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European High-performance
Infrastructures in Turbulence
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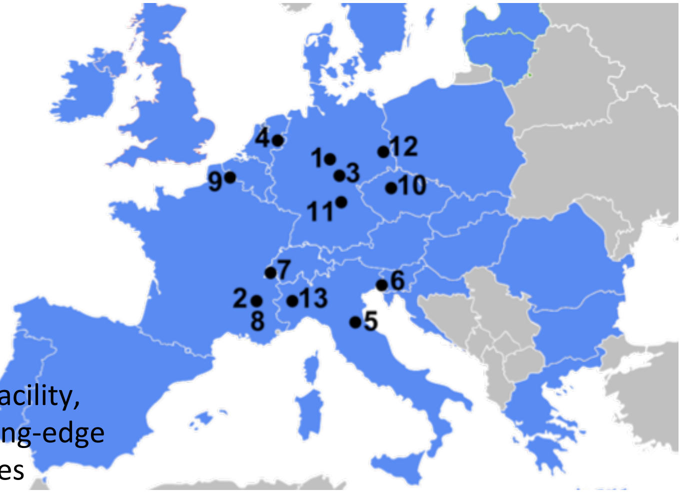
- International scientific mobility program for turbulence research
- 24 Partners from academia and industry
- Funding budget of 7 Mio EUR by the European Commission over 4 years (April 2013 – March 2017)
- Free trans-national access to 14 cutting-edge European turbulence facilities
- Joint research activities, networking and educational program
- Innovation and exploration of new fundamental technologies
- Interaction with numerical modeling and theoretical analysis
- Exchange of instruments, techniques, data and new ideas

The Göttingen Turbulence Facility, one of 14 international cutting-edge European turbulence facilities



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- Tel Aviv University, Israel

Turbulence a Perspective

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ENS DE LYON

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Stiftung / Foundation

Joint work with:
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Grisha Falkovich (Weizmann I.S.), Guido Boffetta (Torino),Hua Xia, Nicolas François,
Michael Shats (Canberra)

The cornerstone of turbulence from the Eulerian viewpoint

Based on an ***energy budget*** [+ use homogeneity and isotropy]

➔ obtain the ***von Karman-Howarth-Kolmogorov*** equation:

$$\left\langle (\delta_r u_L)^3 \right\rangle = -\frac{4}{5}\epsilon r + 6\nu \frac{d}{dr} \left\langle (\delta_r u_L)^2 \right\rangle$$

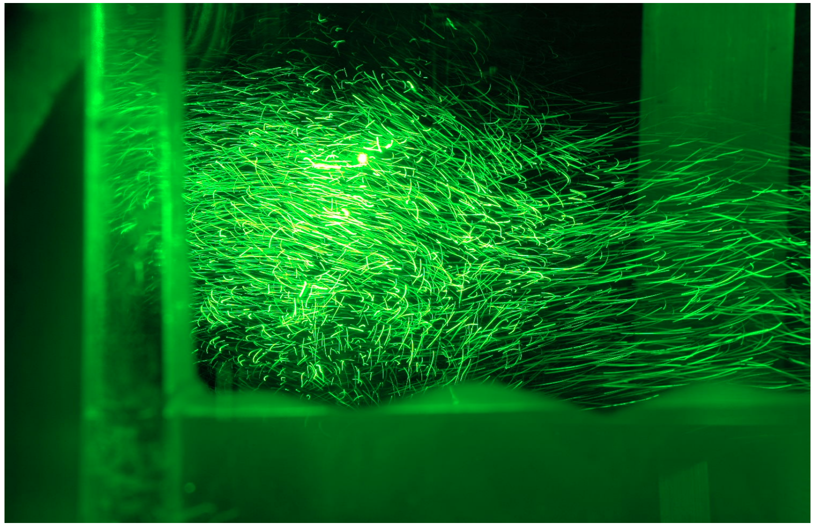
where ε is the flux of energy.

- Viewed at "certain scales", the flow is not smooth…
- *Relation between the structure functions and the energy flux !*
- ***Very useful***, since structure functions have been accessible experimentally for a long time.

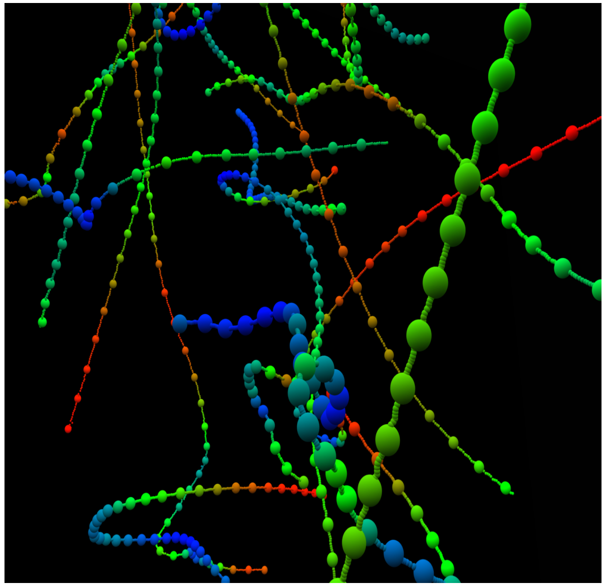
A side-product: strong emphasis on scaling laws in turbulence research.

The Lagrangian point of view

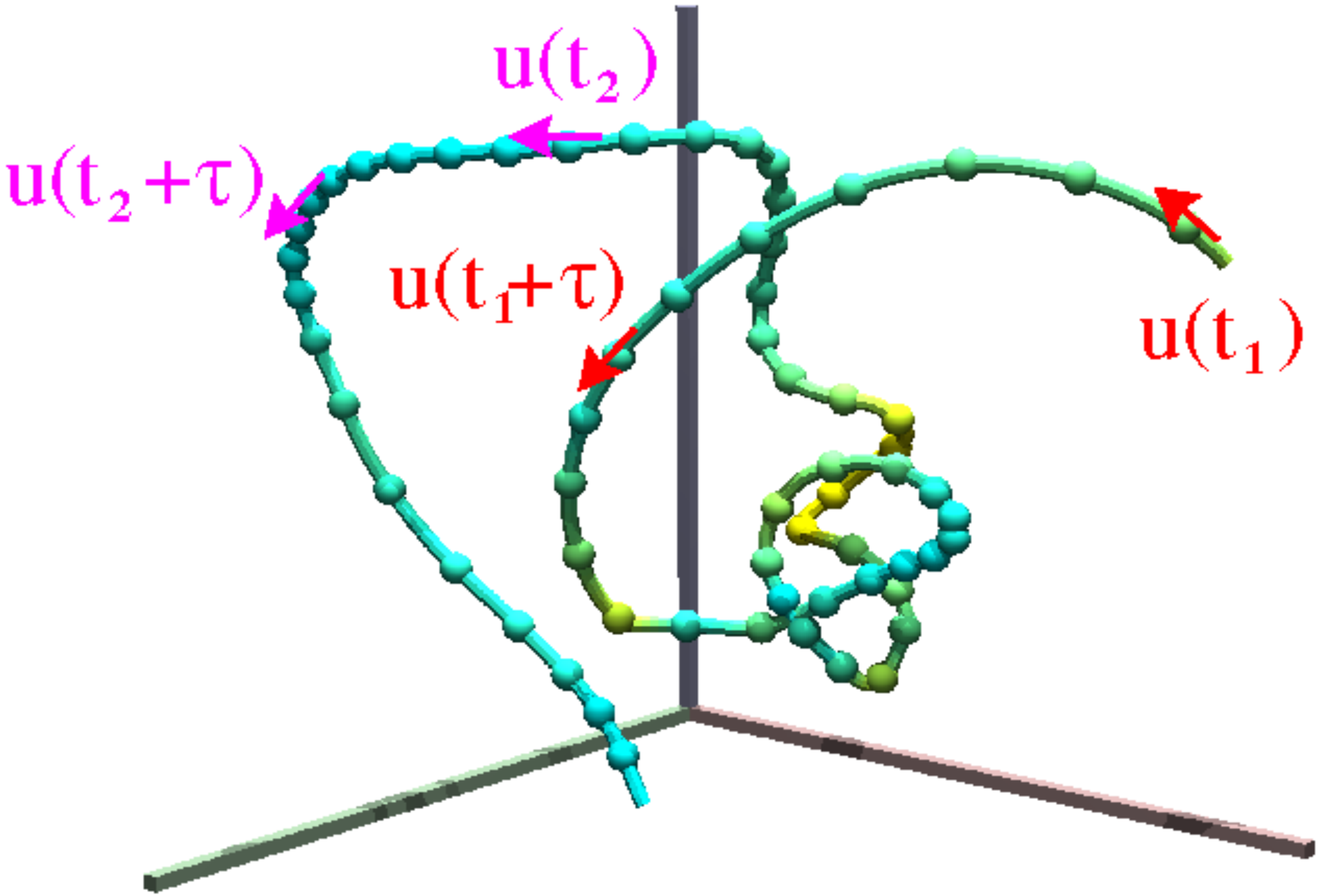
Accurate measurements of particles moving (very fast) in a turbulent flow (Cornell; Göttingen; Lyon; Zürich; Copenhagen …)



➔



What can we learn about turbulence by following particles?



u(t_2)
u(t_2 +τ)
u(t_1+τ)
u(t_1)

Sign of the flux

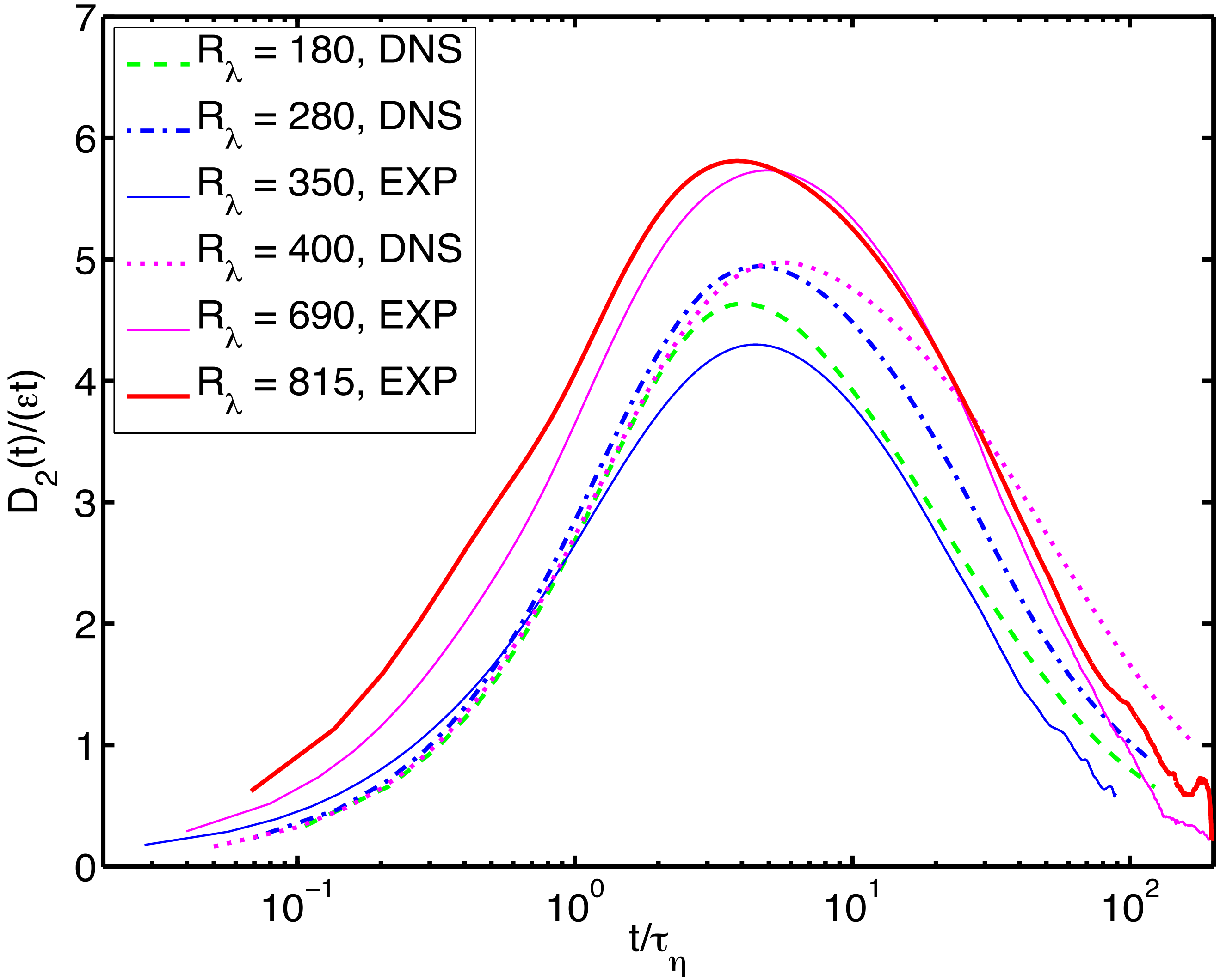
$$\epsilon > 0 \quad \text{for d=3}$$
$$\epsilon < 0 \quad \text{for d=2}$$

But $D_2(\tau) \equiv \langle (\delta_\tau u)^2 \rangle \geq 0$

Moreover, if reverse time $t \to -t$

The sign of $D_2(\tau)$ remain unchanged!

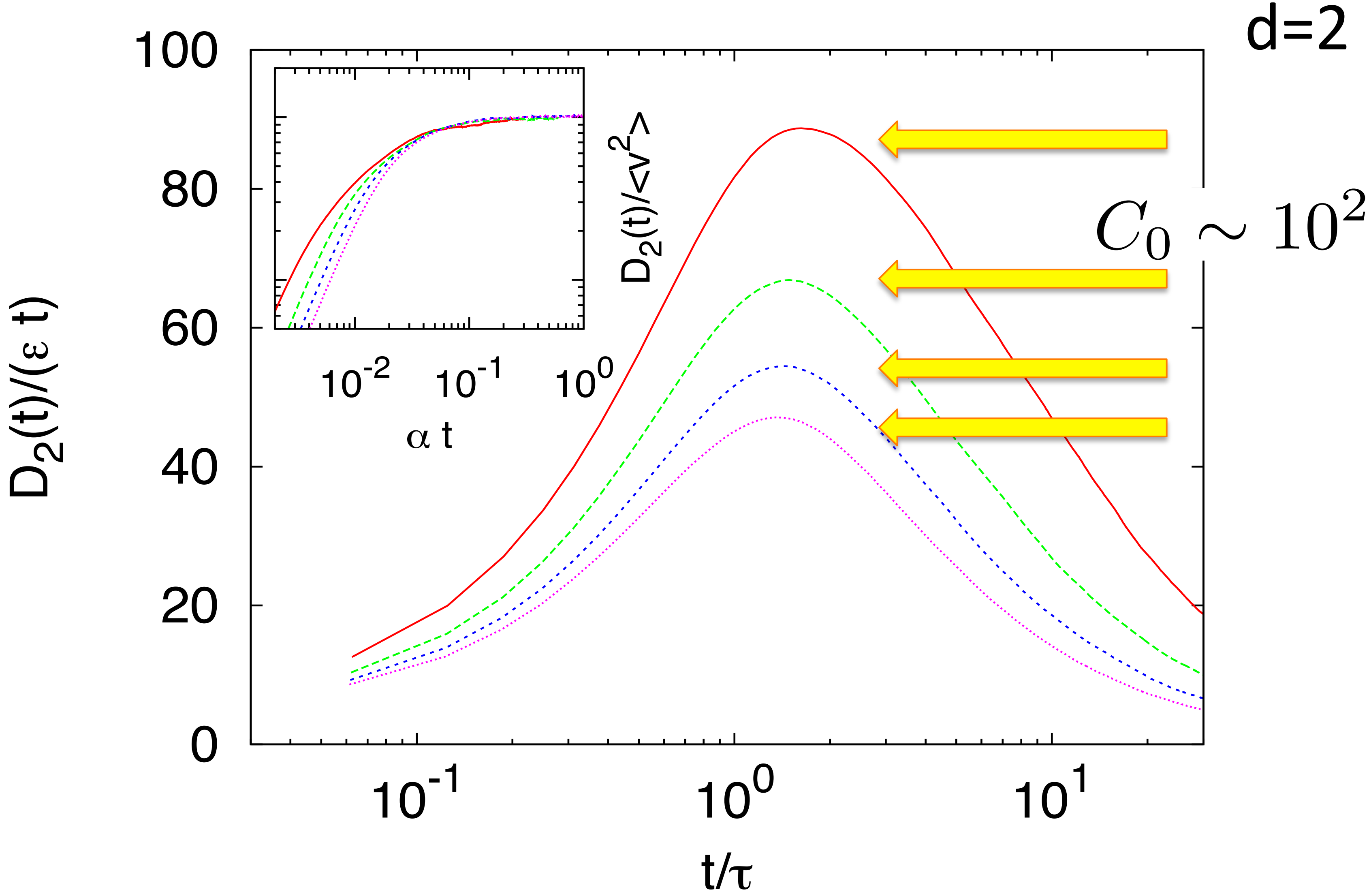
$$D_2(\tau) \equiv \langle (\delta_\tau u)^2 \rangle \sim \epsilon \tau \quad ?$$



