HIGH REYNOLDS NUMBER LARGE EDDY SIMULATION: WHERE REAL AND VIRTUAL TURBULENCE MEET?

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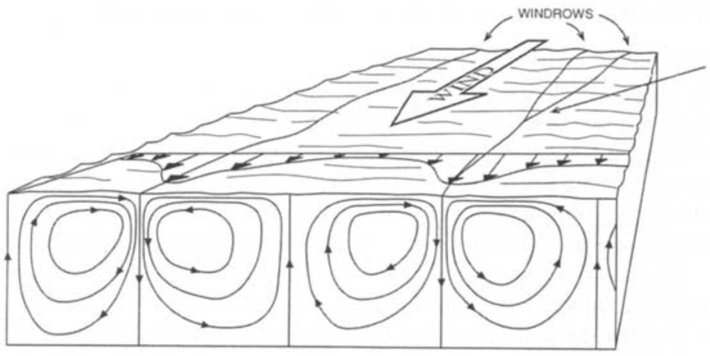
NCAR is sponsored by the National Science Foundation
Support from Office of Naval Research

Gulf of Tehuantepec, U ~ [20 – 25] m/s Courtesy Ken Melville

HIGH RESOLUTION AIR-SEA INTERACTION: U ~ 15 m/s courtesy T. Hristov

LANGMUIR CIRCULATIONS (I. Langmuir, 1938)

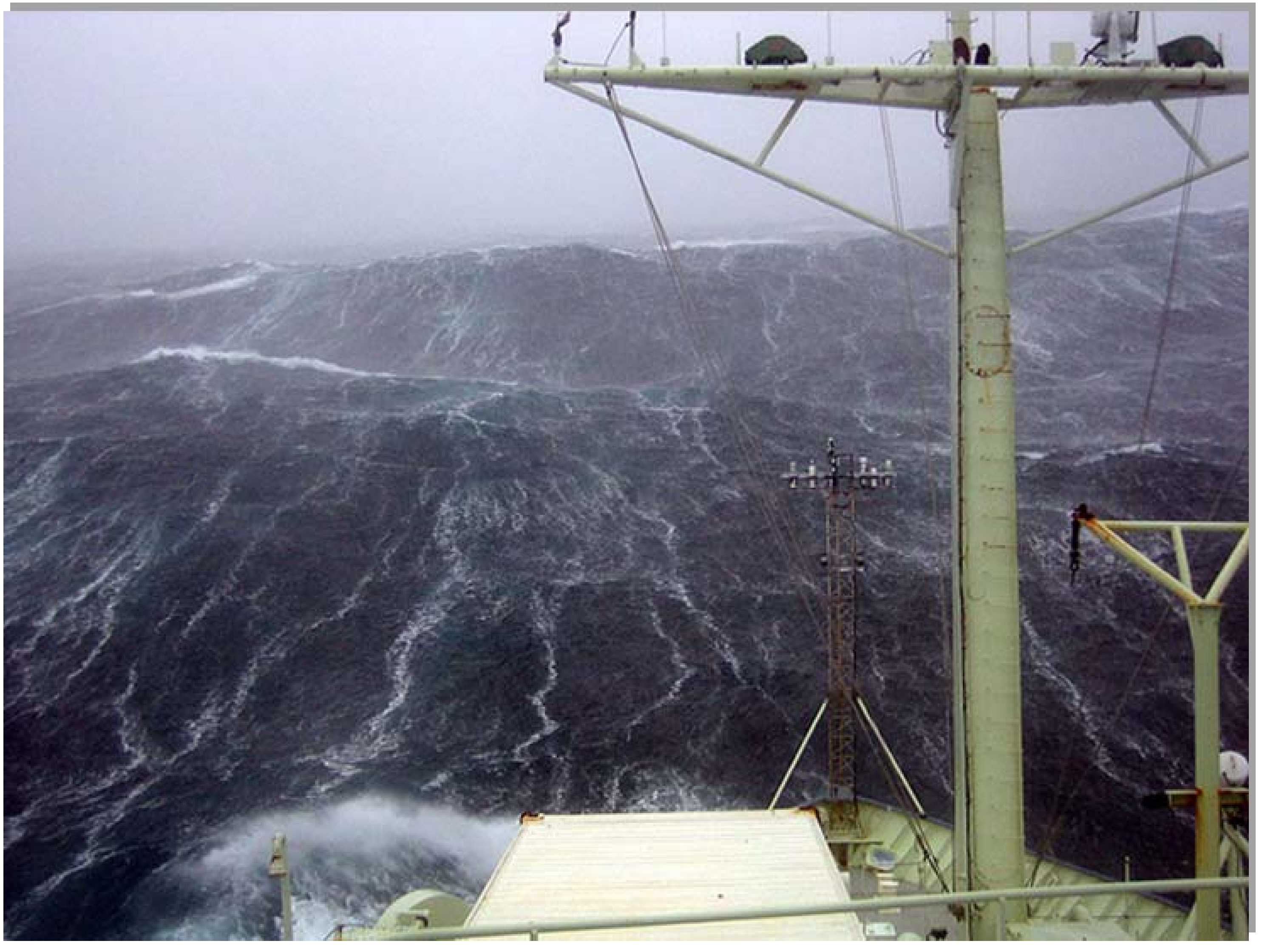


Wave propagation direction

An example of wave-current interaction, foam lines in the Great Salt Lake, courtesy S. Monismith

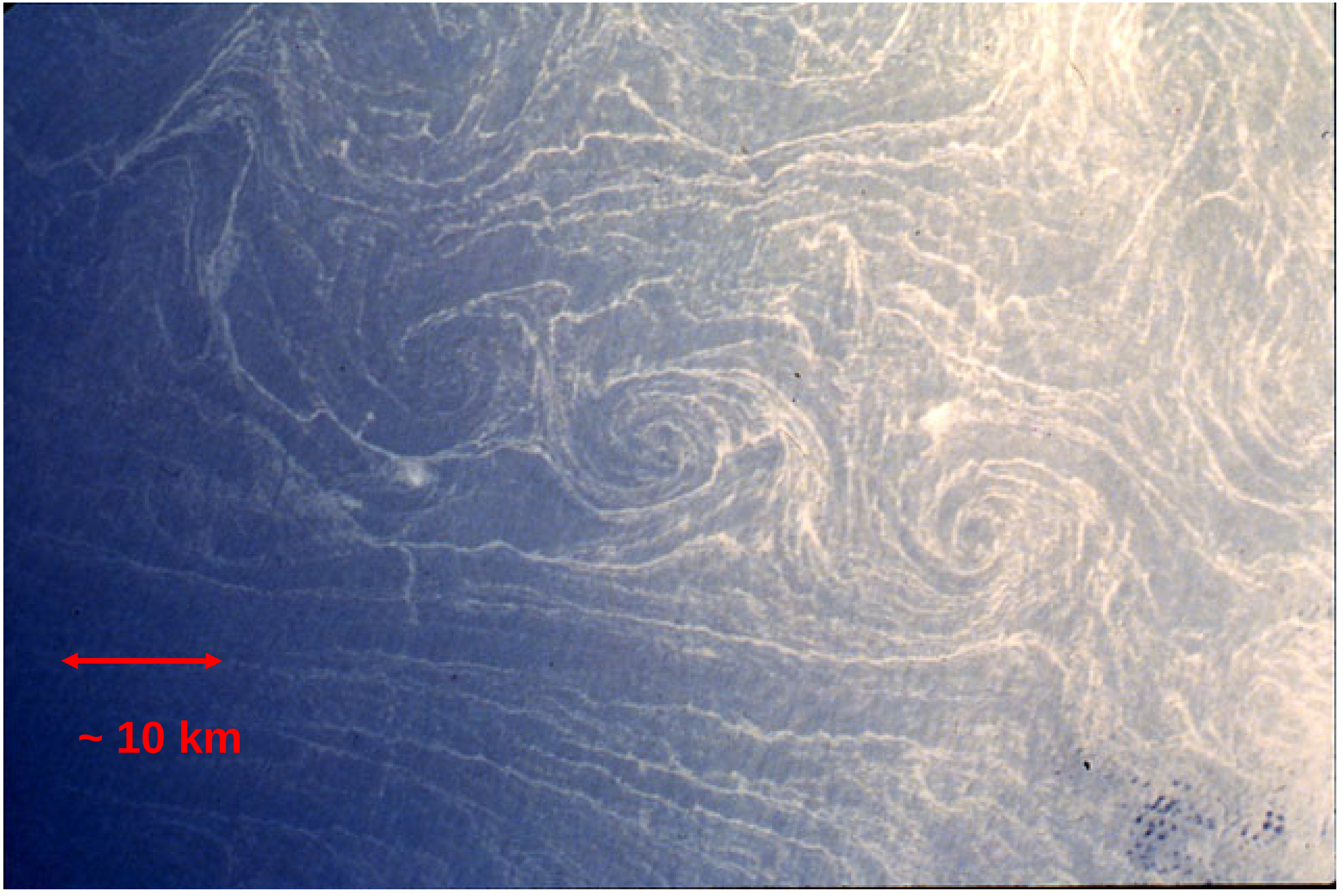
Langmuir circulations in Monterey Bay courtesy Luc Lenain

LANGMUIR CIRCULATIONS IN HIGH WINDS?



