r u_r \; \partial_u \varphi(u_r, r) dr$$

the potential is the stationary solution of the Fokker-Planck equ.

$$\varphi(u_r) = \ln D^{(2)}(u_r, r) - \int_{-\infty}^{u_r} \frac{D^{(1)}(u', r)}{D^{(2)}(u', r)} du'$$

entropy production along trajectory

$$\Delta S = -ln \frac{p_r(u_r)}{p_{r_0}(u_0)}$$

contribution of the path along the steady state

thermodynamics of cascade : entropy balance

2nd law : entropy balance

entropy production in the system (Seifert 2005)

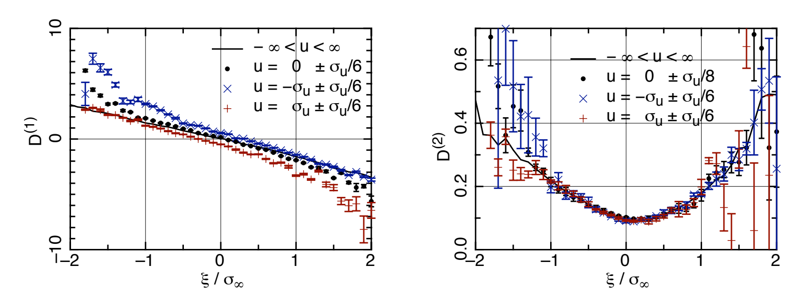
$$S_m(u_r) = -\int_L^\lambda \partial_r u_r \, \partial_u \varphi(u_r, r) dr$$

entropy production along trajectory

$$\Delta S = -\ln \frac{p(u_L, L)}{p(u_\lambda, \lambda)}$$

stoch process

$$D^{(1)}(u,r) = \left[\left(\frac{16}{3}\right)^{\frac{1}{3}} - \frac{19}{9}\right] \frac{1}{r} u \quad , D^{(k)}(u,r) = \frac{1}{k!}\left[\left(\frac{16}{3}\right)^{\frac{1}{3}} - 2\right]^k \frac{1}{r} u^k$$



shift of drift function, no *u*-dependence of diffusion function

thermodynamics of cascade : entropy balance

2nd law : entropy balance

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entropy production along trajectory

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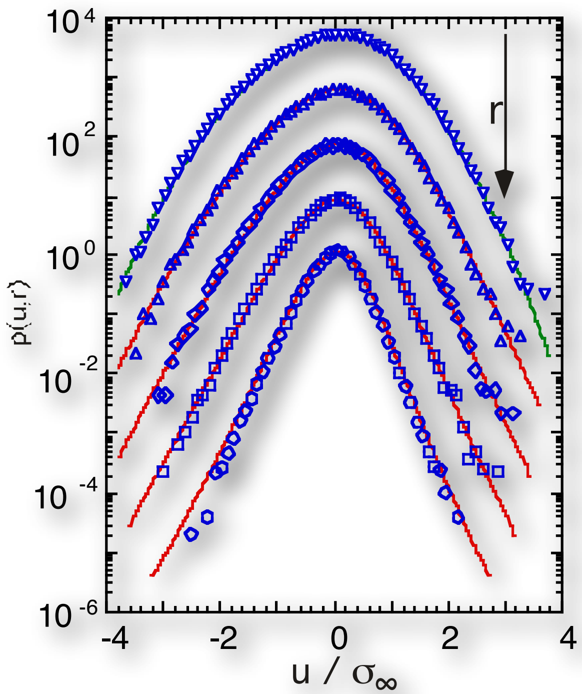
stoch process

experimental quantities



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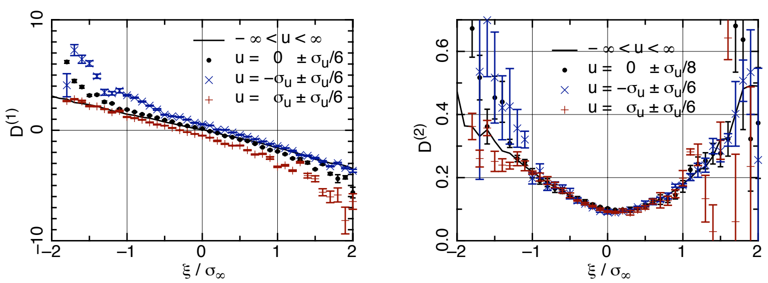