d=3

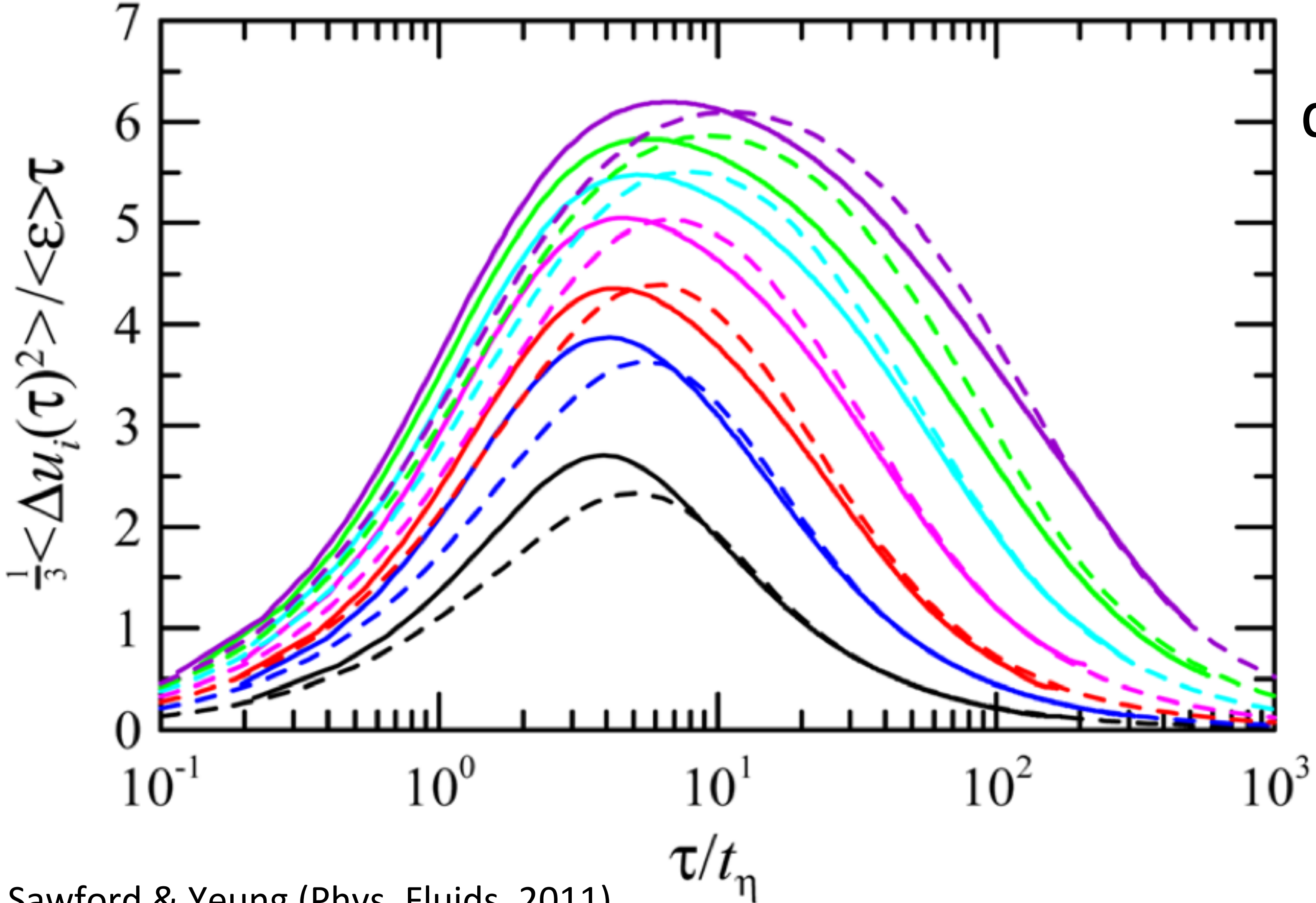
No scaling range observed from currently available DNS and experimental data.

$$D_2(\tau) \equiv \langle (\delta_\tau u)^2 \rangle \sim \epsilon\tau \quad ?$$



No scaling range observed from currently available DNS and experimental data.

$$D_2(\tau) \equiv \langle(\delta_\tau u)^2\rangle \overset{?}{\sim} \epsilon\tau$$



d=3

Sawford & Yeung (Phys. Fluids, 2011)

No scaling range observed from currently available DNS and experimental data.

Acceleration auto-correlation

$$\frac{d}{d\tau}\langle(\delta_\tau u)^2\rangle = 2\langle a(\tau)[u(\tau) - u(0)]\rangle$$
$$= 2\int_0^\tau \langle a(0)a(t)\rangle dt$$

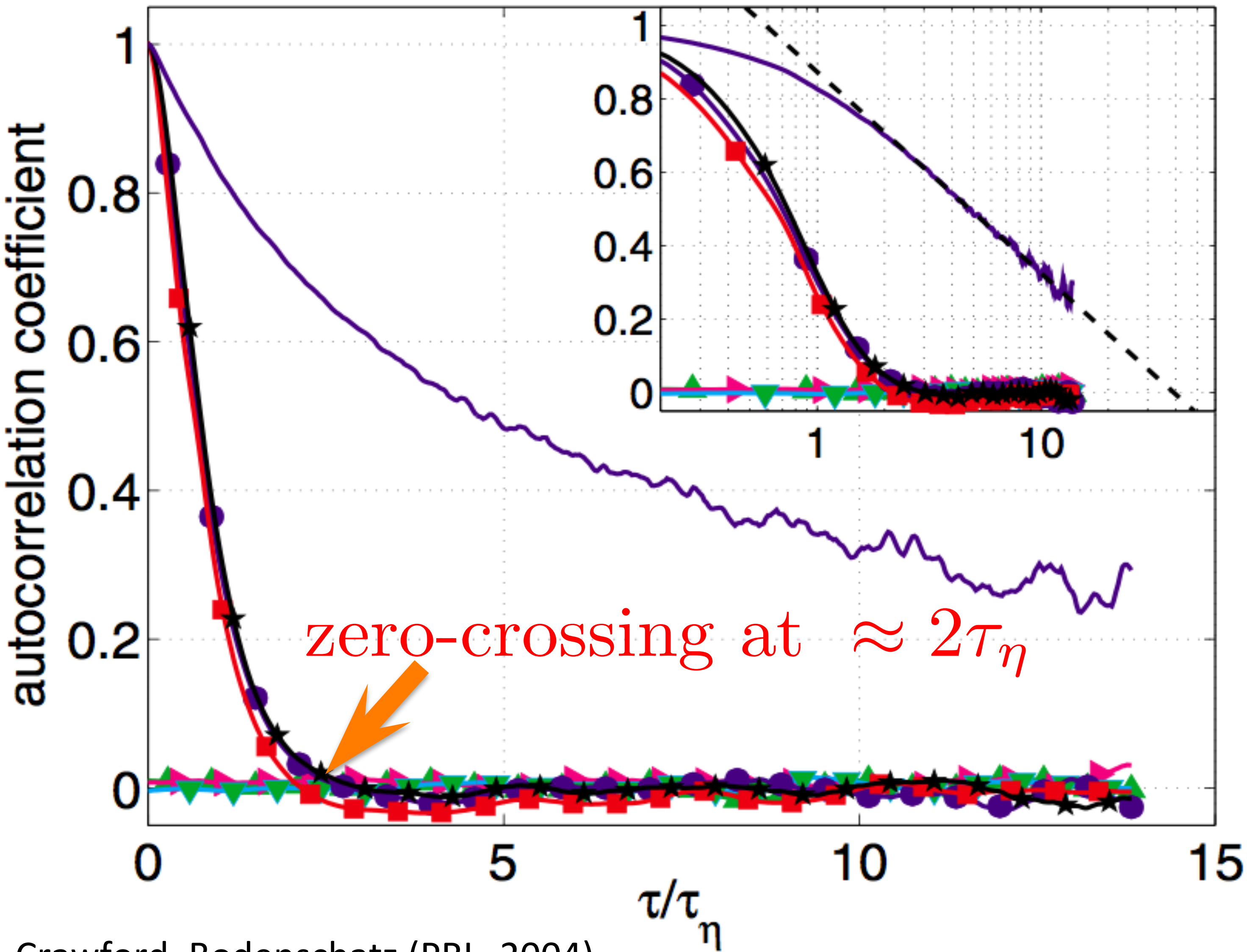
$$(\delta_\tau u)^2 \sim \tau \Rightarrow \text{acceleration is uncorrelated over time-lag } \tau$$

Kinematic constraint (Tennekes & Lumley (1972)):

$$\int_0^\infty \langle a(0)a(t)\rangle dt = 0$$

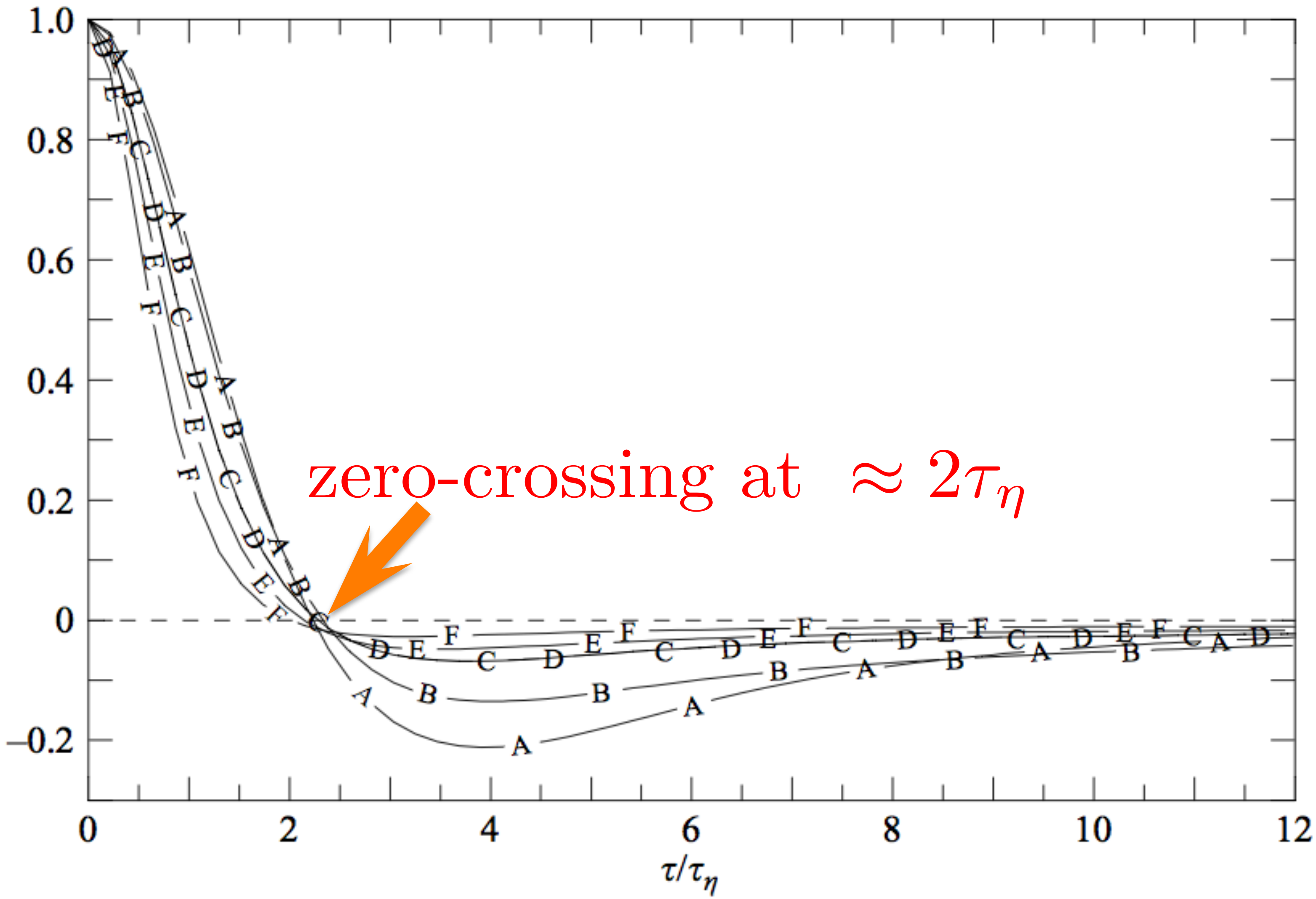
This restrict the shape of the acceleration auto-correlation.

