On an Aw-Rascle type traffic model

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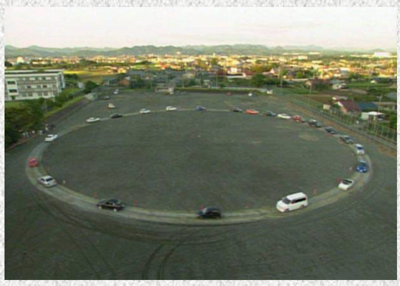
- Introduction
- Observed patterns of traffic flow
- Balanced vehicular traffic

Observed patterns of traffic flow

Instability of traffic flow

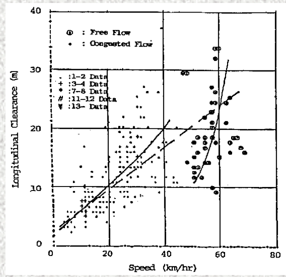
- Phantom traffic jams
  - Sugiyama et al., New Journal of Physics 10 (2008) 033001



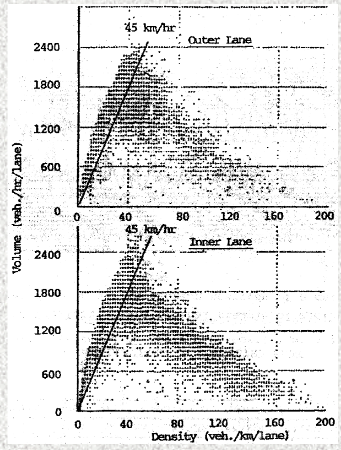
  - Schadschneider (WDR 2006)
- Growing perturbations
  - Schönhof, Helbing, Transportation Science 41 (2007) 135: boomerang effect
  - Treiterer, Myers (1974)
- Conclusions
  - Traffic flow has an unstable regime
  - Lane changes cannot be the reason for these instabilities
  - Traffic models beyond LWR and ARZ are needed

Flow-density data

- Koshi, Iwasaki, Ohkura (1983)



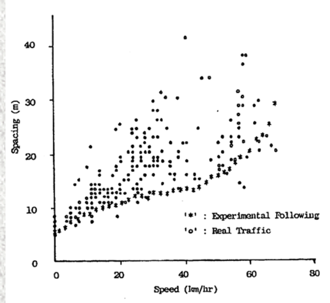
  - reversed λ
  - no homogeneous car following (steady states) for velocities of about 45 km/h



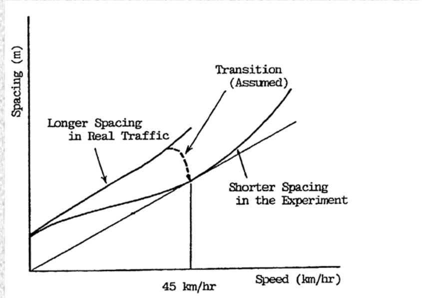
- Conclusion
  - Multivalued fundamental diagrams
  - Kerner (2004): 2D-region of steady states (synchronized flow)

Wide scattering of congested traffic

- Koshi, Iwasaki, Ohkura (1983)



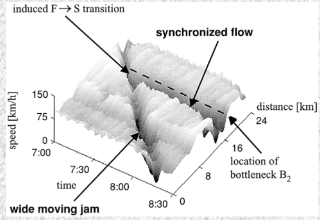
Comparison Between Real and Experimental Spacings

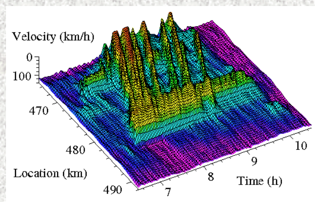


Transition From the Longer Car-Following State To the Shorter

Spatiotemporal patterns of traffic flow

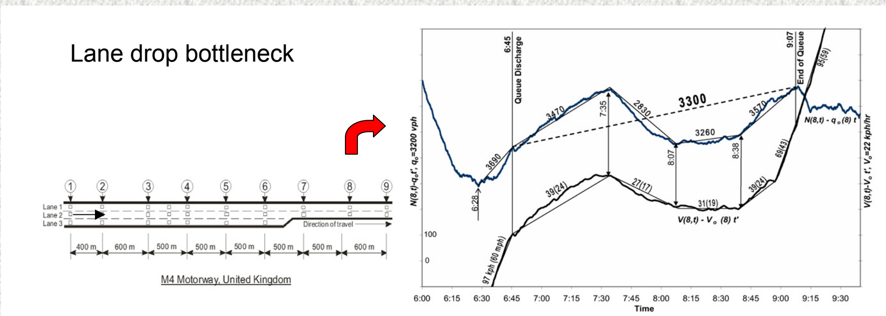
- Kerner et al.
  - ➢ Three-Phase Theory
    - Free flow
    - Synchronized flow
    - Wide moving jams
      - Travel through bottlenecks
      - Constant propagation speed of the downstream jam front
      - Constant outflow to free flow
- Helbing, Treiber et al.
  - ➢ Phase Diagrams
    - Free Traffic (FT)
    - Pinned Localized Clusters (PLC)
    - Moving Localized Clusters (MLC)
    - Stop and Go Waves (SGW)
    - Oscillating Congested Traffic (OCT)
    - Homogenized Congested Traffic (HCT)