Photograph from the research vessel *Knorr* in winds ranging from 60 to 100 knots and 30-40 foot tall waves on an expedition to the Irminger Sea in October 2007. (Photo by Kjetil Vage, Woods Hole Oceanographic Institution)

SPIRALS ON THE SEA



Photograph of a cyclonic spiral-eddy street off the coast of the Egyptian/Libyan border. Eddy radii are ≈ 5 km, and scum convergence lines are ~ 100s m wide. The street configuration suggests a recent vortex roll-up from an unstable submesoscale front or wake. (Scully-Power, 1986), courtesy J. McWilliams.

MARINE BOUNDARY LAYERS WITH WIND-WAVE AND WAVE-CURRENT COUPLINGS

Motivation:

- Do waves matter for the atmospheric and oceanic boundary layers (ABL and OBL)?

- How often are the ABL and OBL in a wave influenced regime, low winds, high winds, etc?

Approach:

- Can we craft and use turbulence resolving simulations plus wave prescriptions that shed light on the coupling processes?

OUTLINE OF LES MODEL EQUATIONS FOR FLOW OVER 3D WAVES $\eta(x,y,t)$

$$\frac{\partial U_i}{\partial \xi_i} = 0$$ ← Continuity

$$\frac{\partial}{\partial t}\left(\frac{1}{J}\right) = \frac{\partial}{\partial \zeta}\left(\frac{\partial z}{\partial t}\right)$$ ← Space conservation

$$\frac{\partial}{\partial t}\left(\frac{\overline{u}_i}{J}\right) + \frac{\partial}{\partial \xi_j}\left[(U_j - \delta_{3j} z_t)\overline{u}_i\right] = \mathcal{F}_i$$ ← Momentum

$$\frac{\partial}{\partial t}\left(\frac{\overline{\theta}}{J}\right) + \frac{\partial}{\partial \xi_j}\left[(U_j - \delta_{3j} z_t)\overline{\theta}\right] = \mathcal{M}$$ ← Scalar (temperature)

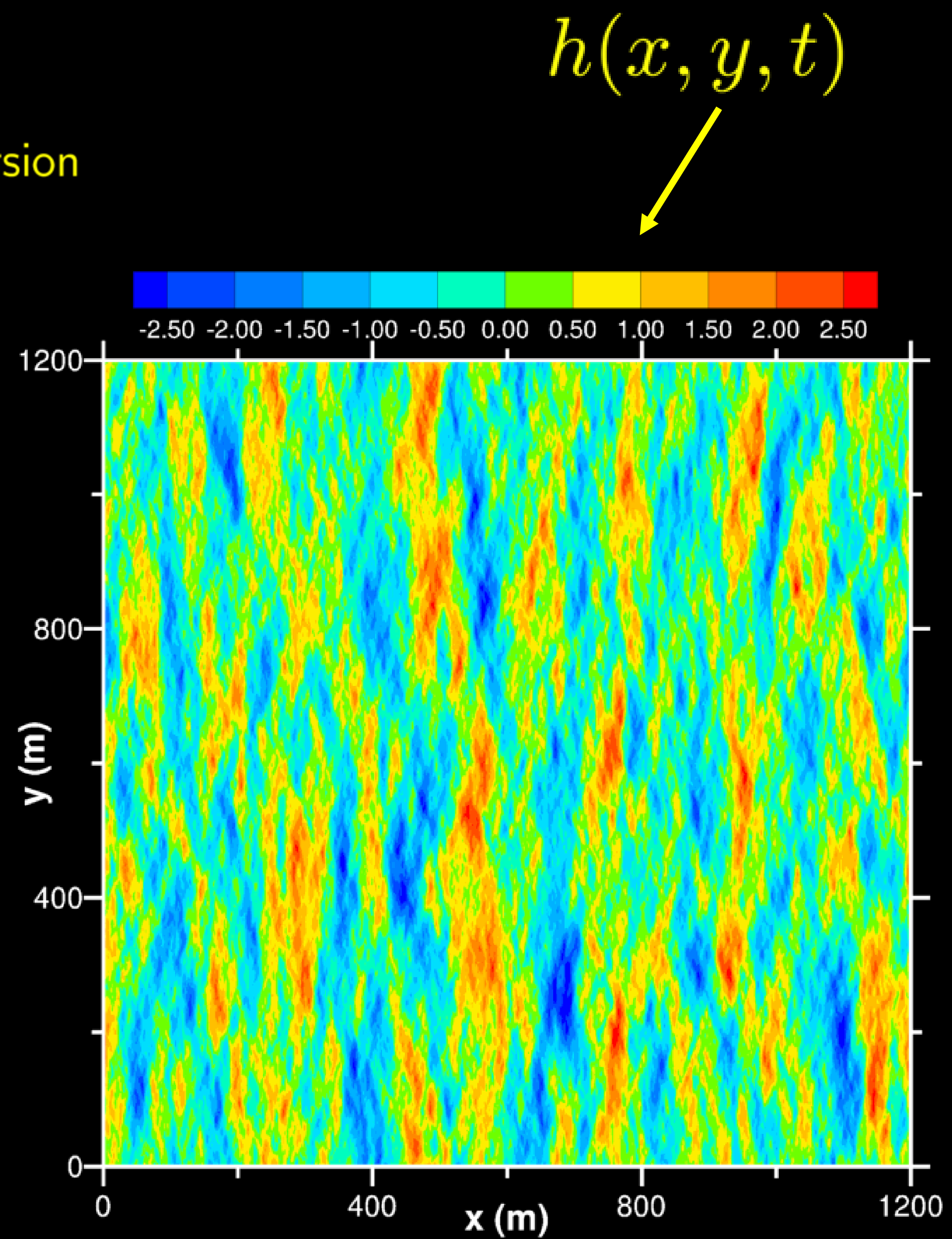
$$\frac{\partial}{\partial t}\left(\frac{e}{J}\right) + \frac{\partial}{\partial \xi_j}\left[(U_j - \delta_{3j} z_t)e\right] = \mathcal{R}$$ ← SGS energy

$$\frac{\partial}{\partial \xi_i}\left[\frac{1}{J}\frac{\partial \xi_i}{\partial x_j}\frac{\partial \xi_m}{\partial x_j}\frac{\partial p^*}{\partial \xi_m}\right] = \mathcal{S}$$ ← Pressure equation