Data from particle tracking measurements



Mordant, Crawford, Bodenschatz (PRL, 2004)

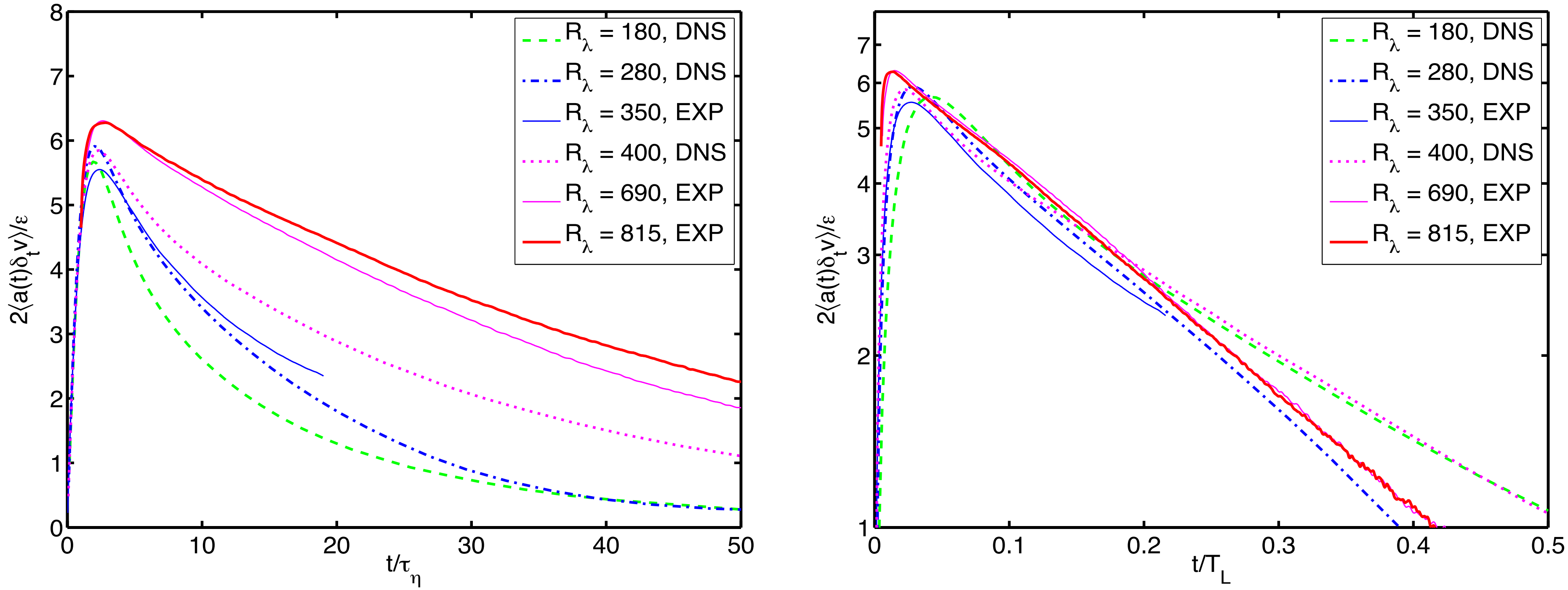
Data from DNS



Yeung et al. (JFM, 2007)

Zero-crossing of the acceleration auto-correlation gives the peak of

$$\frac{d\langle(\delta_\tau u)^2\rangle}{d\tau} = 2\langle a\delta_\tau u\rangle$$



$$\langle(\delta_\tau u)^2\rangle = C_0\epsilon\tau \quad \Rightarrow \quad 2\langle a\delta_\tau u\rangle = C_0\epsilon$$

However, experimental and DNS data suggest an exponential decay after the peak.

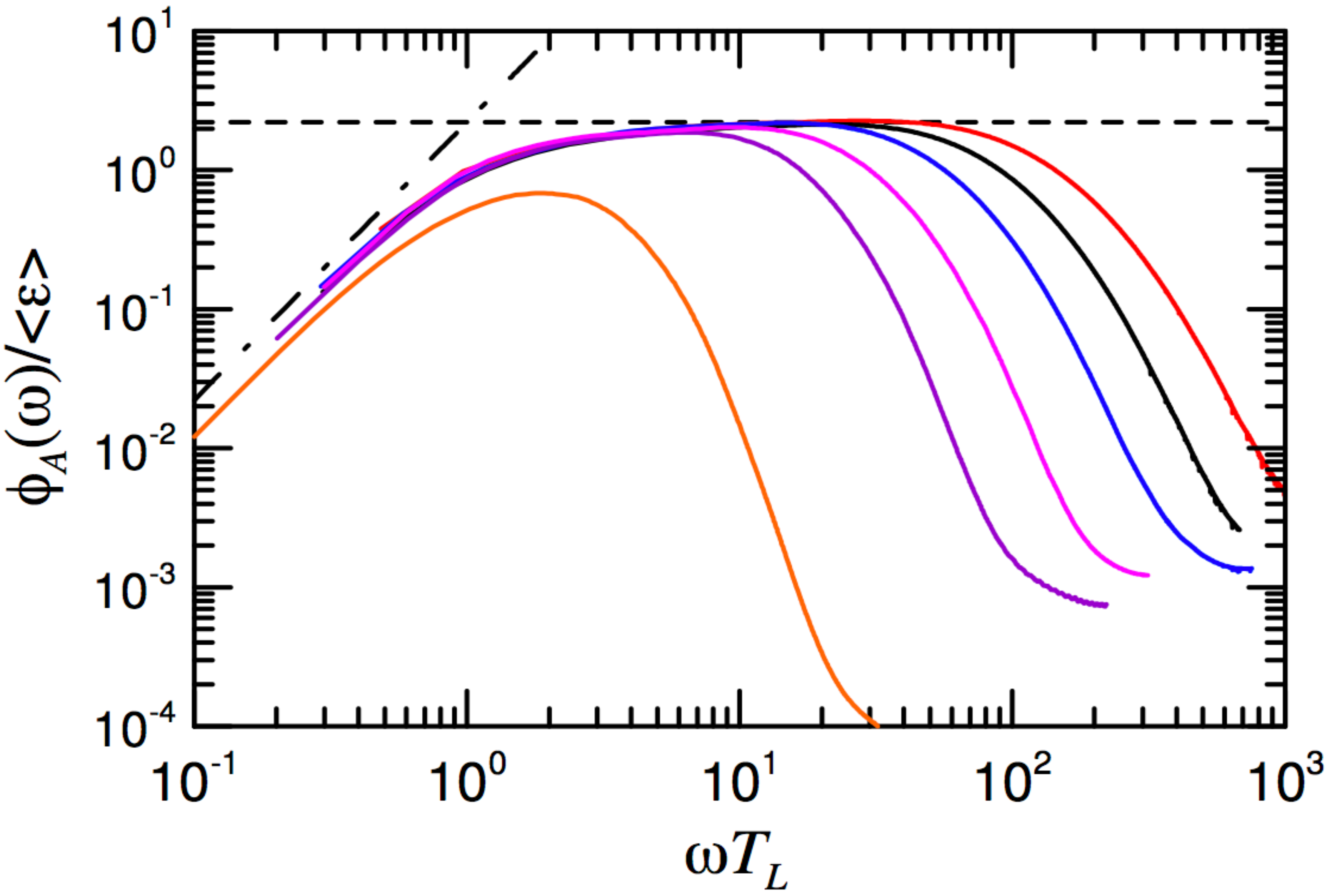
Acceleration spectra

Acceleration spectrum may show a wider scaling range than that of the velocity structure function (Lien & D'Asaro (Phys. Fluids, 2002), Sawford & Yeung (Phys. Fluids, 2011)).

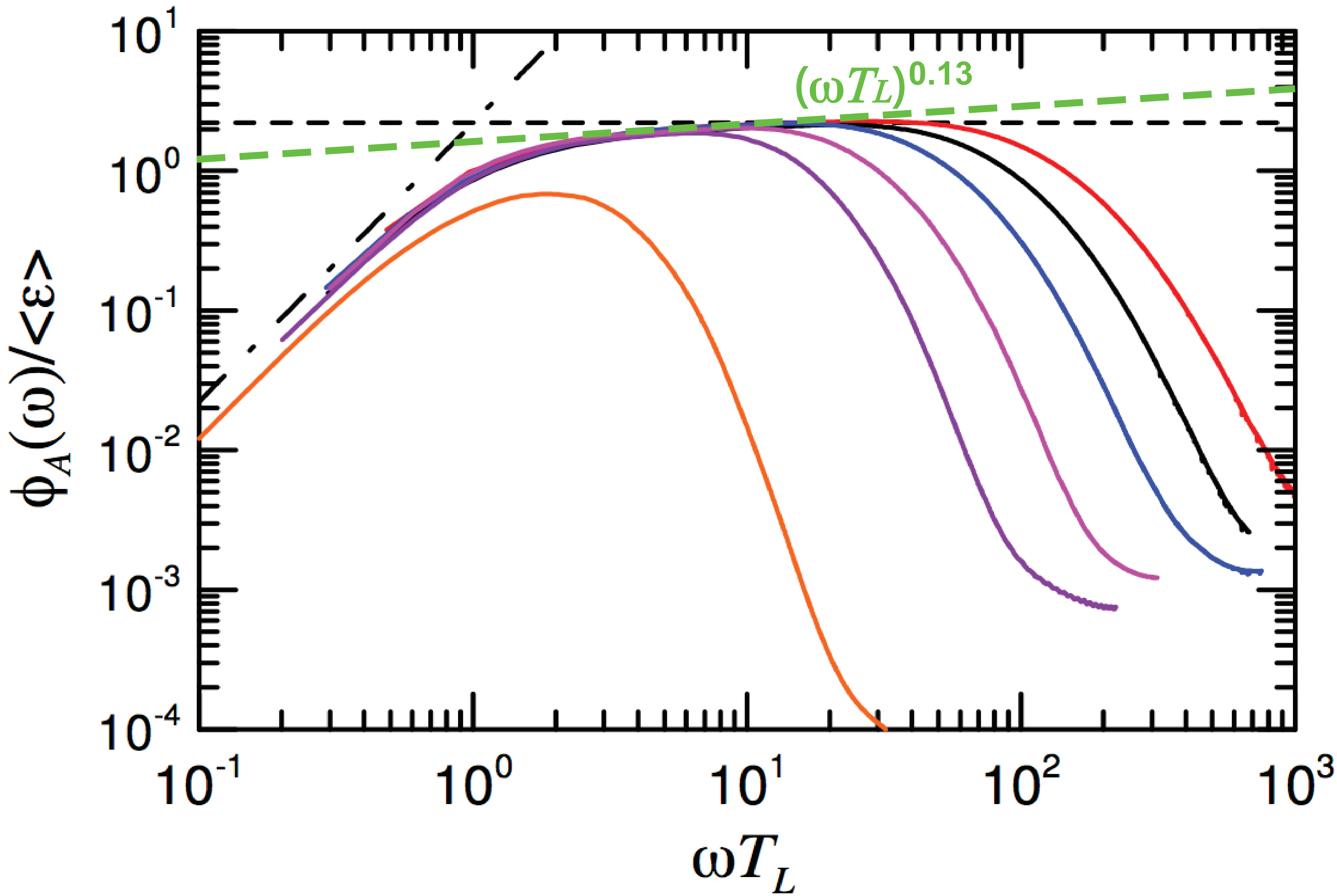
$$\langle (\delta_\tau u)^2 \rangle \sim \tau \quad \Rightarrow \quad \Phi_A(\omega) \sim \omega^0, \quad (1/T_L \lesssim \omega \lesssim 1/\tau_\eta)$$

Remark: A flat acceleration spectrum implies δ-correlated acceleration.

DNS results from Sawford & Yeung (Phys. Fluids, 2011)



DNS results from Sawford & Yeung (Phys. Fluids, 2011)



Acceleration spectra suggest anomalous scaling for velocity increments:

$$\Phi_A(\omega) \sim \omega^{\mu} \quad \Rightarrow \quad \langle(\delta_\tau u)^2\rangle \sim t^{1-\mu}$$

Moreover, acceleration variance is:

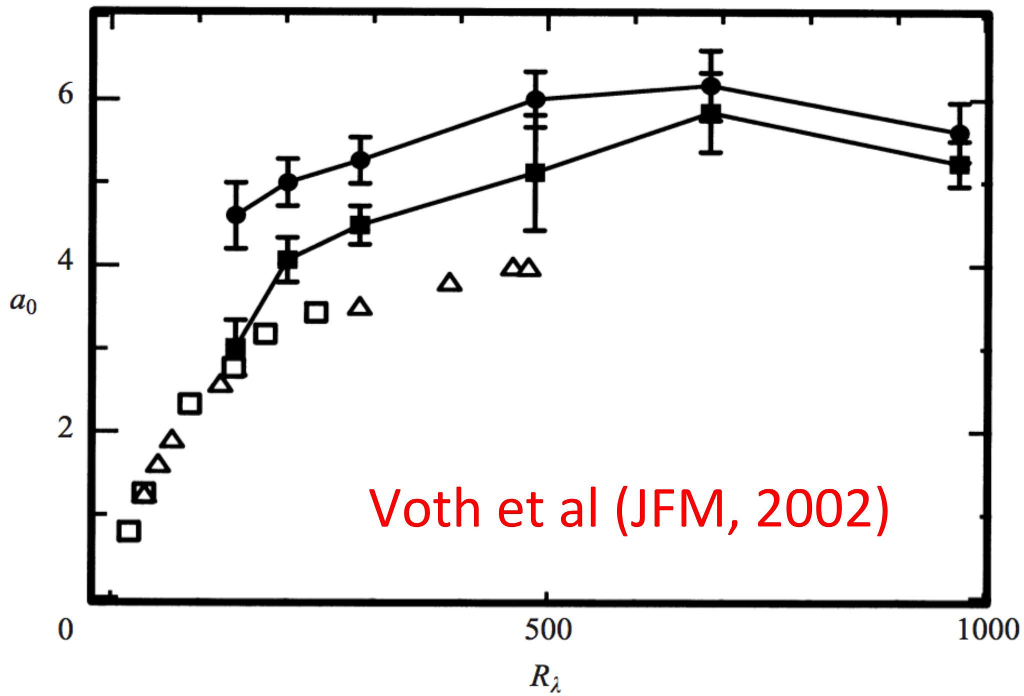
$$\langle a^2 \rangle = \int_0^\infty \Phi_A(\omega) \approx \int_{1/T_L}^{1/\tau_\eta} A_0 \epsilon \omega^\mu d\omega$$

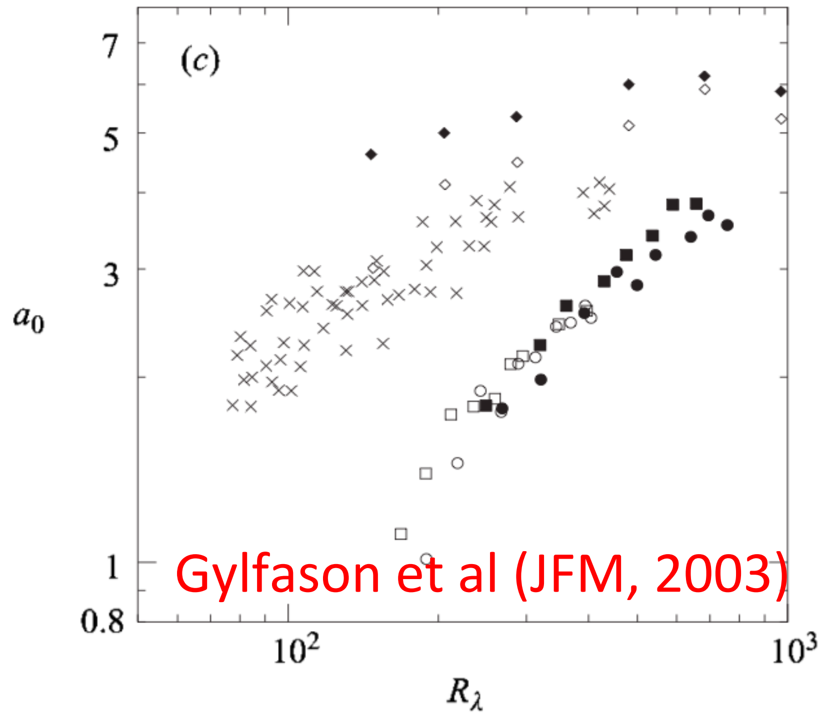
$$\sim \epsilon \tau_\eta^{-(1+\mu)} \sim \frac{\epsilon^{3/2}}{\nu^{1/2}} R_\lambda^\mu$$