Capacity drop

- Bertini, Leal, Journal of Transportation Engineering (2005)



- Capacity drop
  - ➢ The outflow in the downstream section is below the maximum free flow of that section after synchronized flow has formed upstream of the bottleneck

Balanced vehicular traffic

Balanced vehicular traffic model

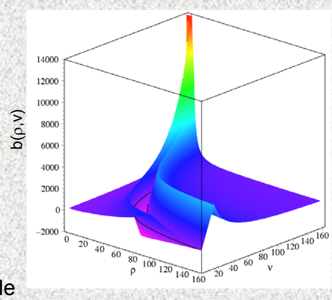
- Hyperbolic system of balance laws

continuity equation: $$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$$

pseudomomentum equation: $$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(v\rho w)}{\partial x} = -b(\rho, v)\rho w$$

ρ: vehicle density
v: dynamical velocity
u(ρ): equilibrium velocity
w=v- u(ρ): distance from equilibrium
b(ρ,v): effective relaxation coefficient

- Characteristic speeds
  - $\lambda_1 = v + \rho u'(\rho) \leq v$: genuinely nonlinear
    - Shocks and rarefaction waves
  - $\lambda_2 = v$: linearly degenerate
    - Contact discontinuities
- Parameterization
  - Equilibrium velocity: u(ρ)
  - Effective relaxation coefficient: b(ρ,v)
    - For b(ρ,v) < 0 equilibrium velocity u(ρ) unstable

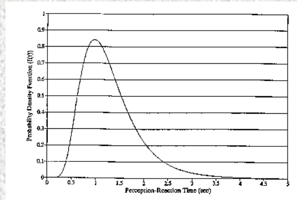


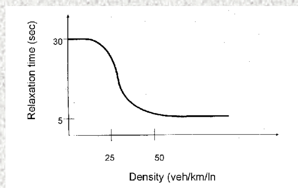
Motivation for negative "effective relaxation"

- Finite reaction time τ in ARZ model:
  - in moving observer coordinates
    - $$\frac{\partial v(t,x)}{\partial \tilde{t}} = \frac{\partial u(\rho(t-\tau, x-v\tau))}{\partial \tilde{t}} - \frac{v(t-\tau, x-v\tau) - u(\rho(t-\tau, x-v\tau))}{T}$$
    - leading order of Taylor series expansion

$$\frac{\partial (v-u)}{\partial \tilde{t}} = -\frac{v-u}{T-\tau}$$

  - instabilities for $T-\tau<0$
- Relaxation time T:
  - typical relaxation times T_{plot}:
    - $T_{plot} \cong 7.5\,T$
  - conclusion: for τ = 1s
    - $T-\tau < 0$ for $\rho>40$ [1/km/lane]

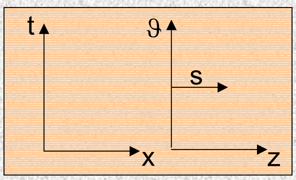


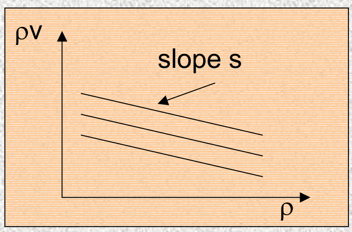


N. Gartner et al. (1997)

Steady-state solutions

- Steady-state solutions:
  - appropriate coordinate system $(z,\vartheta) = (x - s\,t, t)$ with constant s where all time derivates vanish
  - continuity equation:
    $\rho v = q + \rho s, \quad q = const$
    - steady-state solutions lie on straight lines in the flow-density diagram





  - pseudomomentum equation:

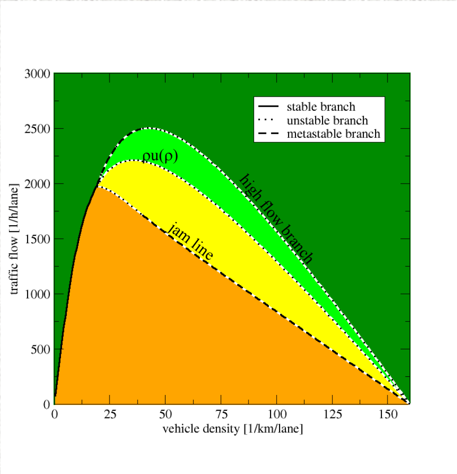
$$(\lambda_1 - s)\frac{dv}{dz} = -b(\rho,v)(v-u(\rho)), \quad \rho = \frac{q}{v-s},$$
$$q = const$$