shift of drift function, no *u*-dependence of diffusion function

thermodynamics of cascade : entropy balance

2nd law : entropy balance

entropy production in the system (Seifert 2005)

$$S_m(u_r) = -\int_L^\lambda \partial_r u_r \, \partial_u \varphi(u_r, r) dr$$

entropy production along trajectory

$$\Delta S = -\ln \frac{p(u_L, L)}{p(u_\lambda, \lambda)}$$

stoch process

experimental quantities

total entropy production

$$S_{tot}(u_r) = S_m(u_r) + \Delta S$$

$$\langle S_{tot}(u_r) \rangle \geq 0$$

2nd law

$$\langle e^{-S_{tot}(u_r)} \rangle = 1$$

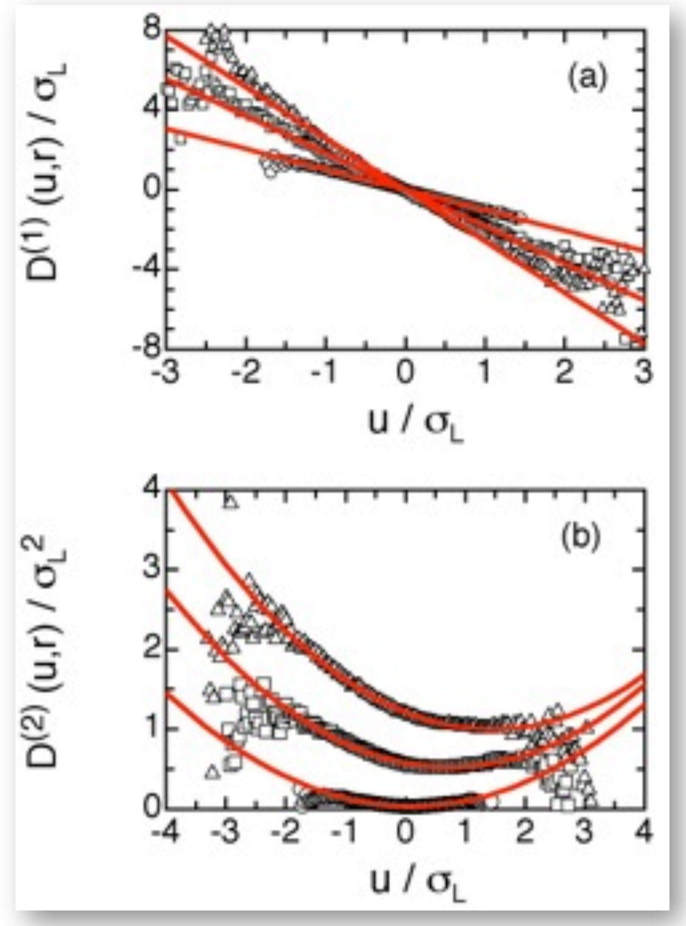
integral fluctuation theorem
Seifert (2005)