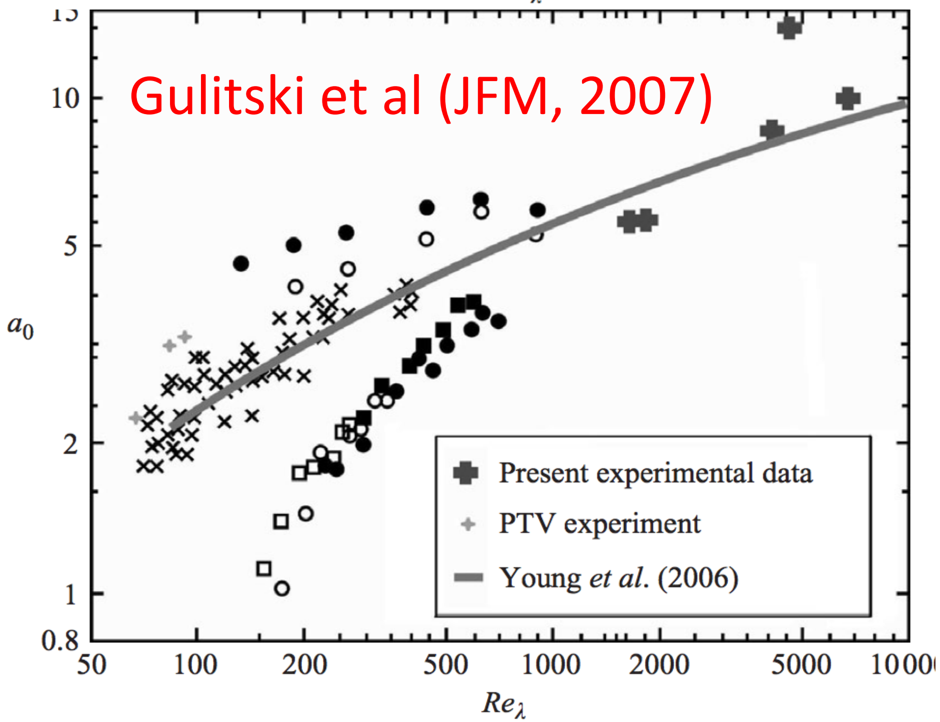
Which implies:

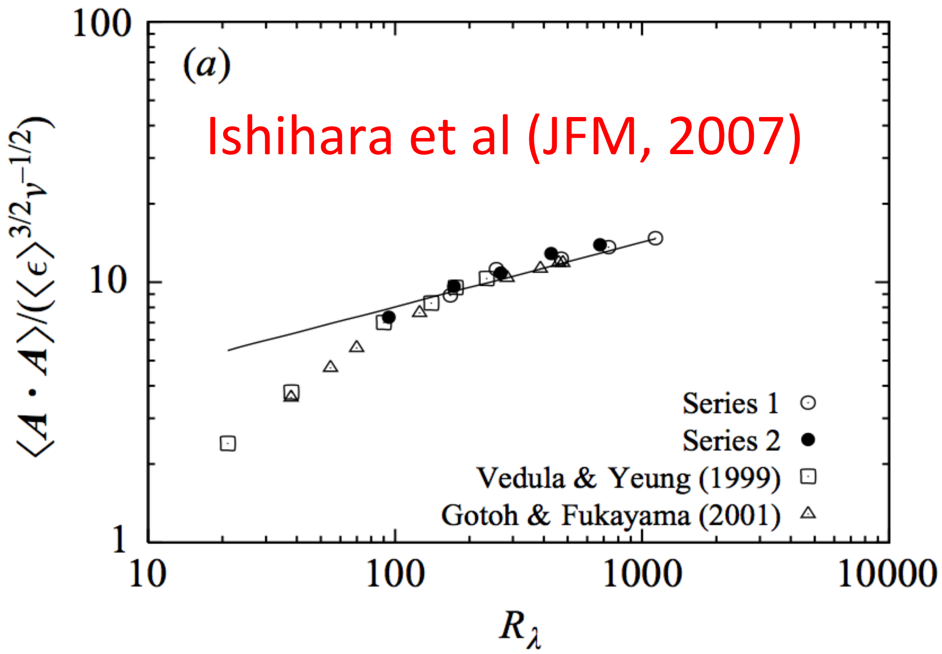
$$a_0 \equiv \frac{\langle a^2 \rangle \nu^{1/2}}{\epsilon^{3/2}} \sim R_\lambda^\mu$$

Consistent with observations from experiments and DNS.


Voth et al (JFM, 2002)
a_0
0
2
4
6
500
1000
R_λ


(c)
Gylfason et al (JFM, 2003)
a_0
0.8
1
3
5
7
10^2
10^3
R_λ


Gulitski et al (JFM, 2007)
a_0
0.8
1
2
5
10
50
100
200
500
1000
2000
5000
10000
Re_λ
Present experimental data
PTV experiment
Young et al. (2006)


(a)
Ishihara et al (JFM, 2007)
$\langle A \cdot A \rangle / (\langle \epsilon \rangle^{3/2} \nu^{-1/2})$
1
10
100
10
100
1000
10000
R_λ
Series 1
Series 2
Vedula & Yeung (1999)
Gotoh & Fukayama (2001)

Summary for part 1

- Dimensional scaling for Lagrangian velocity structure functions is not consistent with either theoretical considerations or experimental/numerical data.
- Using extended-self-similarity to the study of Lagrangian velocity structure functions is questionable.
- Interpolation schemes that bridges viscous, inertial, and large scales can only be used with caution, as the scaling relations are built-in while constructing the scheme.

2. Power Fluctuations and Irreversibility in Turbulence

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Time irreversibility

Turbulent dynamics (Navier-Stokes eqns)

~ *energy flux through scale*

~ ***time irreversible process.***

"A trained eye viewing a movie of turbulence run backwards can tell that something is going wrong…"

(Falkovich & Sreenivasan, Physics Today 2006)

Can one really distinguish "the arrow of time" ?
(see e.g. Pomeau, 1982).

Training the eye ??

Two-particle statistics:

an exact relation

Training the eyes by following pairs of particles

Deduce the equation for the evolution of the relative evolution of two particle relative distance:

$$R(t) = r_2(t) - r_1(t); \;\; \delta R(t) = R(t) - R(0)$$

=> $$\left\langle \delta R(t)^2 \right\rangle = \left\langle u(0)^2 \right\rangle t^2 + \left\langle u(0).\; a(0) \right\rangle t^3 + O(t^4)$$

Identity: (Ott and Mann 2000, Pumir et al 2001, Falkovich et al 2001)

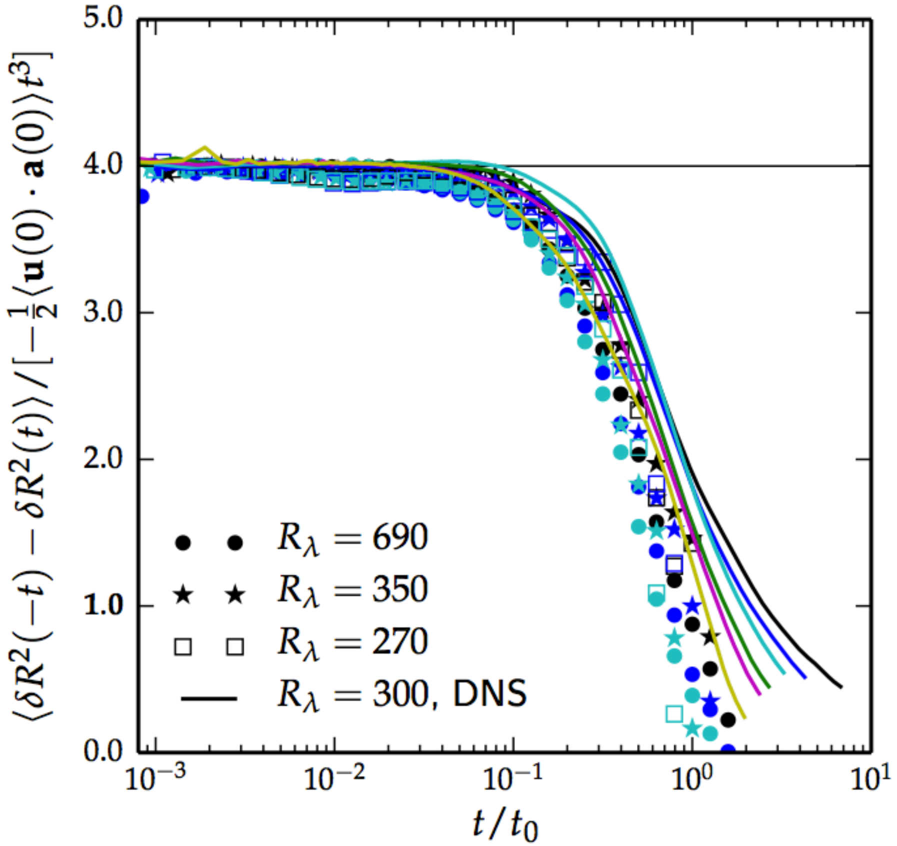
$$\eta \ll |\mathbf{r}_2 - \mathbf{r}_1| \ll L$$

$$\frac{1}{2}\frac{d}{dt}\left\langle (u_2 - u_1)^2 \right\rangle = \left\langle u(0).\; a(0) \right\rangle = -2\varepsilon$$

Training the eyes by following pairs of particles

Consequence:

$$\left\langle \delta R(-t)^2 \right\rangle - \left\langle \delta R(t)^2 \right\rangle = -2\left\langle u(0).\ a(0) \right\rangle t^3 + O(t^5)$$



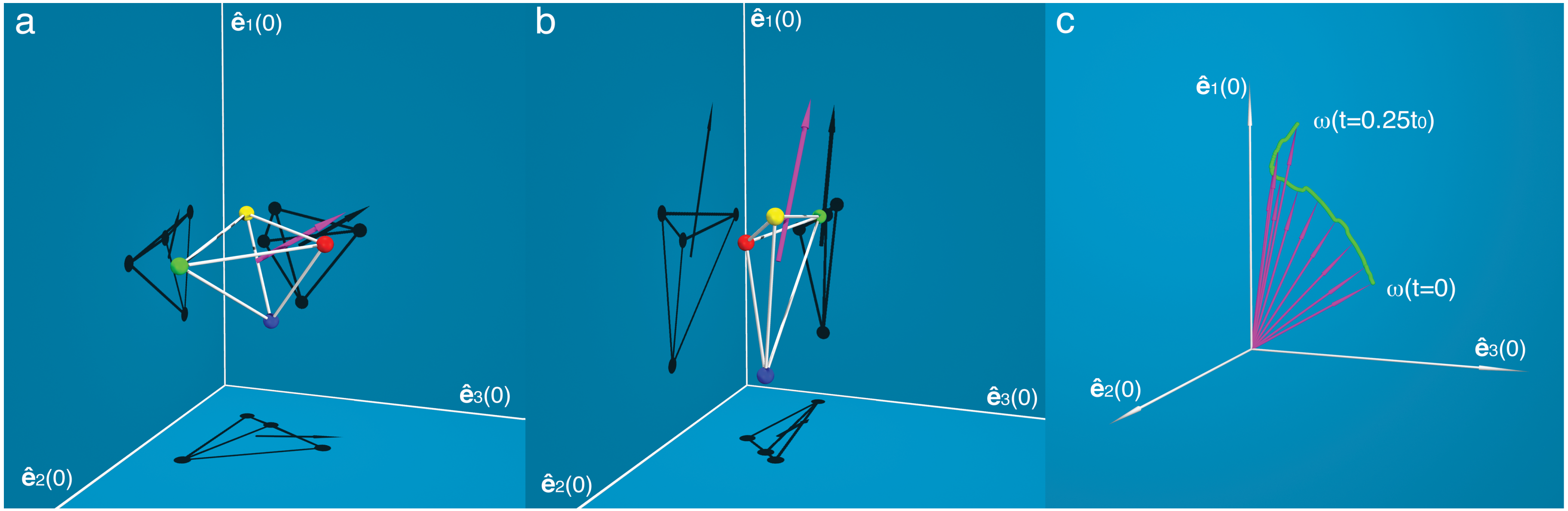
Use different initial separations, R_0, different Reynolds numbers + rescale time with $t_0=(R_0^2/\varepsilon)^{1/3}$

Good data collapse !

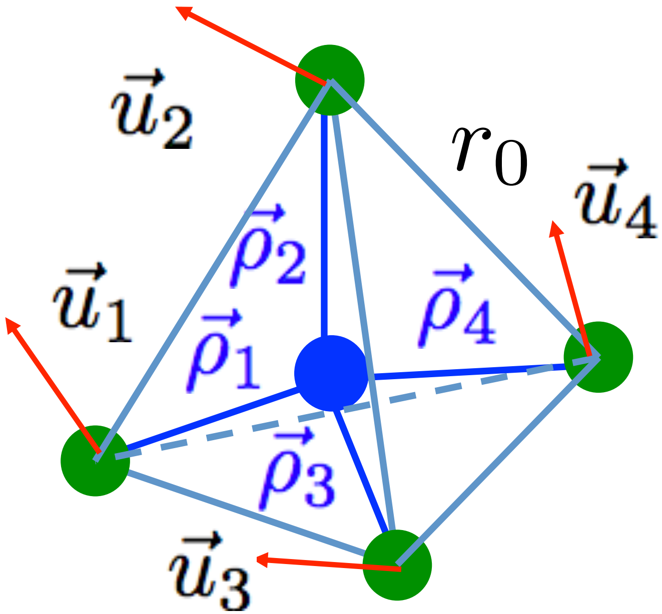
(Jucha et al, PRL 2014)

Using more particles to "train the eye"

More particles => more info !



Analysis: relate the observed time-asymmetry to fundamental properties of the velocity gradient tensor.



Velocity gradient perceived by tetrads:

$$\mathbf{M}: \quad \mathbf{M}\vec{\rho}_i = \vec{u}_i - \frac{1}{4}\sum_i \vec{u}_i$$

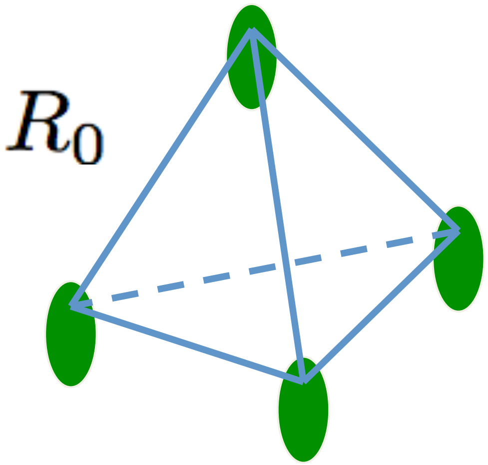
$$\mathbf{M} = \mathbf{S} + \mathbf{\Omega}$$

Perceived strain: $$\mathbf{S} = \frac{1}{2}(\mathbf{M} + \mathbf{M}^T)$$

Perceived vorticity: $$\mathbf{\Omega} = \frac{1}{2}(\mathbf{M} - \mathbf{M}^T)$$

Chertkov, Pumir, Shraiman (**Phys. Fluids**, 1999)

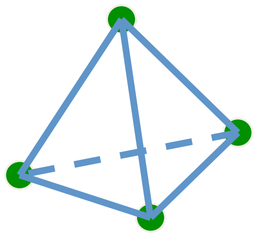
Quantifying the shape evolution



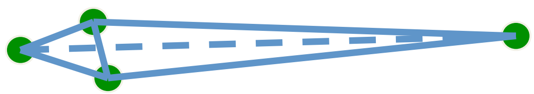
Eigenvalues of the moment of inertia tensor: g_i

$$I_i = \frac{g_i}{g_1 + g_2 + g_3}$$

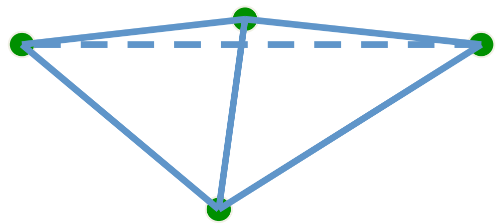
$I_1 = I_2 = I_3 = \frac{1}{3}$



$I_1 \approx 1, I_2 \approx I_3 \approx 0$



$I_1 \geq I_2 > I_3 \approx 0$



Pumir, Shraiman, Chertkov (**Phys. Rev. Lett.**, 2000)

Shape deformation

Evolution for the shape of the set of particles (disregard the motion of the center of mass):

$$d\rho/dt = M \,.\, \rho$$

Take as an initial condition a regular tetrahedron of size R_0

Expand $\boldsymbol{S}$ in Taylor series: $\boldsymbol{S} = \boldsymbol{S}_0 + t\,\boldsymbol{S}_1 + t^2/2\,\boldsymbol{S}_2 + \ldots$