    - trivial (homogeneous) solutions
    - non-trivial (non-homogeneous) solutions

Flow-density diagram

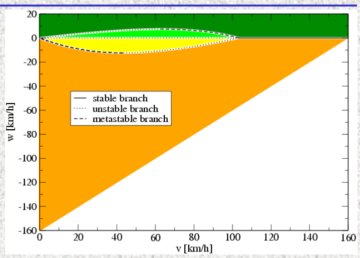
- Trivial steady-state solutions:
  - **equilibrium velocity** u(ρ)
  - **zeros** of the effective relaxation coefficient b(ρ,v)
    - **jam line** $v^j(\rho)$
    - **high-flow branch** $v^h(\rho)$
- Non-trivial steady-state solutions:
  - cover in particular the yellow and bright green regions
- Stability jam line / high flow branch:
  - subcharacteristic condition (Whitham 1974)
    - Linear stability analysis
    - Instability:

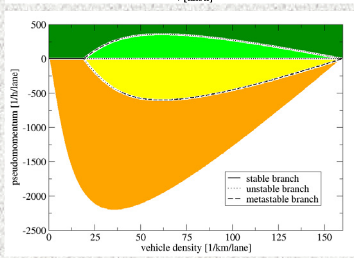
$$\lambda_1 = v + \rho u'(\rho) > \frac{d(\rho v^{j/h}(\rho))}{d\rho}$$



Alternative diagrams

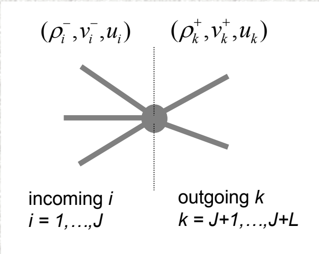
- Velocity (v) - distance from equilibrium (w) - diagram:
  - No homogeneous steady states for velocities of about 50 km/h
  - Stability condition of high-flow branch and jam line: dw/dv ≤ 0
  - High-flow branch: violation of subcharacteristic condition decreases with increasing velocity
- Density (ρ) – pseudomomentum (ρw) - diagram:
  - Invariant regions



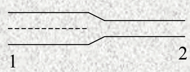


Coupling conditions at intersections and junctions

- Separation in principal part and source term
- Riemann problem for the principal part (Aw-Rascle model)
  - boundary fluxes at a junction
  - macroscopic description: lane changes neglected
  - generalization of the coupling conditions for the LWR model
    - definition of demands and supplies



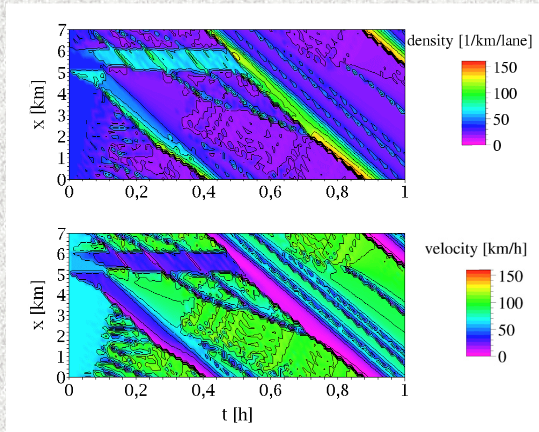
- Principles:
  - flow conservation
  - conservation of pseudomomentum flow
- Example:



$$flow: \quad q_1 = q_2$$
$$pseudomomentum\ flow:$$
$$q_1 w_1 = q_2 w_2 \quad \Rightarrow \quad w_1 = w_2$$

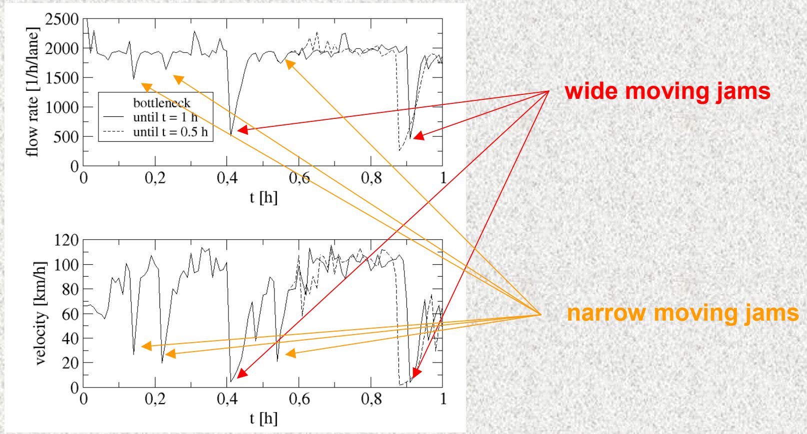
Speed restriction

- Simulation setup:
  - periodic boundaries
  - speed limit between 5 and 6 km
  - initially free flow data
- Traffic dynamics:
  - synchronized flow:
    - bottleneck
    - narrow moving jams
      - pinch region
      - merging
      - catch effect
  - wide moving jam:
    - speed -15 km/h
    - robust