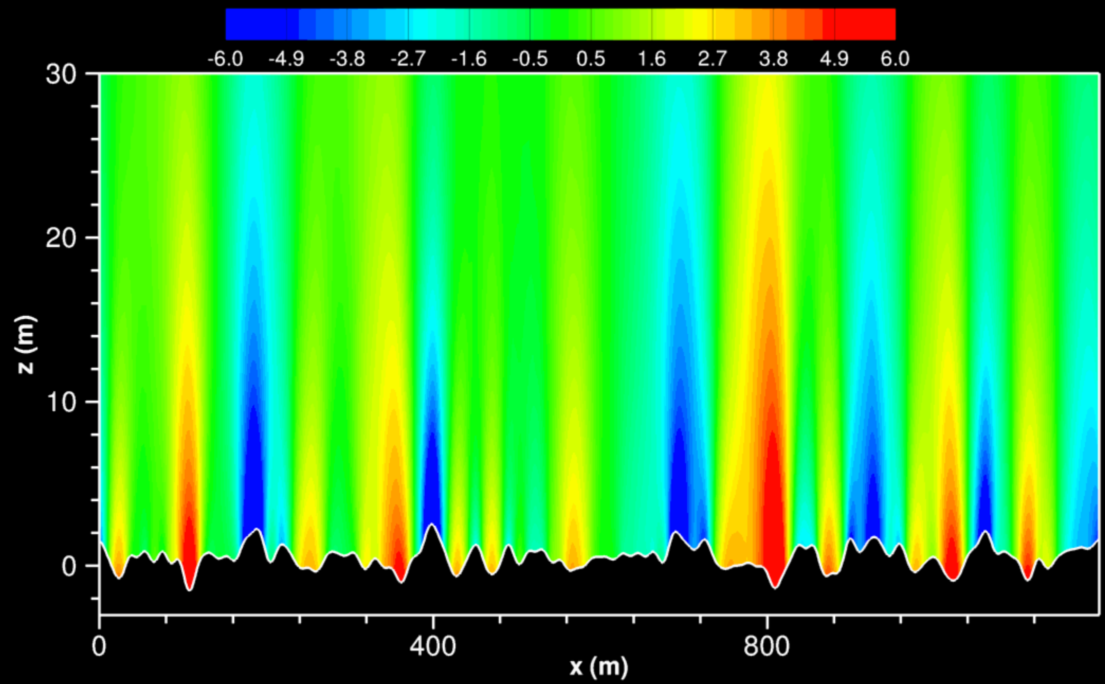
Used as many 16,384 processors on Cray XT4

DETAILS OF LES EXPERIMENTS

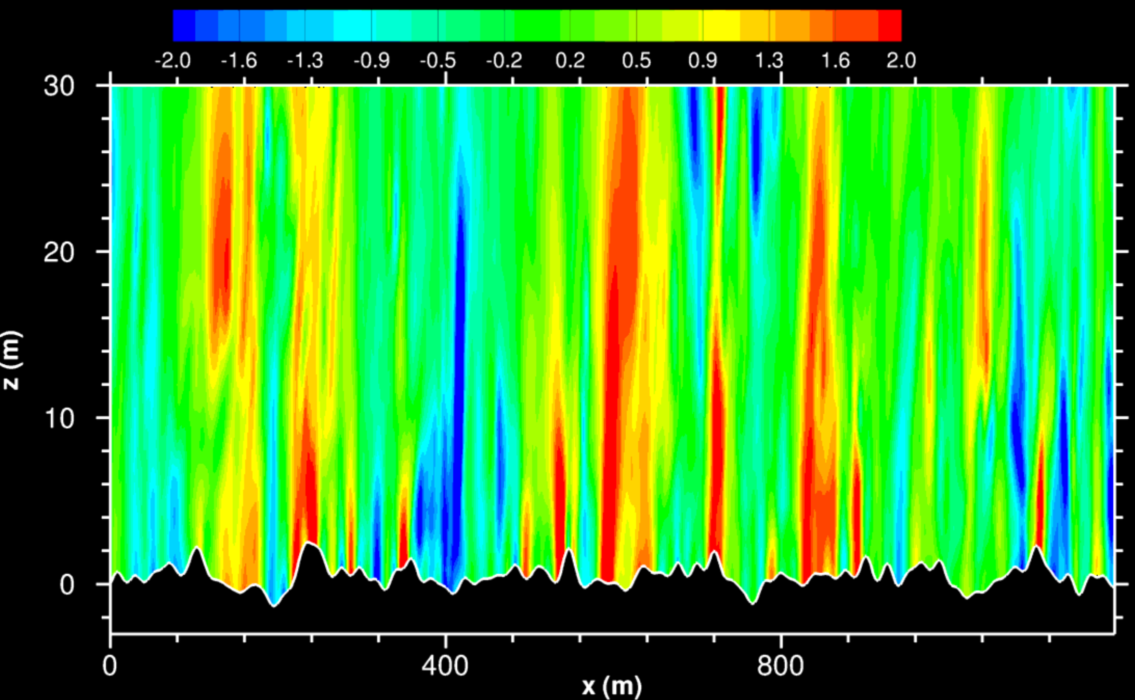
- Neutral flow, overlying temperature inversion
- $z_i = 400$ m, $z_o = 0.0002$ m
- Wave age $C_p/U_{10} = [1.5, 4.8]$
- Geostrophic winds $U_g = [5, 20]$ m s^{-1}
- Pierson-Moskowitz spectrum (held fixed)
- Grid $(512 \times 512 \times 128)$
- $\triangle x = \triangle y = 2.3$ m,
  $\triangle z = 1$ m at surface
- 512 processors $\sim$ 50,000 cpu hours



PRESSURE FLUCTUATIONS IN XZ PLANES NEAR THE WATER SURFACE
(note different scales)

