Schedule

Workshop on "Dispersion and Integrability"

Wolfgang Pauli Institute, Vienna
02nd – 05th October 2018

Organizers: Jean-Claude SAUT (ICP & U.Paris-Sud)
Christian KLEIN (U. Bourgogne)

Tuesday 2nd October

9:30 – 10:30	Jean-Claude Saut, ICP & U. Paris Sud
	"On KP type equations"
10:30 – 11:00	*Coffee break*
11:00 – 12:00	Christian Klein, U. Bourgogne
	"*Numerical study of blow-up in dispersive PDEs*"

Wednesday 3rd October

9:30 – 10:30	Anton Arnold, TU Wien
	"A hybrid WKB-based method for Schrödinger scattering problems in the semi-classical limit"
10:30 – 11:00	*Coffee break*
11:00 – 12:00	Derchyi Wu, Academia Sinica
	"*The Direct Problem of perturbed Kadomtsev-Petviashvili II 1-line solitons*"
12:00 – 14:00	*Lunch*
14:00 – 15:00	Peter Perry, U. Kentucky
	"*Soliton Resolution for the Derivative Nonlinear Schr\"{o}dinger Equation*"

Thursday 4th October

9:30 – 10:30	Patrick Gérard, U. Paris-Sud
	"Growth of Sobolev norms for a weakly damped Szegö equation"
10:30 – 11:00	*Coffee break*
11:00 – 12:00	Thomas Kappeler, U. Zürich
	"Normal form coordinates for the KdV equation having expansions in terms of pseudodifferential operators"
12:00 – 14:00	*Lunch*
14:00 – 15:00	Didier Pilod, U. Bergen
	"Well-posedness for some dispersive perturbations of Burger's equation"

Friday 5th October

9:00 – 10:00	Nikola Stoilov, U. Bourgogne
	"Electric Impedance Tomography"
10:00 – 11:00	Ilaria Perugia, U. Wien
	"Trefftz finite element methods"

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Abstracts

Jean-Claude Saut, ICP & U. Paris Sud

Title: On KP type equations

Abstract: After recalling the known results on the KP I and KP II equations, we survey some open problems on the KP equations, both from the PDE and IST aspects, and also on some relevant KP type equations.

Christian Klein, U. Bourgogne

Title: Numerical study of blow-up in dispersive PDEs

Abstract: We study numerically the stability of solitons and a possible blow-up of solutions in dispersive PDEs of the family of Kortweg-de Vries and nonlinear Schr\"odinger equations. The biow-up mechanism in the L^2 critical and supercritical case is studied.

Anton Arnold, TU Wien

Title: A hybrid WKB-based method for Schrödinger scattering problems in the semi-classical limit

Abstract: We are concerned with 1D scattering problems related to quantum transport in (tunneling) diodes. The problem includes both oscillatory and evanescent regimes, partly including turning points. We shall discuss the efficient numerical integration of ODEs of the form epsilon^2 u" + a(x) u = 0 for 0 < epsilon << 1 on coarse grids, but still yielding accurate solutions. In particular we study the numerical coupling of the highly oscillatory regime (i.e. for given a(x) > 0) with evanescent regions (i.e. for a(x) < 0). In the oscillatory case we use a marching method that is based on an analytic WKB-preprocessing of the equation. And in the evanescent case we use a FEM with WKB-ansatz functions.

We present a full convergence analysis of the coupled method, showing that the error is uniform in epsilon and second order w.r.t. h, when h = O(epsilon^1/2). We illustrate the results with numerical examples for scattering problems for a quantum-tunnelling structure. The main challenge when including a turning point is that the solution gets unbounded there as epsilon -> 0. Still one can obtain epsilon-uniform convergence, when h = O(epsilon^7/12).

Derchyi Wu, Academia Sinica

Title: The Direct Problem of perturbed Kadomtsev-Petviashvili II 1-line solitons

Abstract: Boiti-Pempinelli-Pogrebkov's inverse scattering theories on the KPII equation provide an integrable approach to solve the Cauchy problem and the stability problem of the KPII equation for perturbed multi-soliton solutions. In this talk, we will present rigorous analysis for the direct scattering theory of perturbed KPII one line solitons, the simplest case in Boiti-Pempinelli-Pogrebkov's theories. Namely, for generic small per-turbation of the one line soliton, the existence of the eigenfunction is proved by establishing uniform estimates of the Green function and the Cauchy integral equation for the eigenfunction is justified by non-uniform estimates of the spectral transform. Difficulties and outlooks for the inverse problem will be discussed as well.

Peter Perry, U. Kentucky

Title: Soliton Resolution for the Derivative Nonlinear Schr\"{o}dinger Equation

Abstract: This talk reports on joint work with Robert Jenkins, Jiaqi Liu, and Catherine Sulem. The derivative nonlinear Schr\"{o}dinger equation (DNLS) is a completely integrable, dispersive nonlinear equation in one space dimension that arises in the study of circularly polarized Alfv\'{e}n waves in plasmas, and admits soliton solutions. In 1978, Kaup and Newell showed that the DNLS is completely integrable, and in the 1980's, J.-H. Lee used the Beals-Coifman approach to inverse scattering to solve the DNLS. In the work to be described, drawing on recent advances in the Riemann-Hilbert formulation of inverse scattering due to Dieng-McLaughlin (2008) and Borghese-Jenkins-McLaughlin (2017), we use the inverse scattering formalism to show that, for a spectrally determined generic set of initial data, the solution decomposes into the sum of 1-soliton solutions with calculable phase shifts plus radiation.