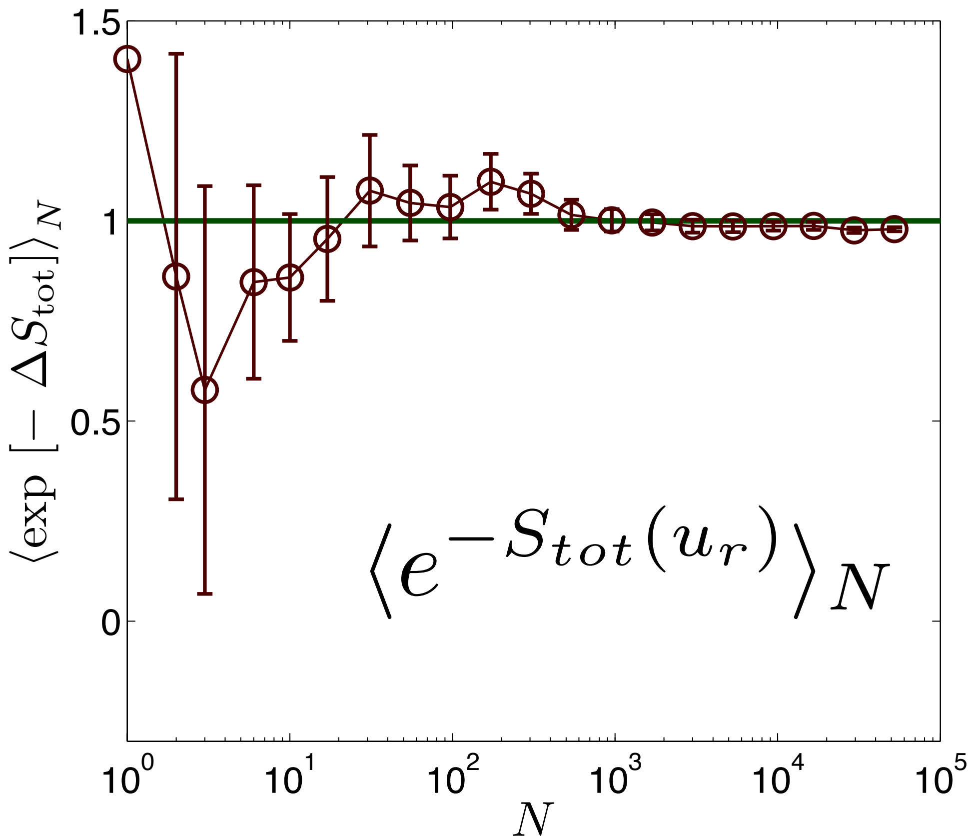
Renner et al JFM (2001)

test of validity of the fluctuation theorem

$$D^{(1)}(u,r) = -a_0 r^{0.6} - a_1 r^{-0.67} u + a_2 u^2 - a_3 r^{0.3} u^3$$
$$D^{(2)}(u,r) = b_0 r^{0.25} - b_1 r^{0.2} u + b_2 r^{-0.73} u^2$$

$$a_0 = 0.0015\,,\quad a_1 = 0.61\,,\quad a_2 = 0.0096\,,\quad a_3 = 0.0023\,,$$
$$b_0 = 0.033\,,\quad b_1 = 0.009\,,\quad b_2 = 0.043\,.$$





Nickelsen Engel PRL (2013)

results for turbulent cascade:

- 1st fulfillment of fluctuation theorem

$$\langle e^{-S_{tot}(u_r)} \rangle = 1$$

“This integral fluctuation theorem - a generalized Jarzynski’s relation -

is truly universal since it holds for

- any kind of initial condition,

- any time dependence of force and potential, with and without detailed balance,

- any length of trajectory without the need for waiting for relaxation.”

Seifert (2005)

- 1st fulfillment of fluctuation theorem

$$\langle e^{-S_{tot}(u_r)} \rangle = 1$$

- 2nd test of the validity of turbulence models

$$\Delta S = -\ln \frac{p(u_L, L)}{p(u_\lambda, \lambda)} \qquad S_m(u_r) = -\int_L^\lambda \partial_r u_r \, \partial_u \varphi(u_r, r) dr$$