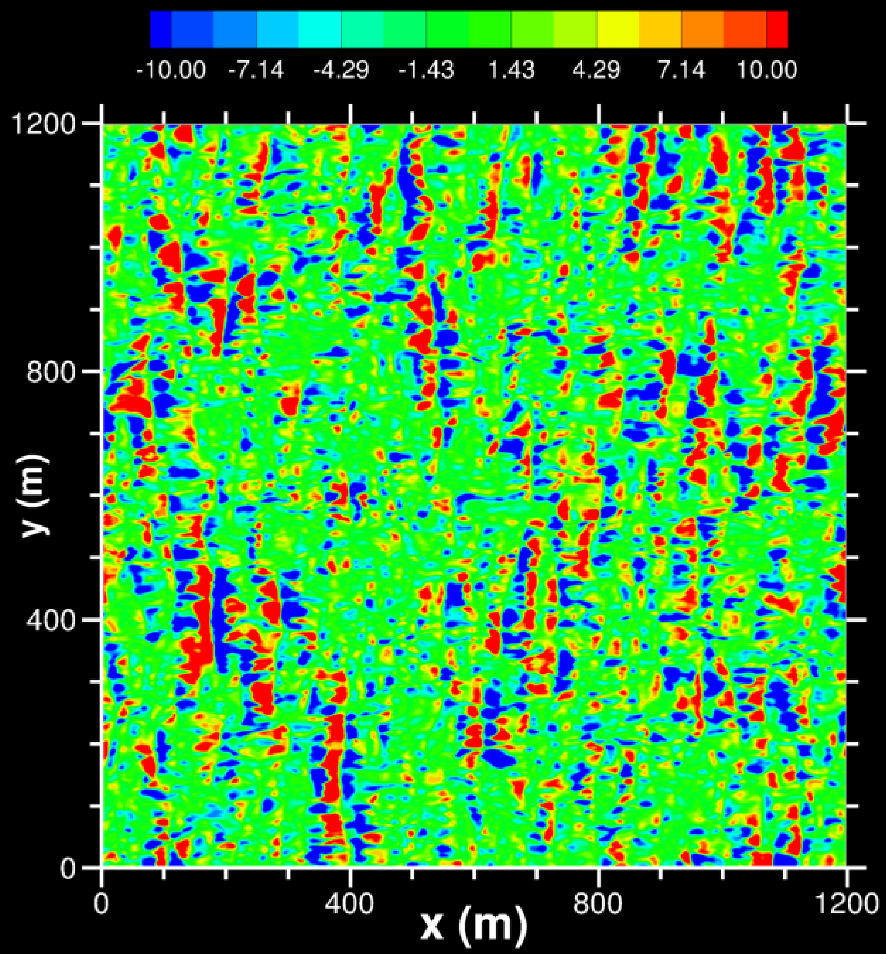


$$U_g = 5\ m/s$$
$$C_p/U_{10} \sim 4.8$$



$$U_g = 20\ m/s$$
$$C_p/U_{10} \sim 1.5$$

RESOLVED MOMEMTUM FLUX NEAR WATER SURFACE



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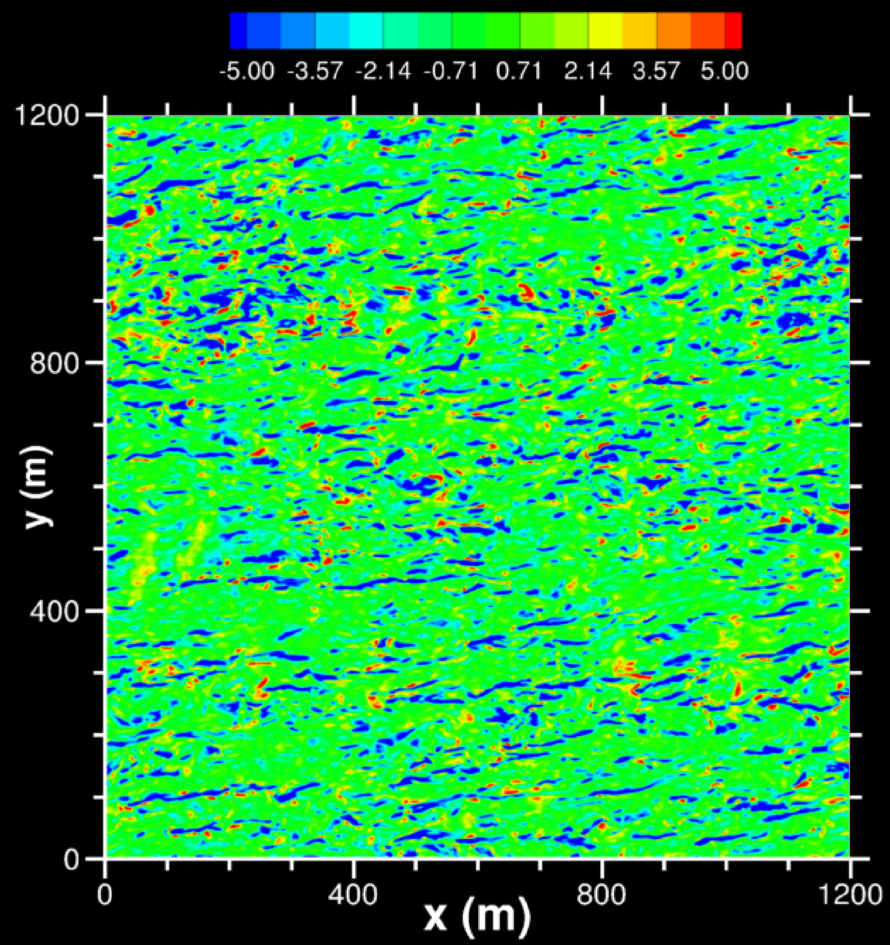
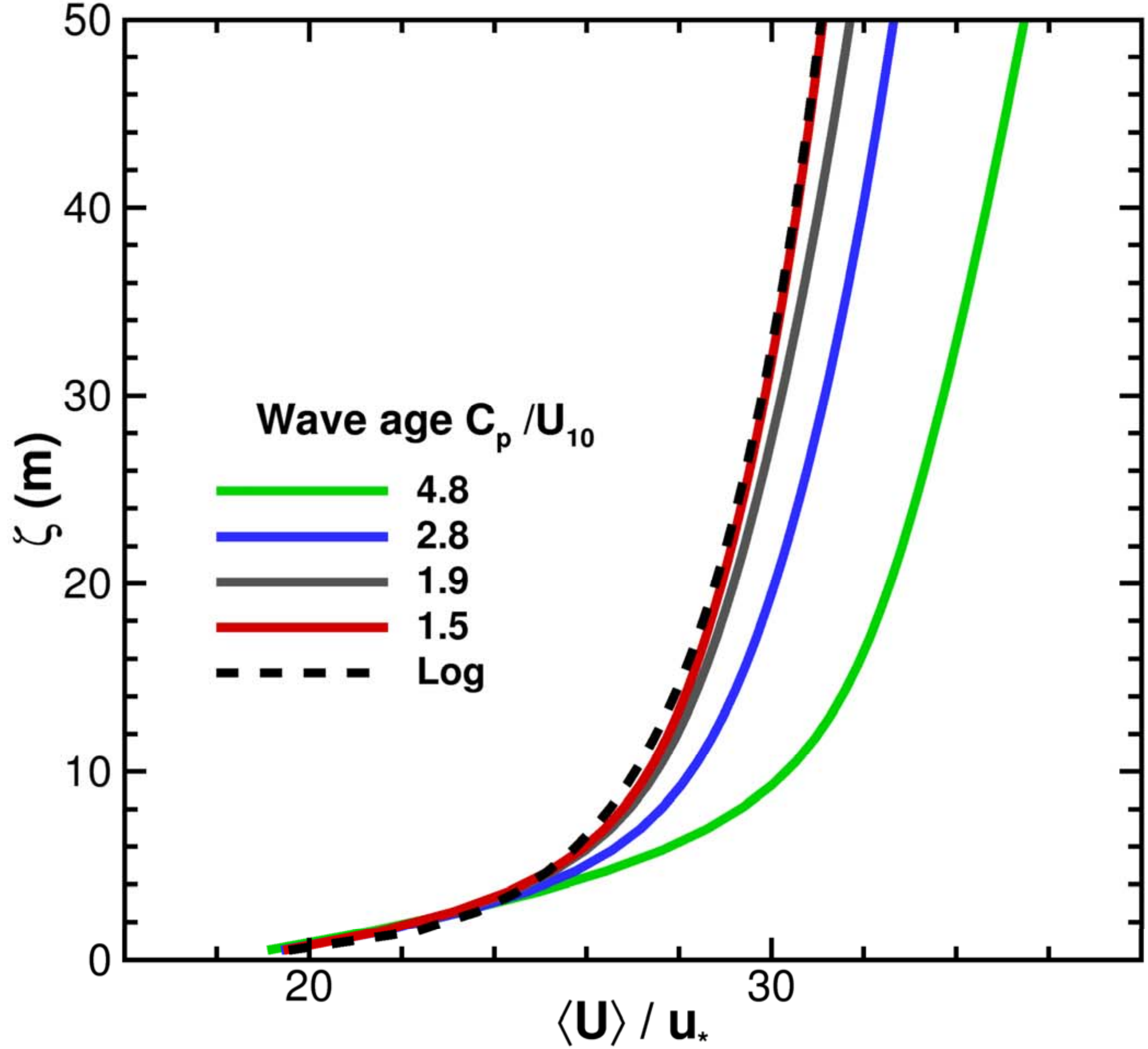


$C_p/U_{10} \sim 1.5$

note different scales

VERTICAL PROFILE OF MEAN WIND



LES MODEL FOR AN OBL WITH WAVE EFFECTS

- Craik-Leibovich equations with phase-averaged wave-current interactions $\Rightarrow$ depend on Stokes drift $\mathbf{u}^{St}$
  - Vortex force
  - Coriolis-Stokes term
  - Scalar advection by Stokes drift
  - Stokes production