Patrick Gérard, U. Paris-Sud

Title: Growth of Sobolev norms for a weakly damped Szegö equation

Abstract: The Szegö equation is an integrable model for lack of dispersion on the circle.
An important feature of this model is the existence of a residual set --- in the Baire sense--- of initial data leading to unbounded trajectories in high Sobolev norms. It is therefore natural to study the effect of a weak damping on such a system.

In this talk I will discuss the damping of the lowest Fourier mode, which has the specificity of saving part of the integrable structure. Somewhat surprinsingly, we shall show that such a weak damping leads to a wider set of unbounded trajectories in high Sobolev norms.

This is a jointwork in collaboration with Sandrine Grellier.

Thomas Kappeler, U. Zürich

Title: Normal form coordinates for the KdV equation having expansions in terms of pseudodifferential operators

Abstract: Complex normal coordinates for integrable PDEs on the torus can be viewed as 'nonlinear Fourier coefficients'.
Based on previous work we construct near an arbitrary finite gap potential a real analytic, 'nonlinear Fourier
transform' for the KdV equation having the following two main properties:
(1) Up to a remainder term, which is smoothing to any given order, it is a pseudodifferential operator of order 0
with principal part given by the Fourier transform.
(2) It is canonical and the pullback of the KdV Hamiltonian is in normal form up to order three. Furthermore,
the corresponding Hamiltonian vector field admits an expansion in terms of a paradifferential operator.
Such coordinates are a key ingredient for studying the stability of finite gap solutions, i.e., periodic multisolitons,
of the KdV equation under small, quasi-linear perturbations. This is joint work with Riccardo Montalto.

Didier Pilod, U. Bergen

Title: Well-posedness for some dispersive perturbations of Burger's equation

Abstract: We show that the Cauchy problem associated to a class of dispersive perturbations of Burgers' equations containing the low dispersion Benjamin-Ono equation
$$ \partial_tu-D_x^{\alpha}\partial_xu+u\partial_xu=0\, ,$$
with $0<\alpha \le 1$, is locally well-posed in $H^s(\mathbb R)$ for $s>s_\alpha: = \frac 32-\frac {5\alpha} 4$.

As a consequence, we obtain global well-posedness in the energy space $H^{\frac{\alpha}2}(\mathbb R)$ as soon as $\frac\alpha 2> s_\alpha$, i.e. $\alpha>\frac67$.

Nikola Stoilov, U. Bourgogne

Title: Electric Impedance Tomography

Abtract: Electric Impedance Tomography (EIT) is a medical imaging technique that uses the re-sponse to voltage difference applied outside the body to reconstruct tissue conductivity. As different organs have different impedance, this technique allows to produce images of the inner body without exposing the patient to potentially harmful radiation. In mathematical terms, EIT is as an inverse problem, whereby data inside a given domain is recovered from data on its boundary. In contrast with techniques like X-ray tomography (based on a linear problem), the particular inverse problem employed in EIT is non-linear - it reduces to a so-called D-bar problem. Such problems also find application in the area of Integrable Systems, specifically in the inverse scattering problem associated with 2+1 dimensional integrable equations such as the Davey - Stewartson and Kadomtsev Petviaschvili equations. I will discuss the design of nu-merical algorithms based on spectral collocation methods that address D-bar problems found in both integrable systems and medical imaging. Successfully implementing these methods in EIT should allow us to achieve images with much higher resolutions at reduced processing times. We take advantage of the fact our approach is highly parallelisable by implementing on graphical processing units (GPUs) to gain efficiency and speed without increasing the cost of the process. Finally I will describe the route towards the full development of the technology, and the hope that EIT will emerge as an effective, fast, convenient and less intrusive and distressing form of medical imaging.

Ilaria Perugia, U. Wien

Title: Trefftz finite element methods

Abtract: Over the last years, finite element methods based on operator-adapted approximating spaces have been developed in order to better reproduce physical properties of the analytical solutions, and to enhance stability and approximation properties. They are based on incorporating a priori knowledge about the problem into the local approximating spaces, by using trial and/or test spaces locally spanned by functions belonging to the kernel of the differential operator (Trefftz spaces). These methods are particularly popular for wave problems in frequency domain. Here, the use of oscillating basis functions allows to improve the accuracy vs. computational cost, with respect to standard polynomial finite element methods, and breaks the strong requirements on number of degrees of freedom per wavelength to ensure stability.

In this talk, the basic principles of Trefftz finite element methods for time-harmonic wave problems will be presented. Trefftz methods differ from each other by the way interelement continuity conditions are imposed. We will focus on discontinuous Galerkin approaches, where the approximating spaces are made of completely discontinuous Trefftz spaces, and on the recent virtual element framework.