Work in the eigen-basis of $\boldsymbol{S}_0$ (eigenvalues of $\boldsymbol{S}_0$: $S_{0,i}$, with

$$S_{0,1} > S_{0,2} > S_{0,3}$$

Obtain the following expression for $<g_i(t)>$:

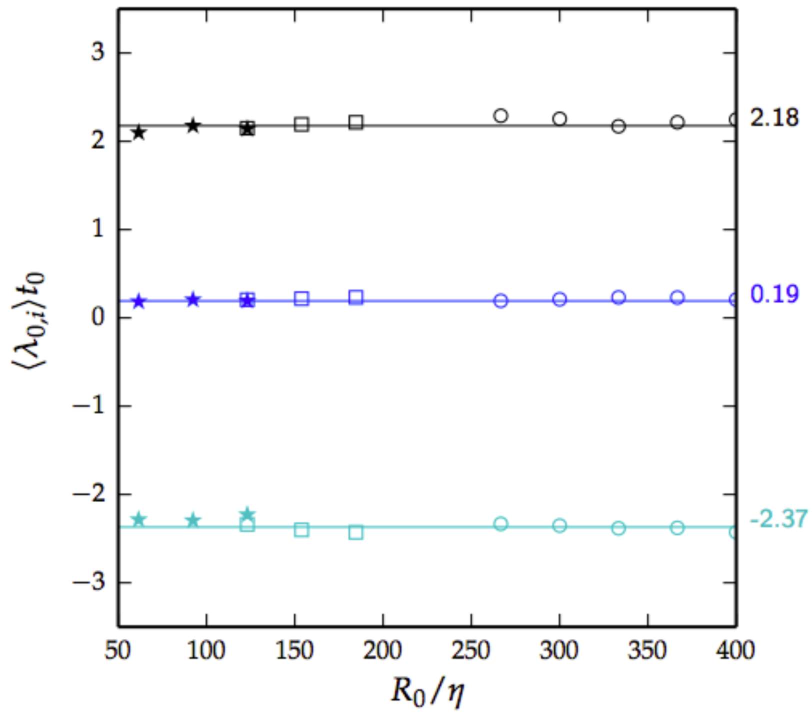
$$<g_i(t)> = 1/2\, R_0^2\, [\, 1 + 2 <S_{0,i}> t \\ + <2\, S_{0,i}^2 + S_{1,i}> t^2 + O(t^3)\,]$$

Shape deformation and time asymmetry t -> -t

The distribution of the eigenvalues of the $\boldsymbol{S_0}$, the strain tensor based on tetrads, is skewed; such that

$$<tr(S_{0,2})> \; > 0$$

~ a fundamental property of turbulence (Betchov 1956, Siggia 1981, Ashurst et al, 1987)..

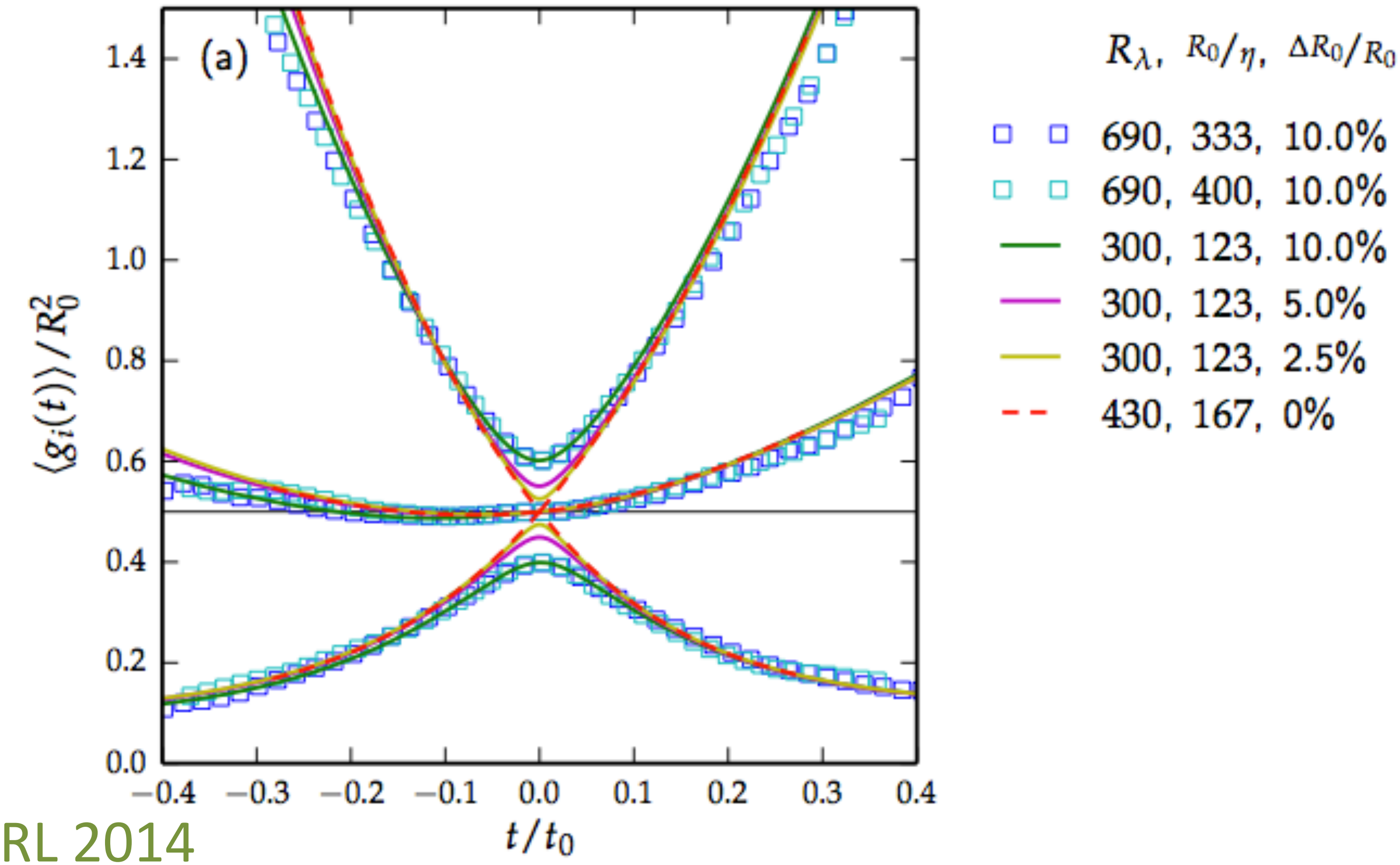


Jucha et al, 2014

Shape deformation and time asymmetry t -> -t

Consequence:

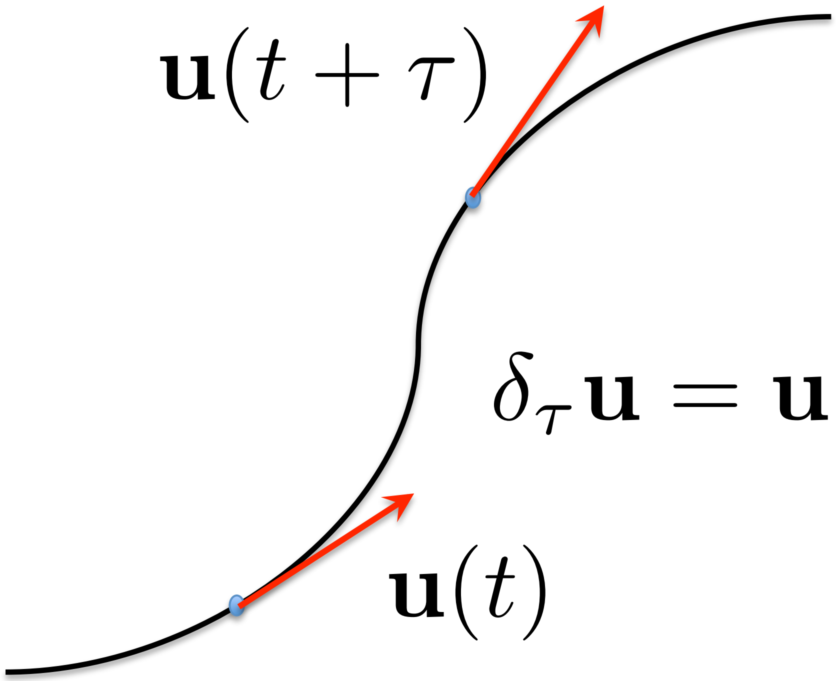
The intermediate eigenvalue *<$g_2(t)$>* is sensitive to the t -> -t asymmetry, to ***first*** order !



Jucha et al, PRL 2014

Can one “train the eye” by following one particle only ?

Lagrangian velocity increments



$$\delta_\tau \mathbf{u} = \mathbf{u}(t+\tau) - \mathbf{u}(t)$$

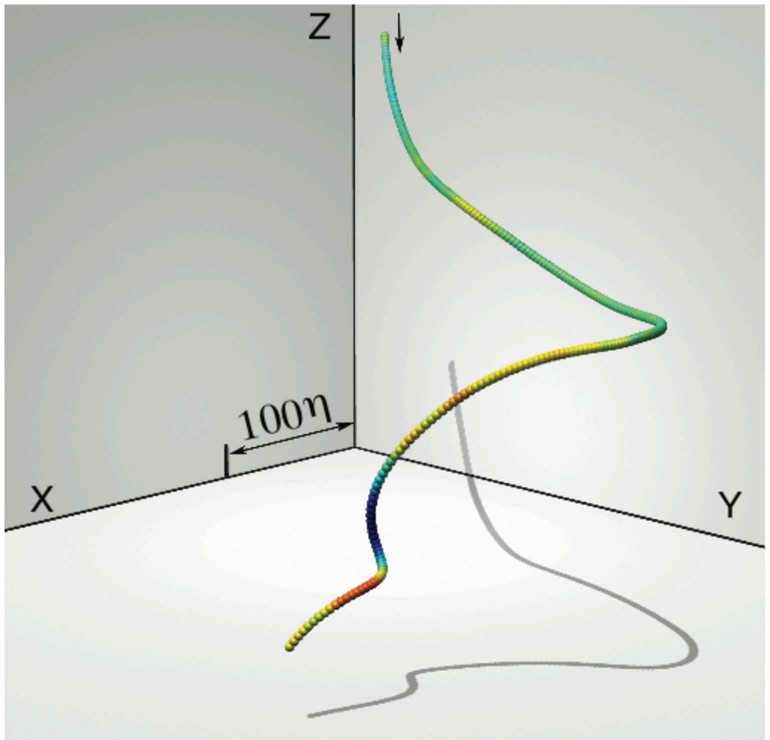
Lagrangian structure functions: $D_n(\tau) = <(\delta_\tau\ u)^n>$.

If one flips the direction of time: $t \to -t$,

$D_2(\tau) = \left\langle (\delta_\tau u)^2 \right\rangle$ is unchanged !

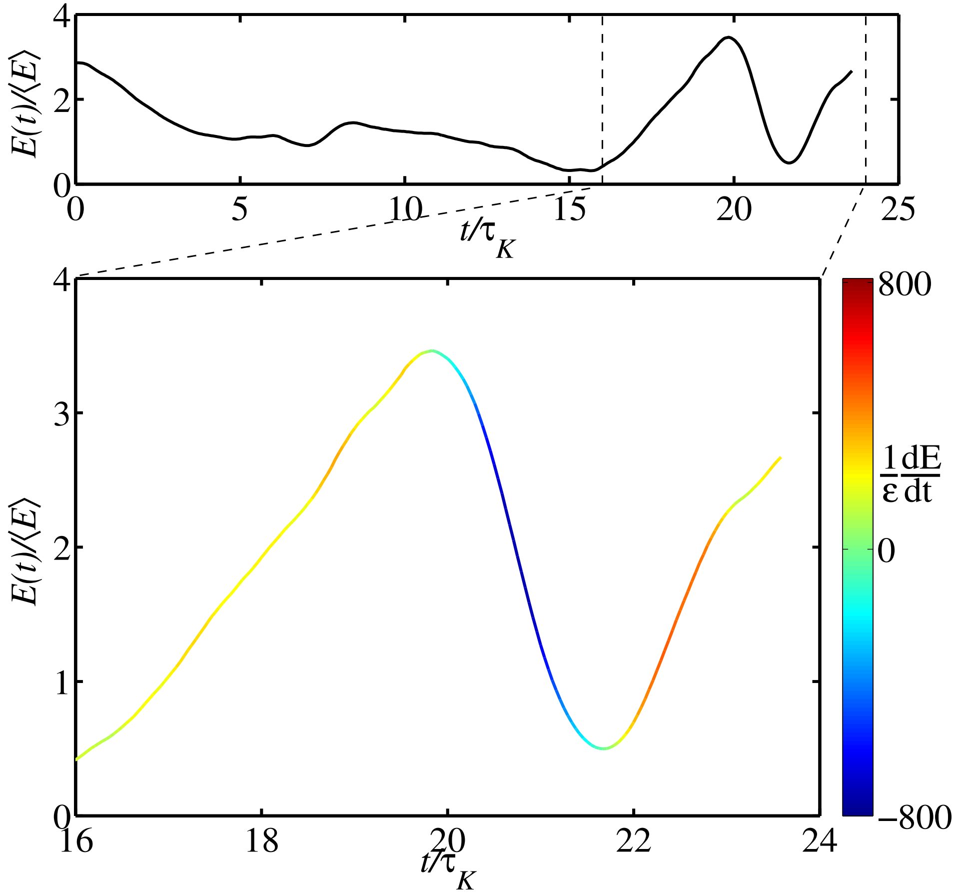
Can one detect irreversibility from one single particle trajectory ?

Observation: large velocity jumps of one given trajectory are associated with a stronger particle deceleration than acceleration.



Data from particle-tracking experiment.

$R_\lambda = 690, \;\; L/\eta \approx 2300$



Detecting time irreversibility from single-particle trajectory

Consider the *kinetic energy increments*:

$$W(\tau) = E(t+\tau) - E(t)$$

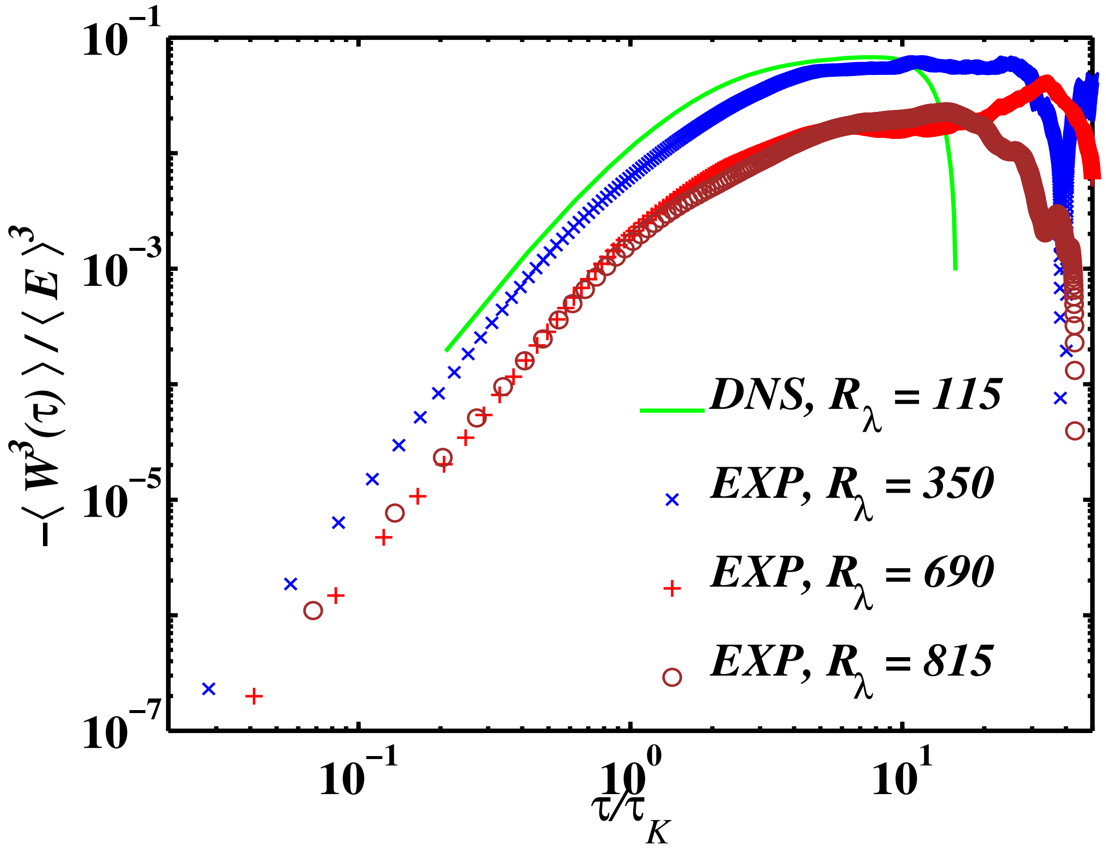
and their moments: $\langle W^n(\tau) \rangle$

The odd moments are not invariant under $t \to -t$
They can pick up the lack of symmetry seen experimentally !