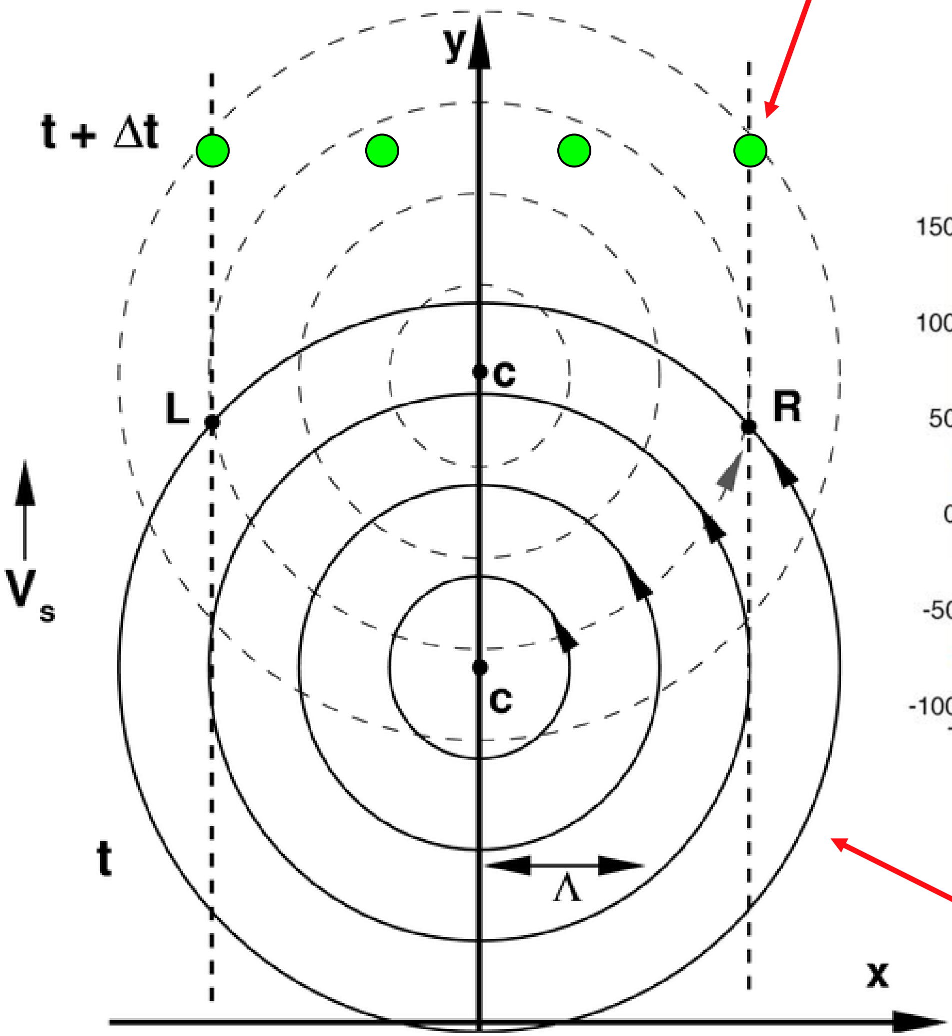
- Discrete stochastic wave breaking model replaces uniform stress τ_o
  - Compact momentum $\mathbf{A}$ and energy W impulses
  - PDF of breaking matches the atmospheric inputs with a dependence on wave age and wind speed

$$
\begin{aligned}
\frac{\partial \mathbf{u}}{\partial t} &= \ldots\ \mathbf{u}^{St} \times (f\hat{\mathbf{z}} + \boldsymbol{\omega}) + \sum_{i=1}^{n} \mathbf{A}^{(i)} \\
\frac{\partial c}{\partial t} &= \ldots\ \mathbf{u}^{St} \cdot \boldsymbol{\nabla}\, c \\
\frac{\partial e}{\partial t} &= \ldots\ \mathbf{u}^{St} \cdot \boldsymbol{\nabla}\, e - \tau_{ij} \frac{\partial u_i^{St}}{\partial x_j} + \sum_{i=1}^{n} W^{(i)}
\end{aligned}
$$

