On an Aw-Rascle type traffic model

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We review the macroscopic "balanced vehicular traffic model" that includes a generalized source term in the momentum equation of the Aw-Rascle model. We discuss the observational and theoretical motivation for such a generalization and point out the consequences of this source term. Moreover we deal with numerical simulation results for highway bottlenecks.

I. INTRODUCTION

In the first part of the talk we introduce the basic partial differential equations of balanced vehicular traffic [1, 2]. As the traffic model of Aw, Rascle [3], Greenberg [4] and Zhang [5] (ARZ model in the following) and the classical LWR model [6, 7], the basic equations form a hyperbolic system of balance laws. Moreover, due to a generalized source term, the model can describe an unstable regime of traffic flow.

II. OBSERVED PATTERNS OF TRAFFIC FLOW

In the second part of the talk observed patterns of traffic flow are presented that show the need for traffic models beyond the LWR and ARZ model. In particular, we focus on observational evidence for instabilities [8, 9], flow-density diagrams and the wide scattering of congested traffic [10, 11], complex spatiotemporal patterns of traffic flow [9, 11] and the capacity drop [12].

III. BALANCED VEHICULAR TRAFFIC

In the main part of the talk we discuss the balanced vehicular traffic model [1, 2]. We focus on the motivation for the effective relaxation coefficient that generalizes the source term of the pseudomomentum equation of the ARZ model, determine the steady-state solutions of the model and analyze the flow-density diagram. Within the model, there are two additional branches of homogeneous steady-state solutions that correspond to the zeros of the effective relaxation coefficient. The stability properties of these steady-state branches are determined by the subcharacteristic condition [13]. The model reproduces a wide scattering of data for congested traffic. Moreover, the stable and metastable solution branches show the observed reversed-λ shape of the fundamental diagram. The coupling conditions for the ARZ model can be applied to model intersections or junctions [14]

Finally, we present numerical simulation results for simple bottlenecks [2, 14]. Simulation results for a lane-drop bottleneck show that the balanced vehicular traffic model can reproduce a capacity drop. An overcompressed region downstream of shock fronts with similarities to detonation theory [15] can be observed in the simulations.

IV. REFERENCES

Only the most relevant references are listed below, additional references could (and probably should) be added.

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