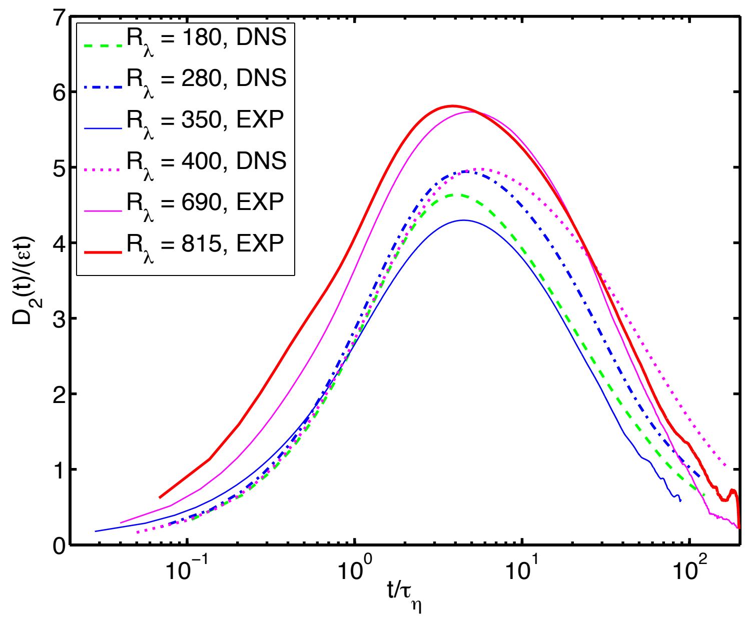
n.b.: the moments $\langle W^n(\tau) \rangle$ *cannot* be expressed in terms of velocity increments only

Detecting time irreversibility from single-particle trajectory

3D



The third moment of W(τ) is negative, and remains ~ constant when τ/τ_K is larger than ~2.



Note that plateau range is much more significant than that of the velocity structure functions.

A quantitative measure of irreversibility

How to measure irreversibility in a turbulent flow ?

- When the Reynolds number increases, the range of excited scale becomes larger.

Can one quantify the irreversibility, and get a notion as how it depends on the Reynolds number ?

First problem: find a proper measure of irreversibility !

At short times:

$$\langle W^3(\tau)\rangle = \left\langle \left(\frac{dE}{dt}\right)^3 \right\rangle \tau^3 + h.o.t.$$

$$= \langle (\mathbf{a}\cdot\mathbf{v})^3\rangle \tau^3 + h.o.t.$$

The observation of nonzero odd moments of W suggests that time-irreversibility should also be reflected in the statistics of the instantaneous power on a fluid particle

$$p \equiv \mathbf{a}\cdot\mathbf{v}$$

In particular, it implies that the PDF of power *p* is *negatively skewed*.

A quantity measuring Irreversibility (Ir)

A naïve suggestion:

Origin of irreversibility: the energy flux ε.

$$Ir = \varepsilon \,?$$

Problem: ε is dimensional – ie, it can be made arbitrarily large or small by a change of units.

A better suggestion:

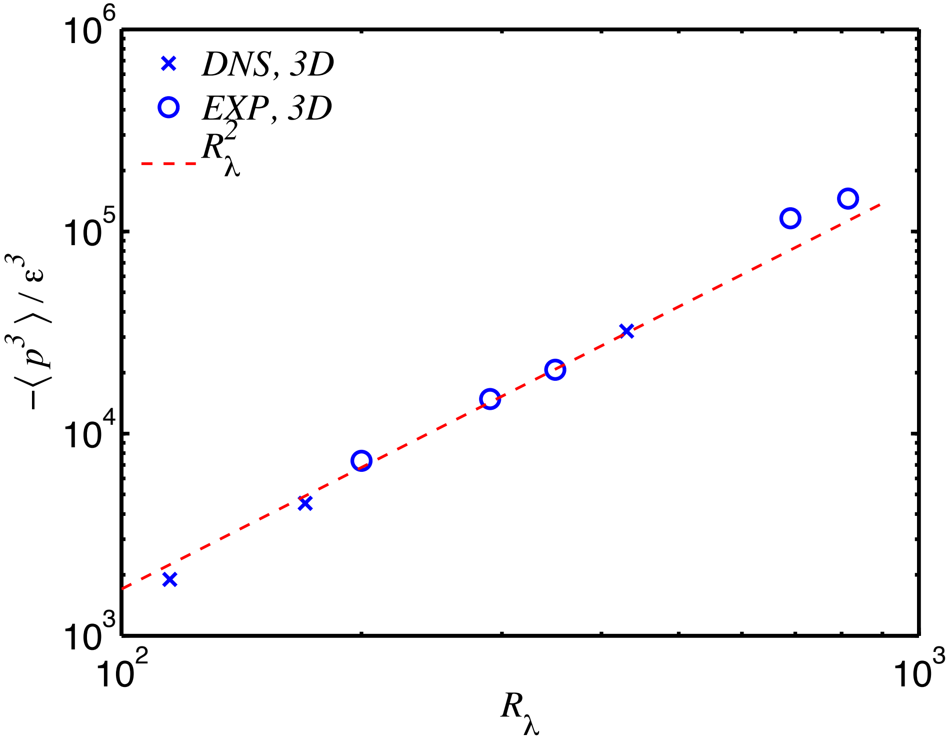
Use the simplest non-obvious moment of p, $\langle p^3 \rangle$, made properly dimensionless.

~notice that p and ε have the same dimension.

$$Ir = \langle p^3 \rangle / \varepsilon^3$$

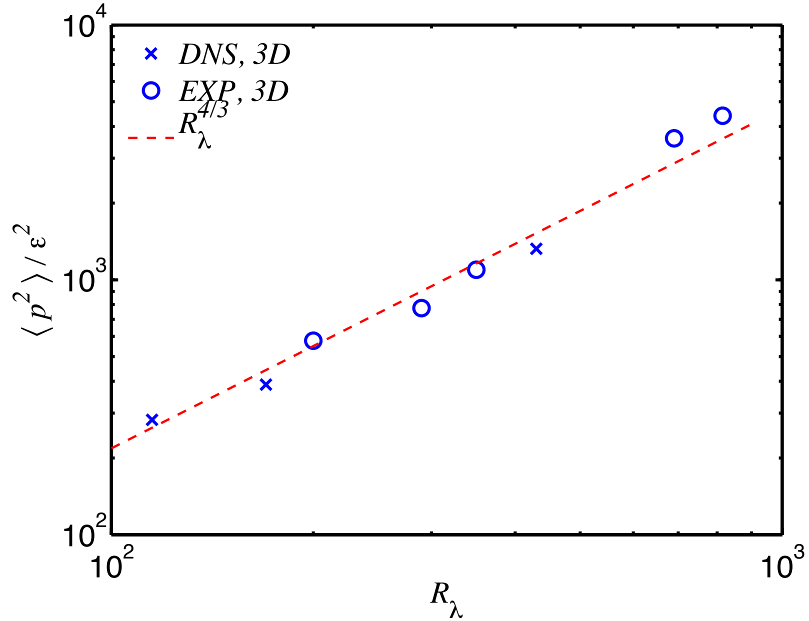
Scaling of *Ir*

3rd moment



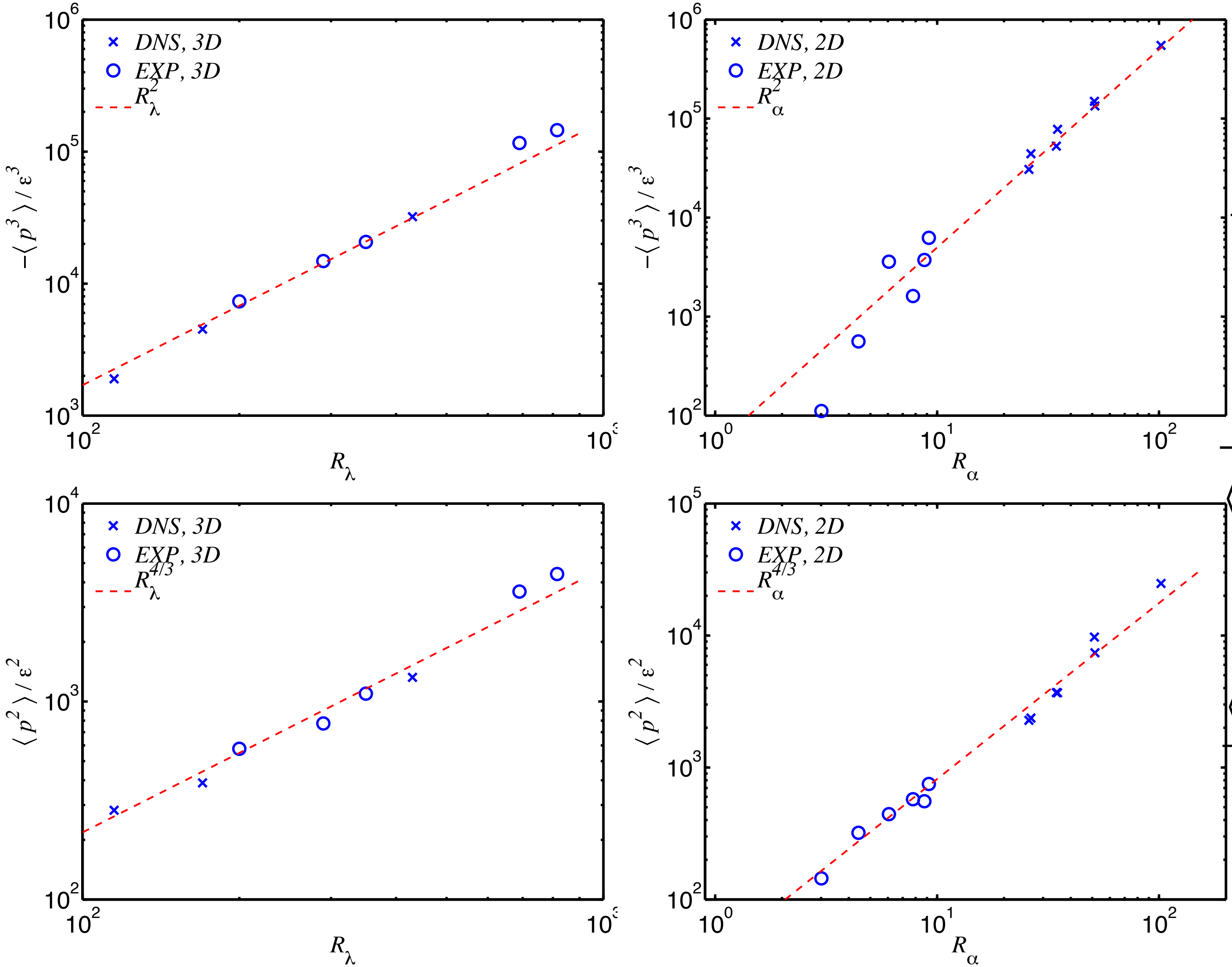
$Ir \sim R_\lambda{}^2$

nb: 2nd moment ~ $R_\lambda{}^{4/3}$



Statistics of the instantaneous power

3D 2D



$$\frac{\langle p^3\rangle}{\epsilon^3} \sim R_\lambda^2, R_\alpha^2$$

$$\frac{\langle p^3\rangle}{\langle p^2\rangle^{\frac{3}{2}}} \approx -0.5$$

$$\frac{\langle p^2\rangle}{\epsilon^2} \sim R_\lambda^{\frac{4}{3}}, R_\alpha^{\frac{4}{3}}$$

Asymmetry of the PDF of *p* and breaking of detailed balance

The observed lack of symmetry $p \rightarrow -p$ from the PDFs shows that:

$$P(E \to E + \Delta E) \neq P(E + \Delta E \to E)$$

Detailed balance is broken !!

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Redistribution of Kinetic Energy in Turbulent Flows

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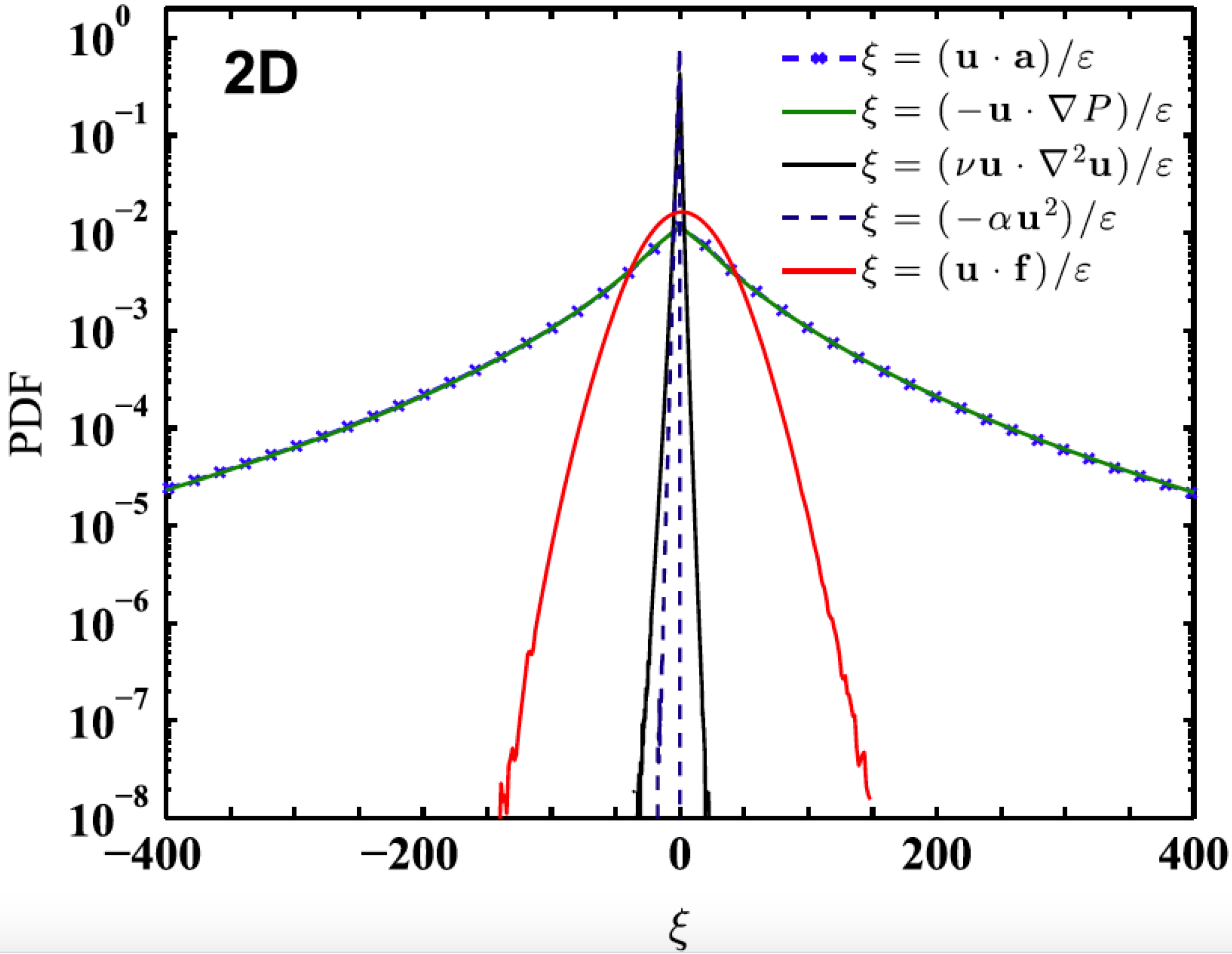
In statistically homogeneous turbulent flows, pressure forces provide the main mechanism to redistribute kinetic energy among fluid elements, without net contribution to the overall energy budget. This holds true in both two-dimensional (2D) and three-dimensional (3D) flows, which show fundamentally different physics. As we demonstrate here, pressure forces act on fluid elements very differently in these two cases. We find in numerical simulations that in 3D pressure forces strongly accelerate the fastest fluid elements, and that in 2D this effect is absent. In 3D turbulence, our findings put forward a mechanism for a possibly singular buildup of energy, and thus may shed new light on the smoothness problem of the solution of the Navier-Stokes equation in 3D.