$$
La_t = \sqrt{\frac{u_*}{u^{St}}}
$$

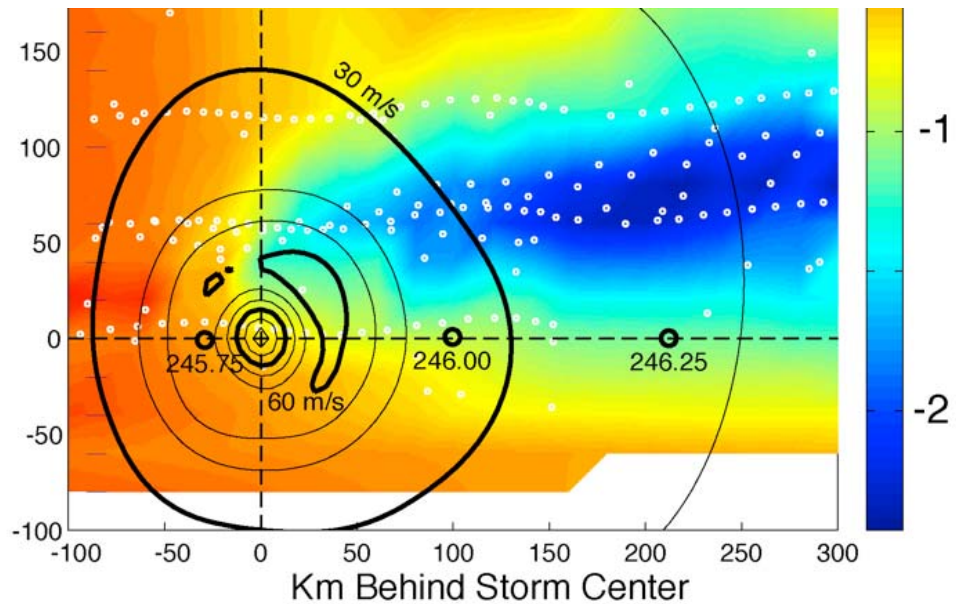
LES OF OBLs DRIVEN BY HURRICANE FRANCES

LES Domain

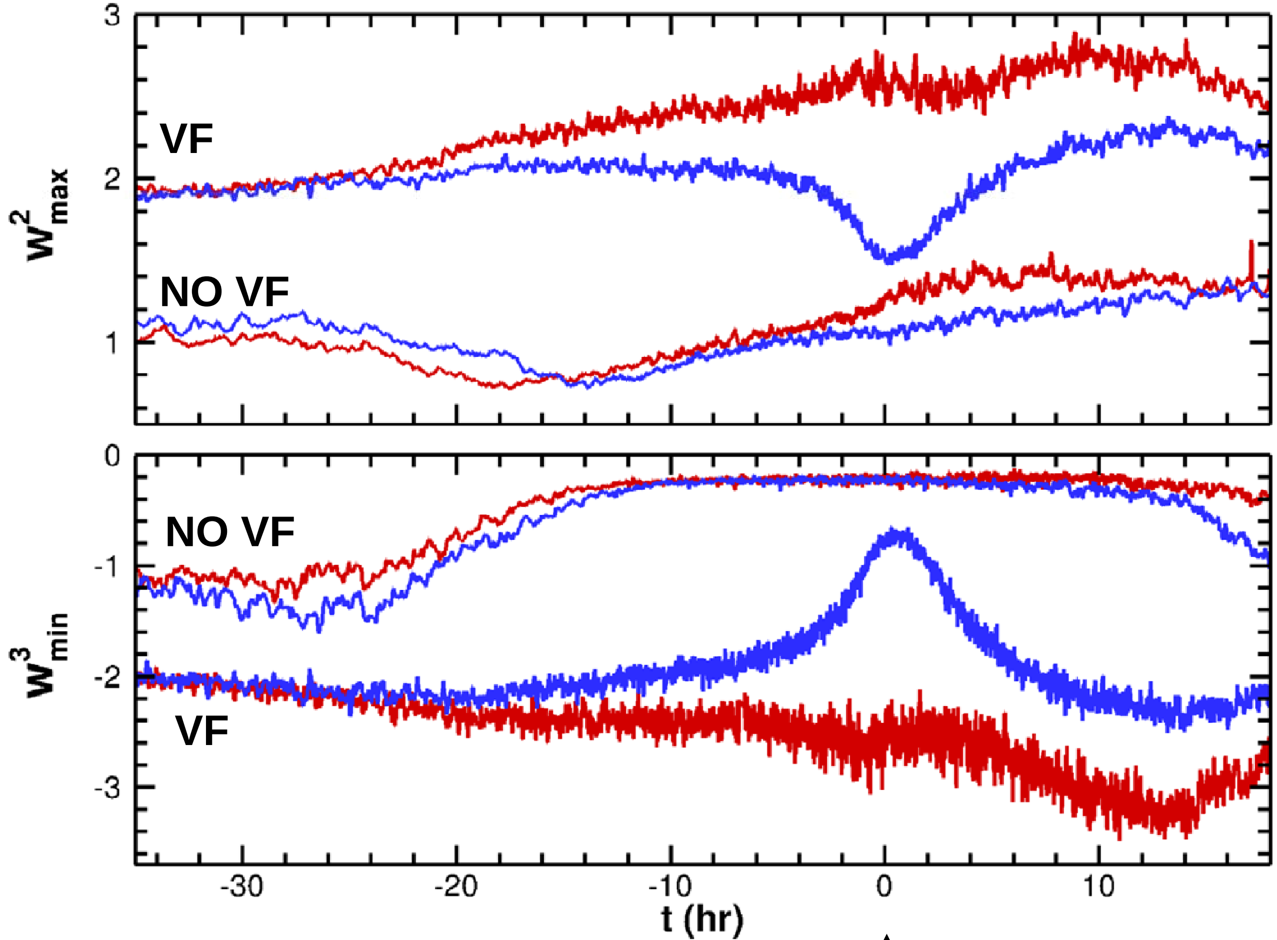


Idealized hurricane winds

COLD WAKE

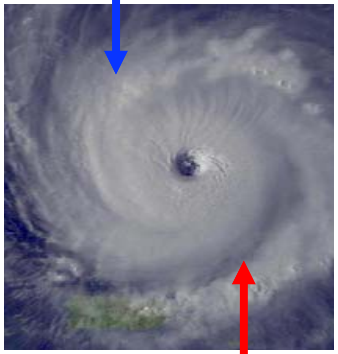


RESOLVED VERTICAL VELOCITY MOMENTS



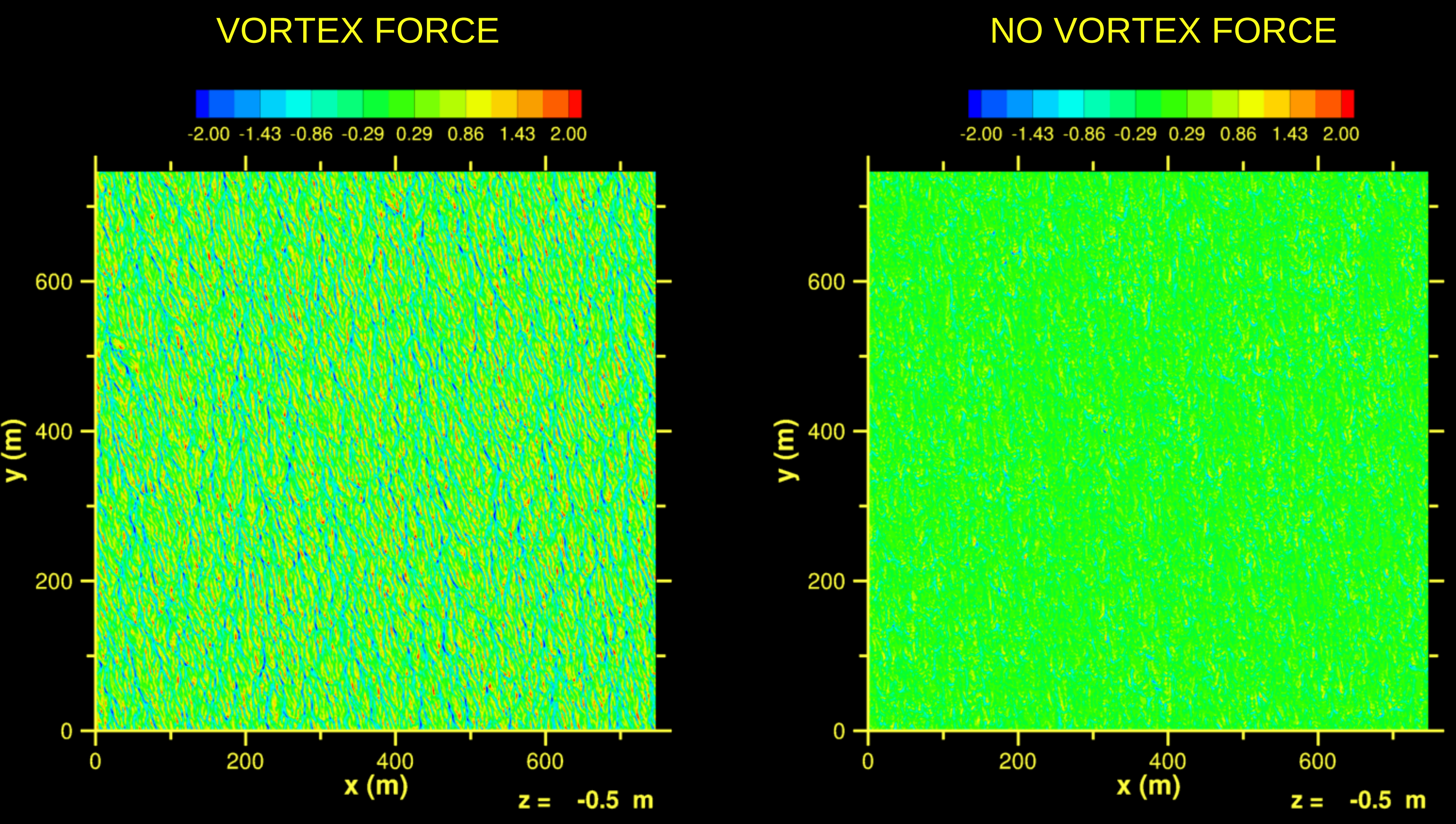
Max Winds

Non-resonant



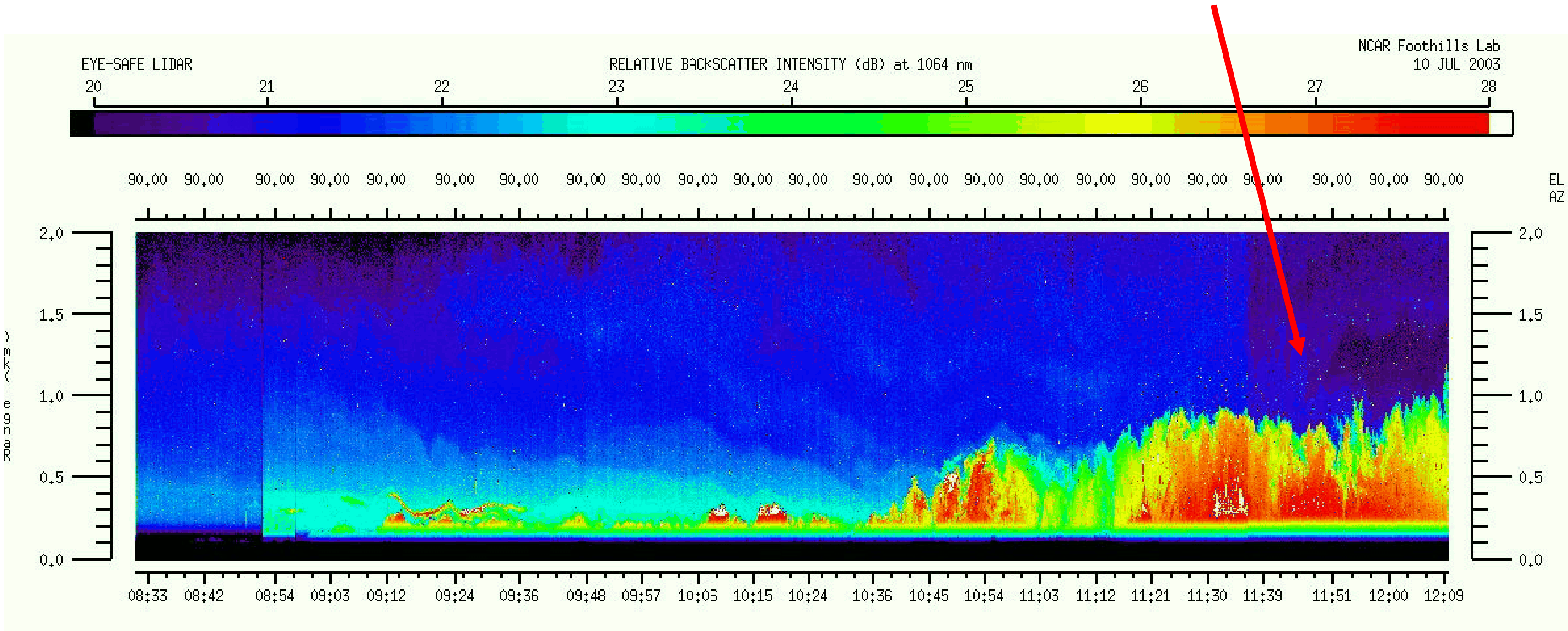
Resonant

VERTICAL VELOCITY FIELD AS FUNCTION OF DEPTH



DO LES STATISTICS CONVERGE WITH MESH REFINEMENT?