=> IFT holds only for "non - scaling" stochastic processes

summary - nonequi. thermodynamics

generalized 2nd law - (entropy maximization) fulfilled for cascade

- non scaling process fits to data

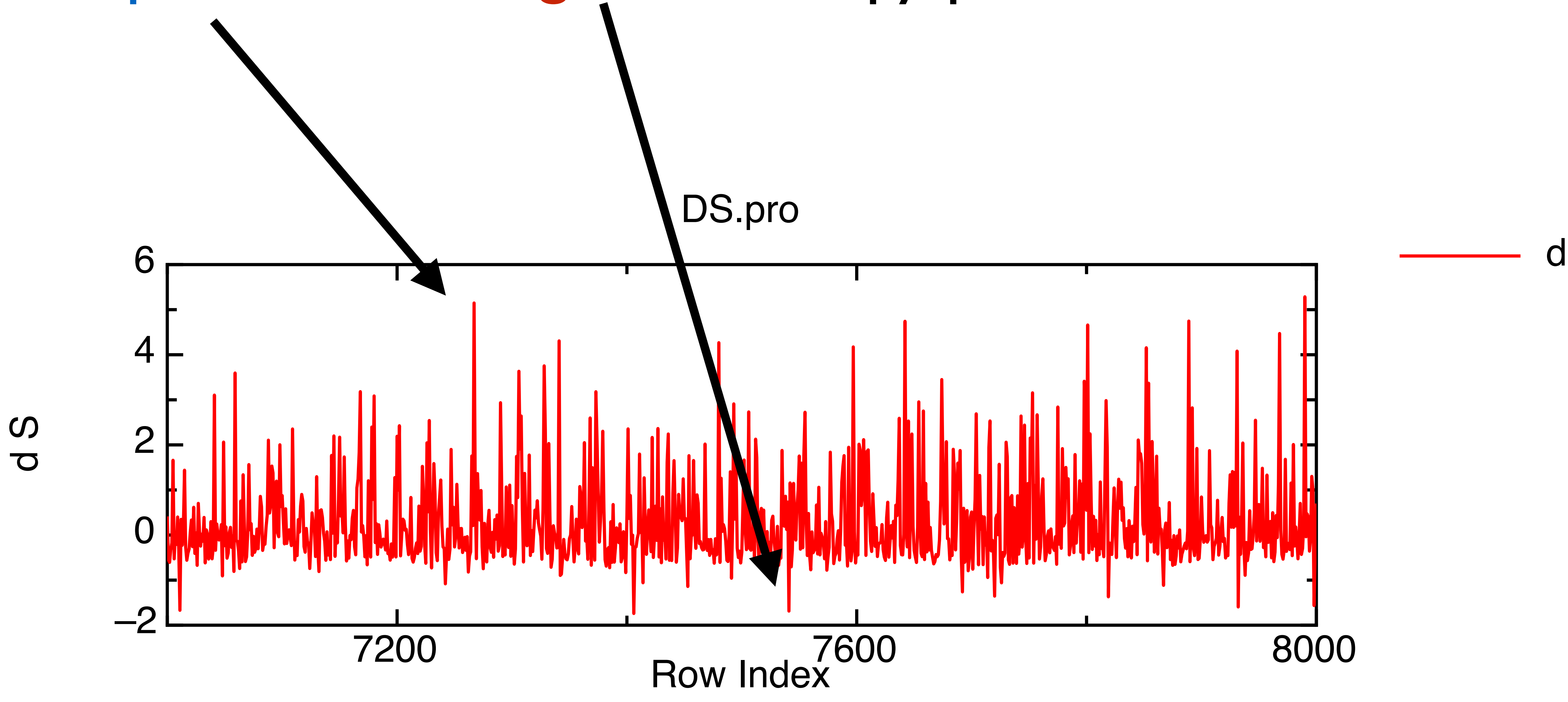
(?? => new window for blow up??>

extreme events and its entropy production $\Delta S_{tot}(u_r)$

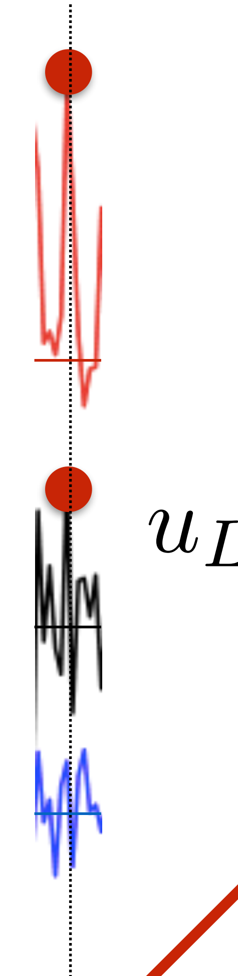
to fulfill the IFT - there must be paths with positive and negative entropy production

extreme events and its entropy production $\Delta S_{tot}(u(r))$

to fulfill the IFT - there must be paths with positive and negative entropy production