Power

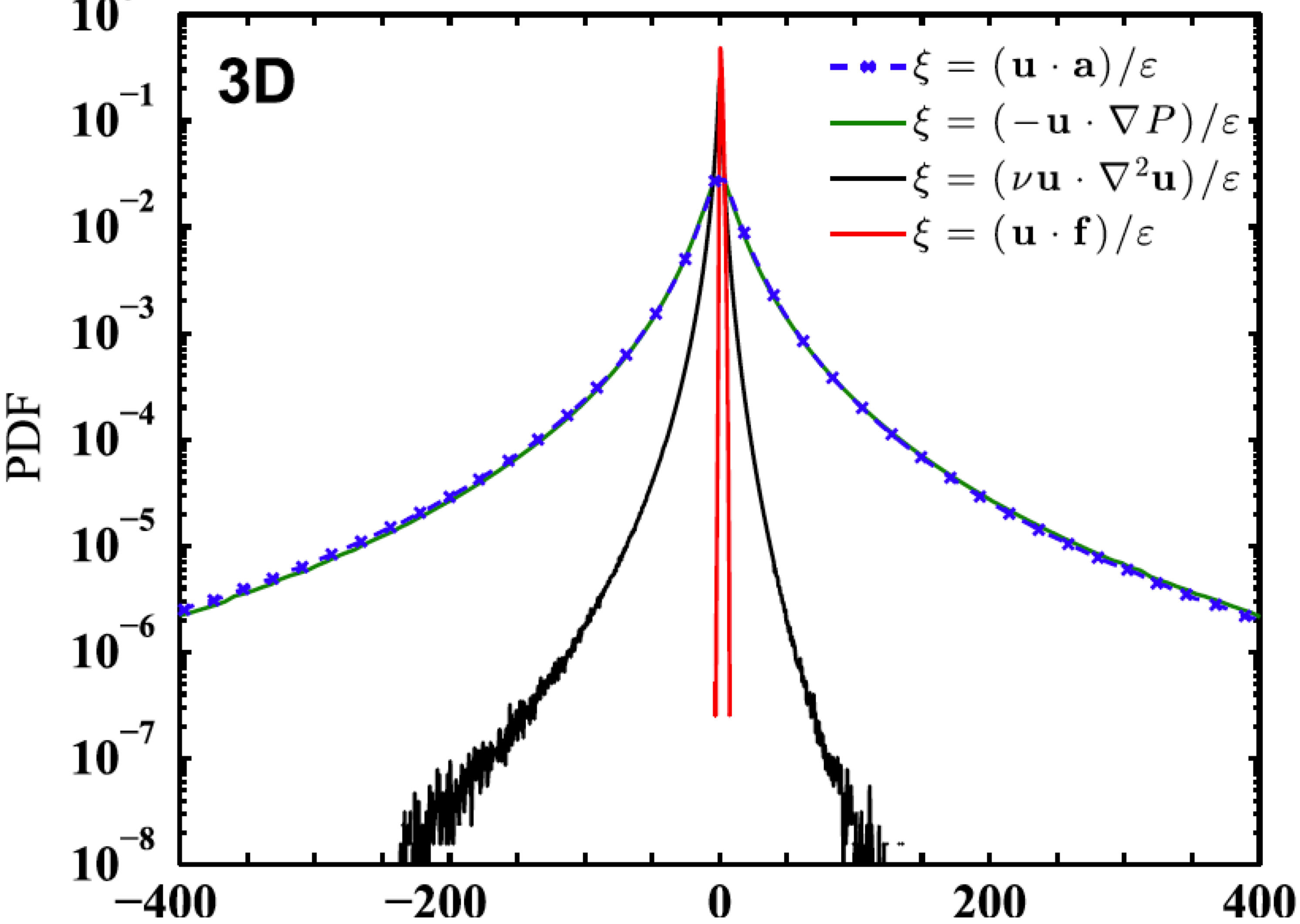
$$p = \mathbf{u} \cdot \mathbf{a} = -\mathbf{u} \cdot \nabla P + \mathbf{u} \cdot \mathbf{f} + \mathbf{u} \cdot \mathbf{D}$$

3D: $$\mathbf{D} = \nu \nabla^2 \mathbf{u}$$

2D: $$\mathbf{D} = \nu \nabla^2 \mathbf{u} - \alpha \mathbf{u}$$



2D
$\xi = (\mathbf{u}\cdot\mathbf{a})/\varepsilon$
$\xi = (-\mathbf{u}\cdot\nabla P)/\varepsilon$
$\xi = (\nu\mathbf{u}\cdot\nabla^2\mathbf{u})/\varepsilon$
$\xi = (-\alpha\mathbf{u}^2)/\varepsilon$
$\xi = (\mathbf{u}\cdot\mathbf{f})/\varepsilon$
PDF
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
−400
−200
0
200
400
ξ



$$\langle(\mathbf{u}\cdot\mathbf{f})^2\rangle \ll \langle(\nu\mathbf{u}\cdot\nabla^2\mathbf{u})^2\rangle \ll \langle(\mathbf{u}\cdot\nabla P)^2\rangle$$

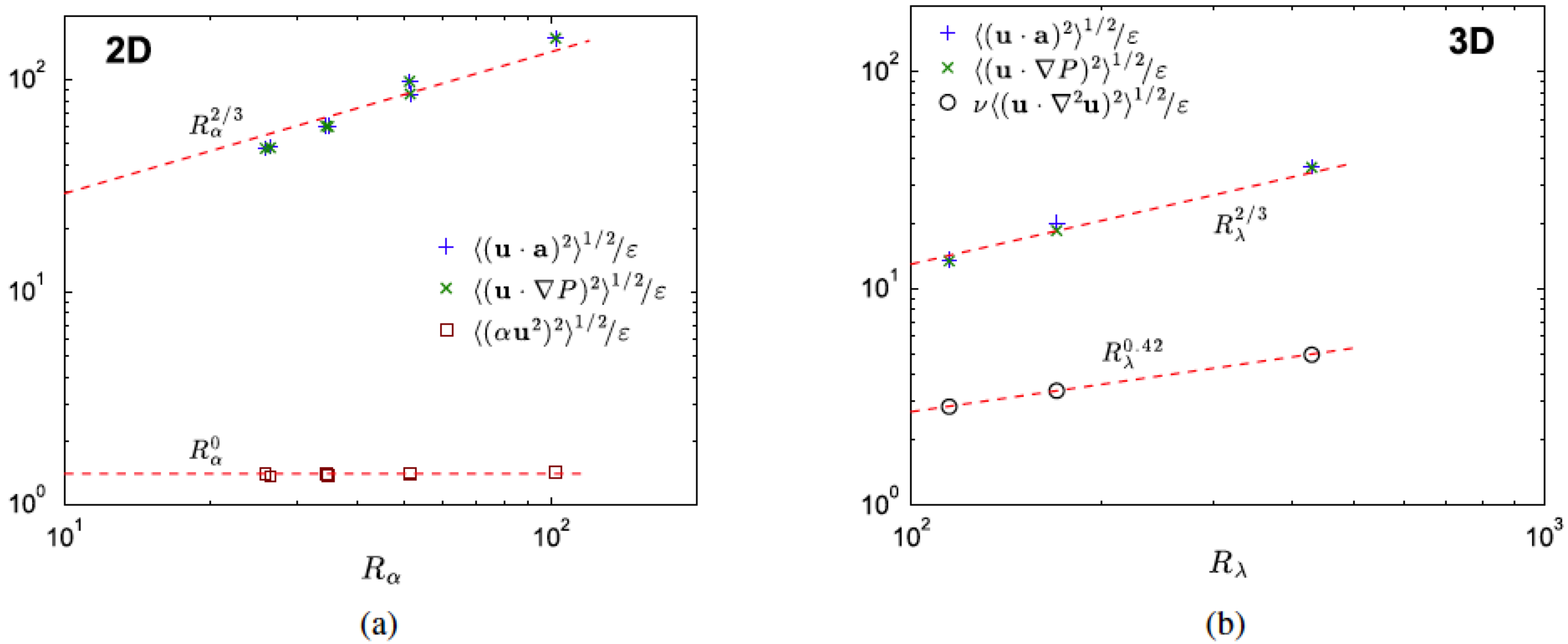


FIG. 2. Variation of the standard deviations of $\mathbf{u}\cdot\mathbf{a}/\varepsilon$ (blue pluses), $\mathbf{u}\cdot\nabla P/\varepsilon$ (green crosses), $\nu\mathbf{u}\cdot\nabla^2\mathbf{u}/\varepsilon$ (black circles), and $\alpha u^2/\varepsilon$ (brown squares) as functions of Reynolds numbers in both (a) 2D and (b) 3D.

One may attempt to evaluate the pressure contribution to power by assuming $\langle(\mathbf{u}\cdot\nabla P)^2\rangle \approx \langle\mathbf{u}^2\rangle\langle(\nabla P)^2\rangle$ and then using the standard estimate $|\nabla P| \approx |\mathbf{a}| \propto (\varepsilon^3/\nu)^{1/4}$ to obtain $\langle(\mathbf{u}\cdot\nabla P)^2\rangle/\varepsilon^2 \propto R_\lambda$ or R_α.