Daytime Convective BL



8 am **Local time** **Noon**

Courtesy Shane Mayor

LES EQUATIONS FOR DRY ATMOSPHERIC PBL

Momentum

$$\frac{D\overline{\mathbf{u}}}{Dt} = -\,\mathbf{f} \times \overline{\mathbf{u}} - \nabla\pi + \hat{\mathbf{z}} g \frac{\overline{\theta}}{\theta_*} - \nabla \cdot \mathbf{T}$$

Scalar

$$\frac{D\overline{b}}{Dt} = -\nabla \cdot \mathbf{B}$$

TKE

$$\frac{De}{Dt} = -\mathbf{T} : \mathbf{S} + \mathbf{B} \cdot \hat{\mathbf{z}} - \mathcal{E} + \nabla \cdot (2\nu_t \nabla e)$$

Subgrid-scale momentum and scalar fluxes

$$\mathbf{T} = \overline{u_i\, u_j} - \overline{u_i}\,\overline{u_j}$$

$$\mathbf{B} = \overline{u_i\, b} - \overline{u_i}\,\overline{b}$$

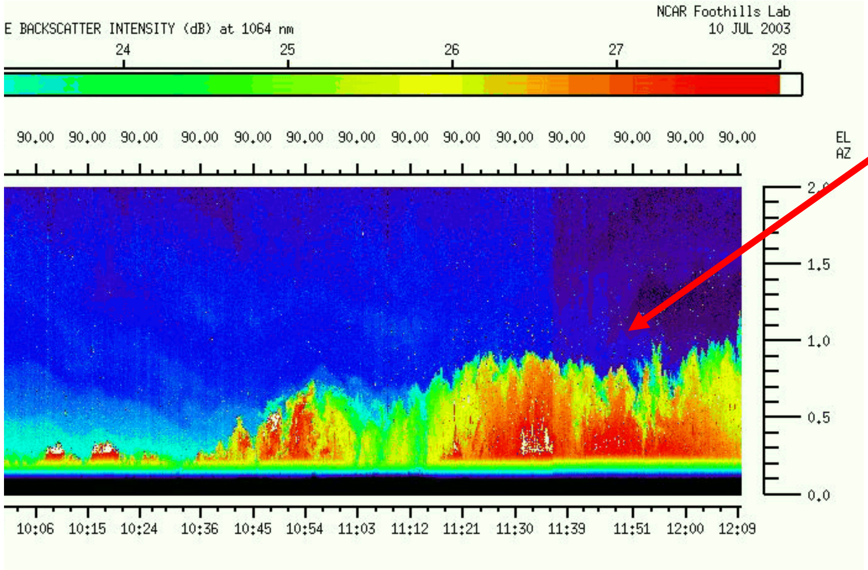
Incompressible Boussinesq flow

$$\nabla \cdot \overline{\mathbf{u}} = 0 \implies \nabla^2 \pi = s$$

DO LES STATISTICS CONVERGE WITH MESH REFINEMENT?

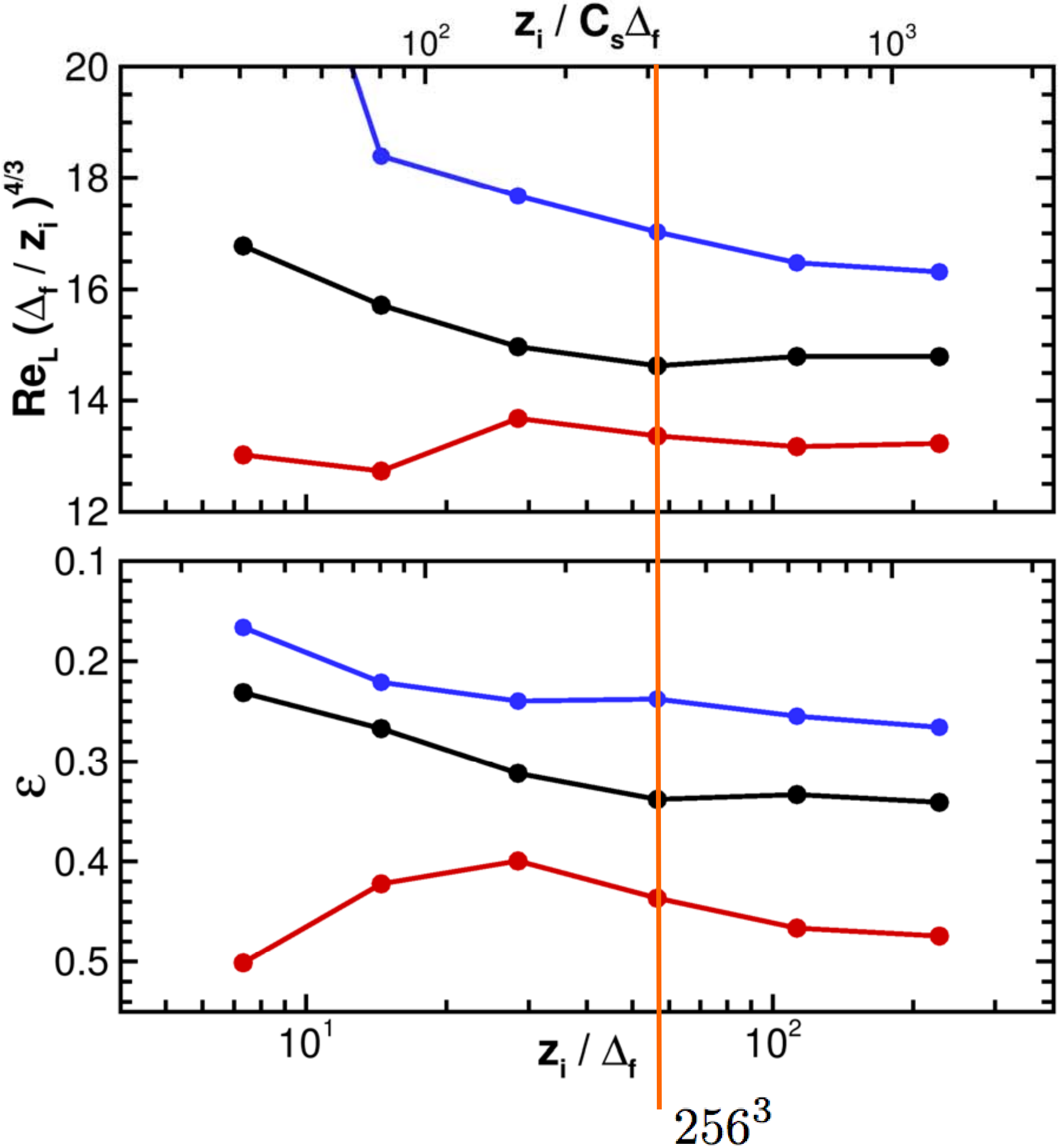
$$(X_L, Y_L, Z_L) = (5, 5, 2)\ km$$

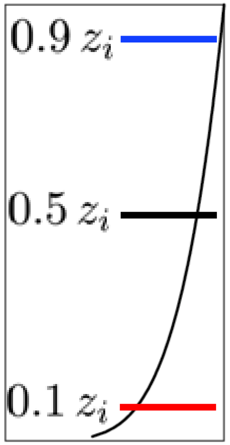
Gridpoints	$(\triangle x, \triangle y, \triangle z)$ (m)	$z_i/\triangle_f$	$z_i/C_s\triangle_f$
32^3	(160,160,64)	7.2	40
64^3	(80,80,32)	14.5	80
128^3	(40,40,16)	28.5	158
256^3	(20,20,8)	56.6	314
512^3	(10,10,4)	113.3	630
1024^3	(5,5,2)	229.0	1272



DO LES STATISTICS CONVERGE WITH MESH REFINEMENT?