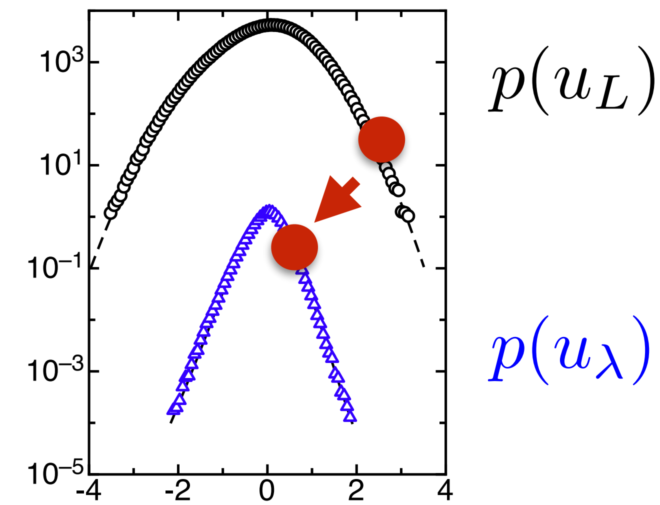
1.) positive entropy production $\Delta S_{tot}(u_r) > 0$

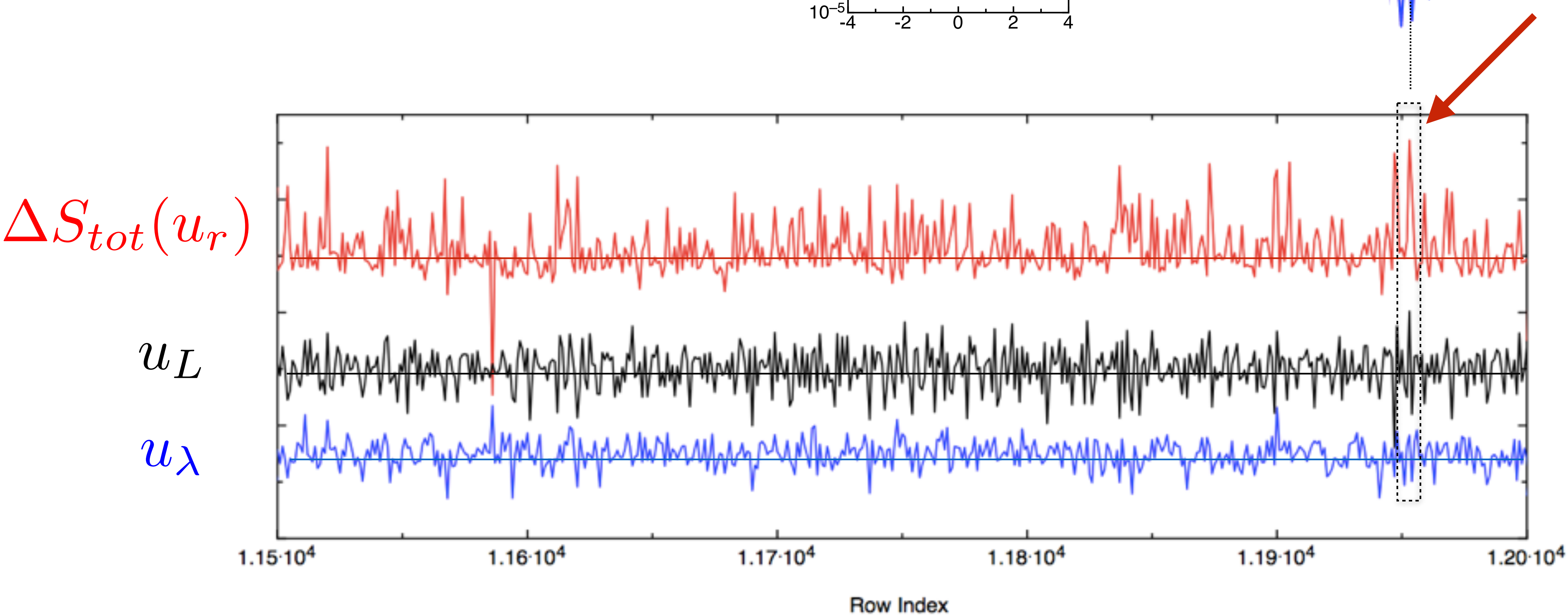


event:

large u_L with low probability

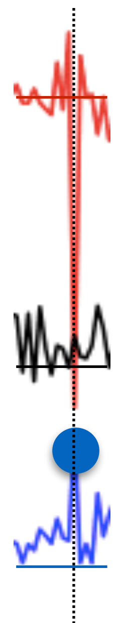
small u_λ with high probability





2.) extreme and negative entropy production $\Delta S_{tot}(u_r) < 0$

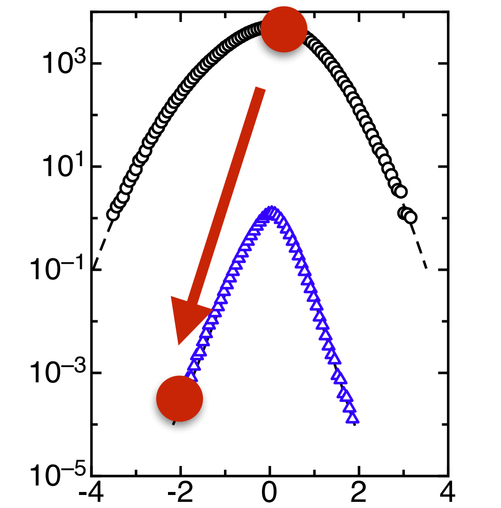
extreme event

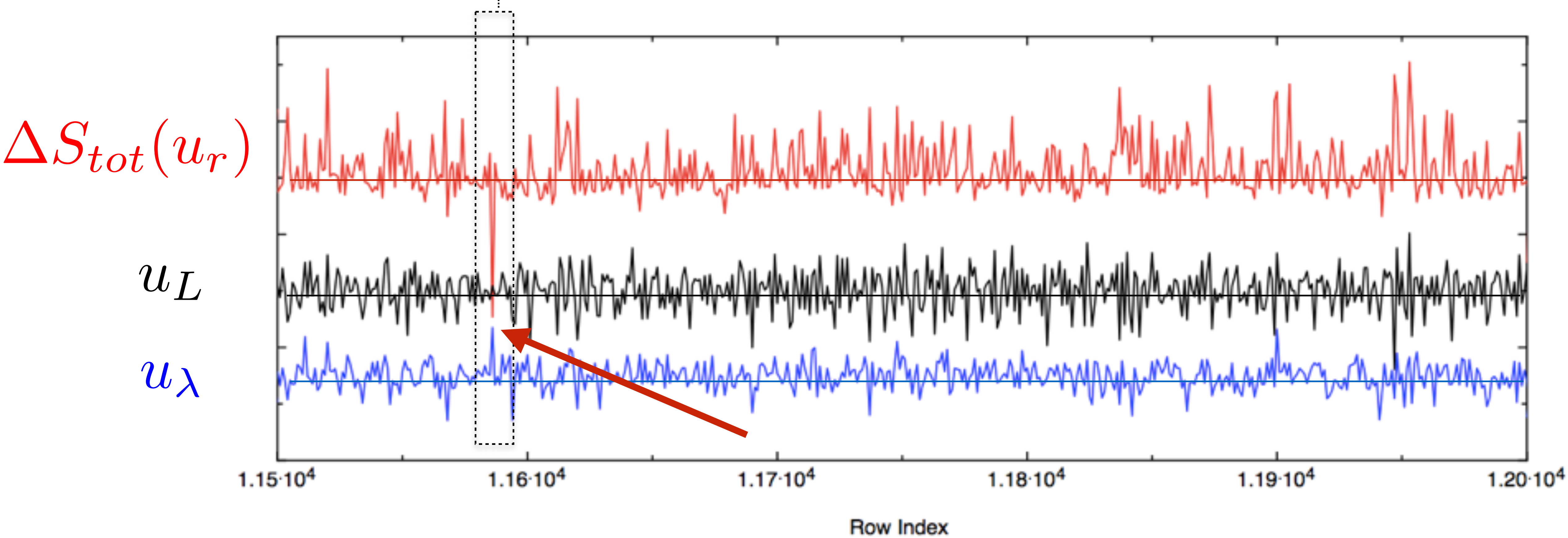


event

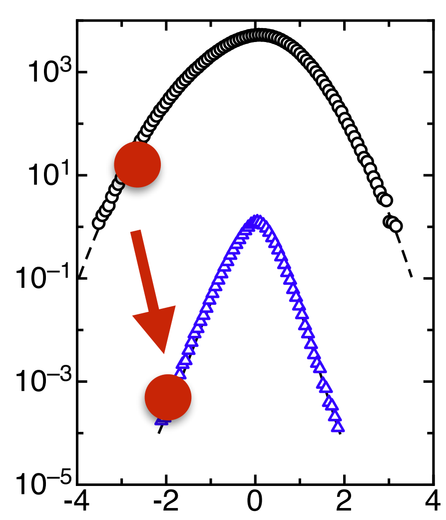
small u_L with high probability

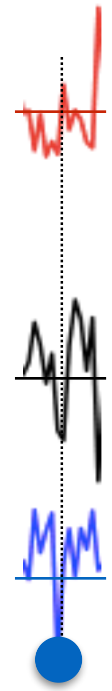