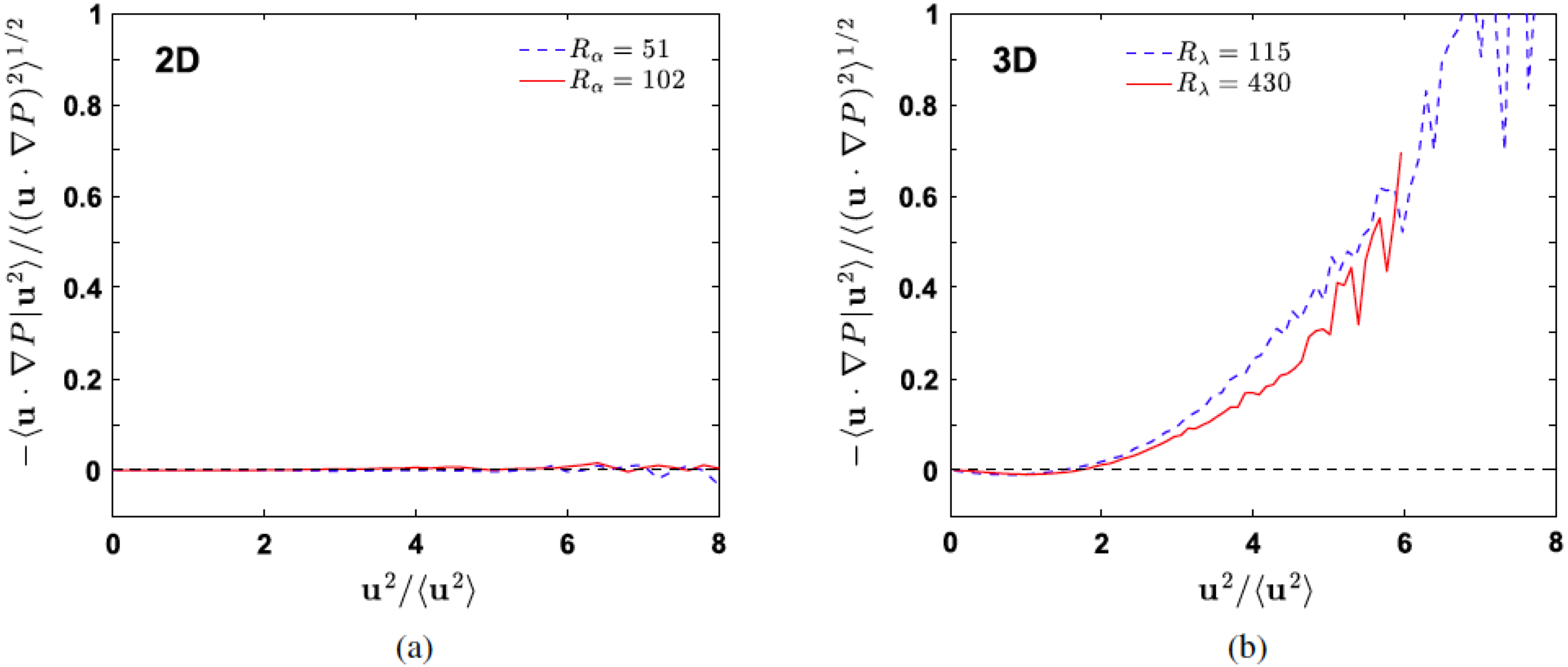
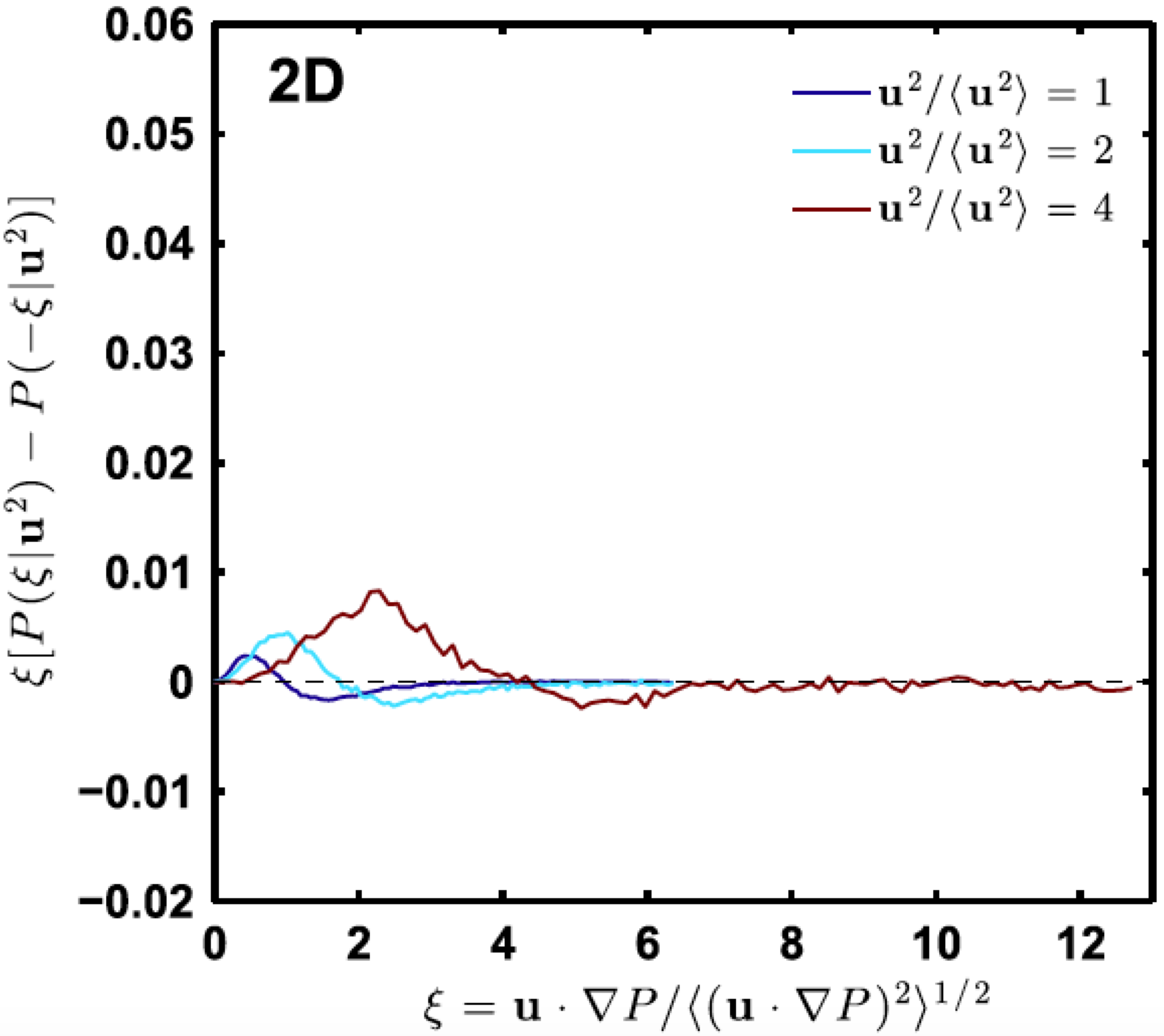
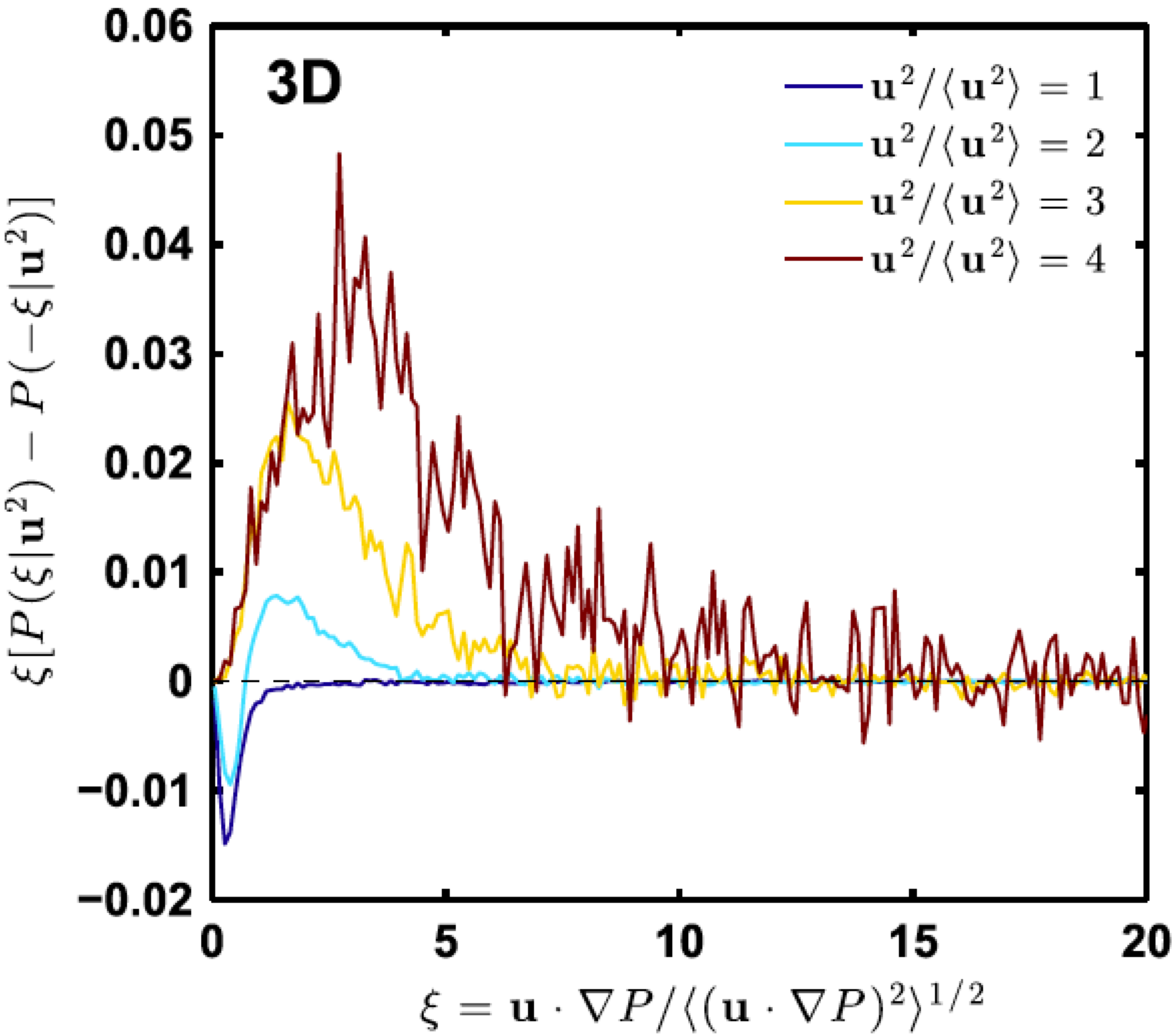


FIG. 3. The role of pressure in redistributing kinetic energy among fluid particles points to the dramatic difference between 2D and 3D turbulence. (a) The average of the pressure-gradient term conditioned on the kinetic energy of the fluid particle, $\langle -\mathbf{u}\cdot\nabla P|u^2\rangle$, in 2D turbulence (DNS data at $R_\alpha = 51$ and $R_\alpha = 102$). The average of $-\mathbf{u}\cdot\nabla P$, conditioned on u^2, is extremely small, and consistent with being 0, implying that, on average, pressure does not redistribute energy among particles in 2D flows. (b) The same conditional average for 3D turbulence (DNS data at $R_\lambda = 115$ and 430). Contrary to the 2D case, the conditional mean of the pressure term is negative for particles with small u^2, and becomes strongly positive for larger values of u^2, which implies that, on average, the pressure term in 3D turbulence takes energy from *slow* particles and gives to *fast* particles. This leads to a runaway effect and can be stopped only by viscous forces.

In particular, it is important to know whether the conditional mean $\langle -\mathbf{u}\cdot\nabla P|\mathbf{u}^2\rangle$ grows faster than $\mathbf{u}^2$, so that it can lead to blowup of particle energy in a finite time.



2D
$\mathbf{u}^2/\langle\mathbf{u}^2\rangle = 1$
$\mathbf{u}^2/\langle\mathbf{u}^2\rangle = 2$
$\mathbf{u}^2/\langle\mathbf{u}^2\rangle = 4$
0.06
0.05
0.04
0.03
0.02
0.01
0
−0.01
−0.02
0
2
4
6
8
10
12
$\xi[P(\xi|\mathbf{u}^2) - P(-\xi|\mathbf{u}^2)]$
$\xi = \mathbf{u}\cdot\nabla P/\langle(\mathbf{u}\cdot\nabla P)^2\rangle^{1/2}$


3D
$\mathbf{u}^2/\langle\mathbf{u}^2\rangle = 1$
$\mathbf{u}^2/\langle\mathbf{u}^2\rangle = 2$
$\mathbf{u}^2/\langle\mathbf{u}^2\rangle = 3$
$\mathbf{u}^2/\langle\mathbf{u}^2\rangle = 4$
0.06
0.05
0.04
0.03
0.02
0.01
0
−0.01
−0.02
0
5
10
15
20
$\xi[P(\xi|\mathbf{u}^2) - P(-\xi|\mathbf{u}^2)]$
$\xi = \mathbf{u}\cdot\nabla P/\langle(\mathbf{u}\cdot\nabla P)^2\rangle^{1/2}$

Summary

- big difference 2D versus 3D

- in 3D fast particles get accelerated even further

- Is it a runaway effect at high Re?

Xu et al, PNAS, 2014 ; Jucha et al, PRL, 2014.

Irreversibility *equals* small-scale generation in 3D turbulent flows

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Significance

Three-dimensional turbulent fluids are stirred at scales much larger than those at which energy is dissipated. This has two major consequences: (i) generating small scales from large ones is essential in turbulence and (ii) turbulent flows are time irreversible. Here we demonstrate analytically with support from numerical simulations that these two phenomena are fundamentally connected by the amplification of vorticity. Remarkably, solely the irreversible motion of an individual fluid element in space and time is sufficient to provide quantitative information on small-scale generation in turbulence.

- Small scales from vortex stretching

$$\langle \omega \cdot \mathbf{S} \cdot \omega \rangle = -\frac{1}{3} \langle \mathrm{tr}(\mathbf{S}^3) \rangle$$

Remarkably, in homogeneous and isotropic flows, $\langle \mathrm{tr}(\mathbf{S}^3) \rangle$, hence vorticity production, can be simply expressed in terms of the third moment of a single component of the velocity gradient tensor, $\partial_x u_x$ [7, 10]. Thus, small scale generation implies an asymmetry of the distribution of the velocity derivative. Available data from experiments using hot-wire anemometry [11, 12] and from direct numerical simulations (DNS) have led to the conclusion that the normalized third moment of $\partial_x u_x$, the skewness, $S_{\partial_x u_x} \equiv \langle (\partial_x u_x)^3 \rangle / \langle (\partial_x u_x)^2 \rangle^{3/2}$ is negative, of the order of ≈ -0.5, with at most a weak dependence on the Reynolds number [13, 14].

Power following a tracer

$$-\langle p^3 \rangle \propto \varepsilon^3 R_\lambda^2, \qquad [1]$$

with the skewness of p being approximately constant, $S_p \equiv \langle p^3 \rangle / \langle p^2 \rangle^{3/2} \approx -0.5$. This result is unexpected, and the aim of this article is to provide an understanding of it. More precisely, we propose here a connection between the cubic moment of the power, $\langle p^3 \rangle$, and the generation of small scales, i.e., vortex stretching.

$$p = \mathbf{a} \cdot \mathbf{u}$$

$$\mathbf{a} = -\nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\mathbf{a} = \mathbf{a}_L + \mathbf{a}_C \quad \mathbf{a}_L = \partial_t \mathbf{u} \quad \mathbf{a}_C = (\mathbf{u} \cdot \nabla)\mathbf{u}$$

$$\langle p_C^3 \rangle \propto -\langle \omega \cdot \mathbf{S} \cdot \omega \rangle$$

Vortex stretching and moments of p_C

$$p_C = \mathbf{u} \cdot (\mathbf{u} \cdot \nabla)\mathbf{u} = \mathbf{u} \cdot \mathbf{S} \cdot \mathbf{u}$$

$\mathbf{u} = \sum_{i=1}^{3} u_i \mathbf{e}_i$, where $u_i = \mathbf{u} \cdot \mathbf{e}_i$ is the coordinate of $\mathbf{u}$ along the direction $\mathbf{e}_i$, and the rate of strain tensor is expressed as $\mathbf{S} = \sum_{i=1}^{3} \lambda_i \mathbf{e}_i \mathbf{e}_i$. Denoting $\hat{x}_i$ the cosines of the angles between the velocity $\mathbf{u}$ and the unit vectors $\mathbf{e}_i$: $\hat{x}_i \equiv \mathbf{u} \cdot \mathbf{e}_i / |\mathbf{u}| = u_i / |\mathbf{u}|$, the expression of p_C reduces to:

$$p_C = \sum_{i=1}^{3} \lambda_i u_i^2 = \mathbf{u}^2 \sum_{i=1}^{3} \lambda_i \hat{x}_i^2. \qquad [5]$$

Let us now assume that S and u are uncorrelated

$$\langle p_C^3 \rangle = \frac{8}{105} \langle |\mathbf{u}|^6 \rangle \langle \mathrm{tr}(\mathbf{S}^3) \rangle = -\frac{8}{35} \langle |\mathbf{u}|^6 \rangle \langle \omega \cdot \mathbf{S} \cdot \omega \rangle. \qquad [6]$$

Let us now assume that S and u are uncorrelated

$$\langle p_C^3 \rangle = \frac{7}{225} S_{\partial_x u_x} R_\lambda^3 \varepsilon^3. \qquad [7]$$

The assumptions of lack of correlation between $\mathbf{u}$ and $\mathbf{S}$, and of a Gaussian distribution of the velocity $\mathbf{u}$, also lead to an exact determination of the variance of p_C: $\langle p_C^2 \rangle = \frac{1}{15} R_\lambda^2 \varepsilon^2$,

$$\langle p^3 \rangle = -\frac{8}{35}(1 - \beta - \zeta)\langle |\mathbf{u}|^6 \rangle \langle \omega \cdot \mathbf{S} \cdot \omega \rangle, \qquad [13]$$

Table 2. Third moments of the distributions of p/ε, p_C/ε and p_L/ε at the three Reynolds numbers studied in this article. The last 8 rows compare the normalized moments with our predictions.

R_λ	193	275	430
$-\langle p^3 \rangle/\varepsilon^3$	3.87×10^3	1.23×10^4	3.21×10^4
$-\langle p_C^3 \rangle/\varepsilon^3$	5.39×10^4	2.40×10^5	1.00×10^6
$\langle p_C^2\, p_L \rangle/\varepsilon^3$	4.54×10^4	2.05×10^5	8.99×10^5
$-\langle p_C\, p_L^2 \rangle/\varepsilon^3$	4.02×10^4	1.84×10^5	8.29×10^5
$\langle p_L^3 \rangle/\varepsilon^3$	3.44×10^4	1.63×10^5	7.63×10^5
$\zeta = \langle p_L^2 p \rangle/\langle p_C^3 \rangle$	0.108	0.088	0.066
$1 - \beta$	0.17	0.14	0.11
$\langle p^3 \rangle/\langle p_C^3 \rangle$	0.072	0.051	0.032
$1 - \beta - \zeta$	0.061	0.052	0.037
$-\langle p_L^3 \rangle/\langle p_C^3 \rangle$	0.64	0.68	0.76
$\beta - 2\zeta$	0.62	0.69	0.77
$\langle p_C\, p_L^2 \rangle/\langle p_C^3 \rangle$	0.75	0.77	0.83
$\beta - \zeta$	0.72	0.77	0.83
$-\langle p_C^2\, p_L \rangle/\langle p_C^3 \rangle$	0.84	0.85	0.90
β	0.83	0.86	0.90

Summary and perspectives

Can one detect time-irreversibility from the motion of particles ?

- With ***2 particles or more***: YES [Jucha et al, PRL, 2014; related to known-properties of turbulent flows]
- With ***only 1 particle*** ? YES [Xu et al, PNAS, 2014]!
- Irreversibility is related to vortex stretching and small scale generation

3D turbulence – pressure leads to "blow out"

Perspectives:

…much to be learned from studying the motion of particles in a turbulent flow !

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