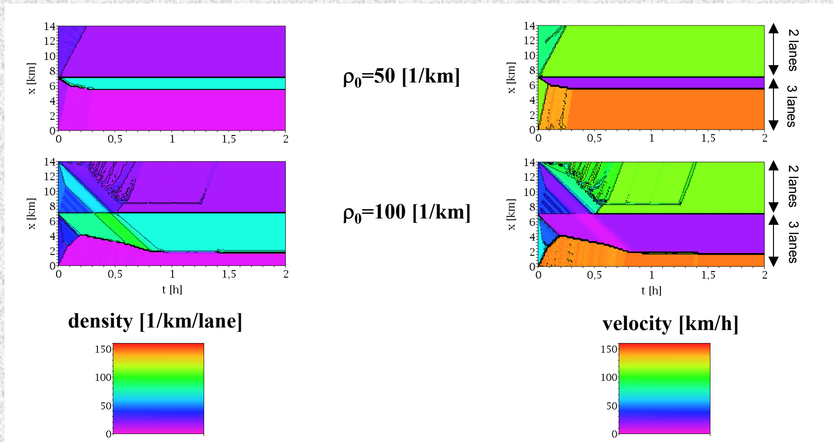
Speed restriction

- Measurements of a virtual detector located at x=0 km:



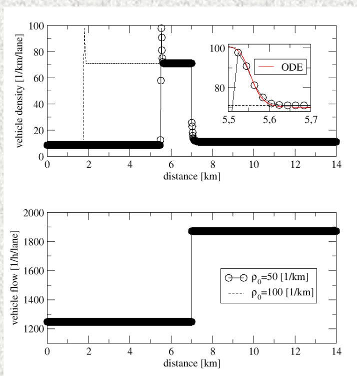
Lane drop bottleneck

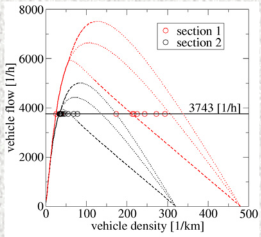
- Lane drop from three lanes (section 1) to two lanes (section 2):



Lane drop bottleneck

- Static solution:
  - von Neumann state downstream of the shock, followed by a section of a nontrivial steady-state solution

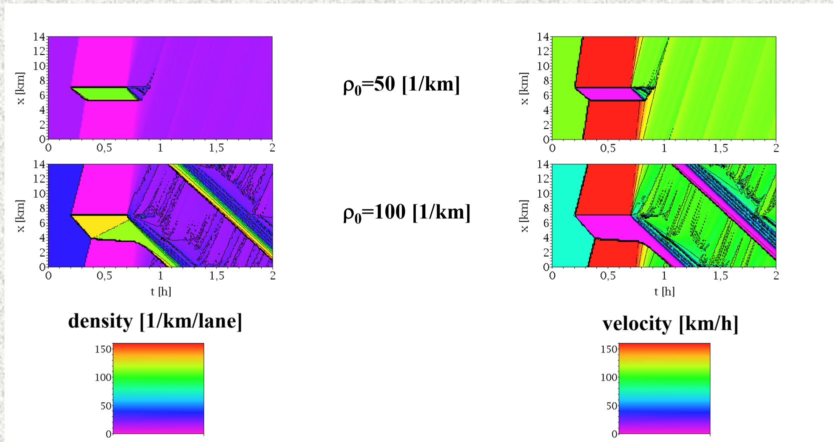




  - **capacity drop**:
    - flow value below maximum in downstream section 2
  - determined by the crossing of the static solutions with the jam line
    - similar to wide cluster solutions:
      - Zhang, Wong (2006), Zhang, Wong, Dai (2006)

Temporary lane closure

- Local closure of 2 lanes on 3-lane highway (between 0.2 and 0.7h):



Conclusion

- Balanced vehicular traffic model
  - hyperbolic system of balance laws
    - macroscopic
    - deterministic
    - effective one lane
    - no distinction between different vehicle types or driving behavior
    - nonlinear dynamics
  - model results
    - multi-valued fundamental diagrams
    - wide scattering of congested traffic
    - metastability of free flow at the onset of instabilities
    - wide moving jams
    - capacity drop