large u_λ with low probability





3.) extreme and normal entropy production $\Delta S_{tot}(u_r) \approx 0$

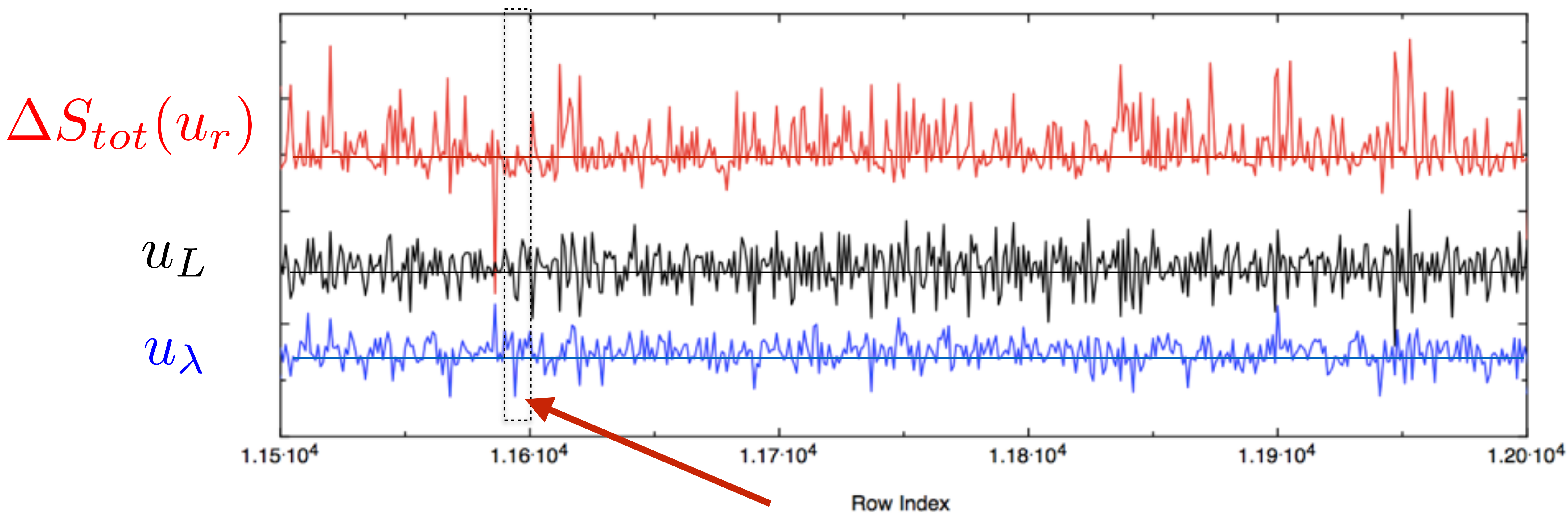




event

large u_L with low probability

large u_λ with low probability



End

concept of stochastic cascade

- multipoint statistics

- non- equilibrium thermodynamics

Thank you

references

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