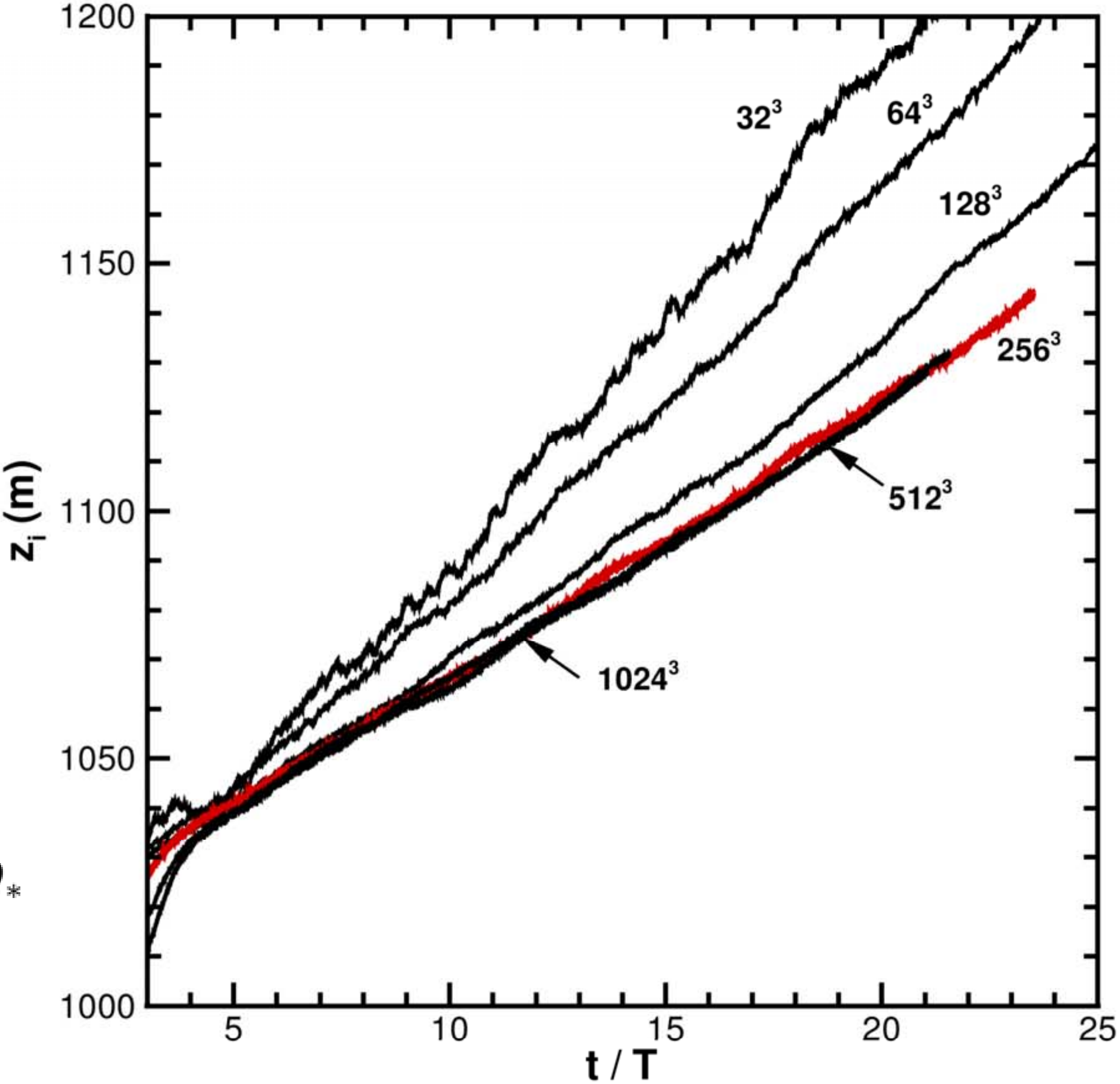
Large eddy
Reynolds number





Dissipation

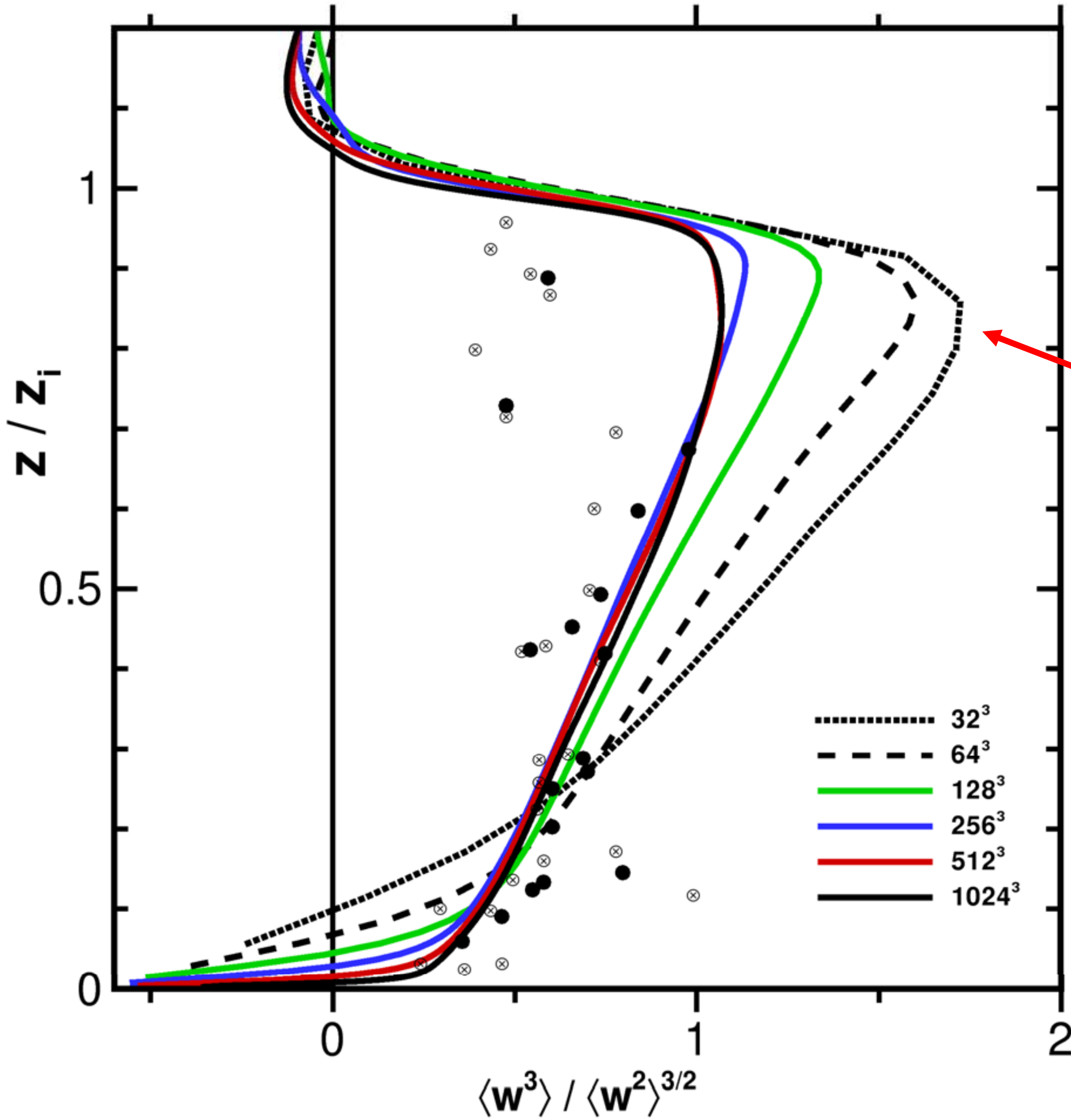
PBL HEIGHT z_i FOR VARYING MESHES



$\langle w\theta \rangle_{min} \sim -0.2Q_*$

Entrainment rate $w_e = dz_i/dt$ decreases with increasing mesh resolution

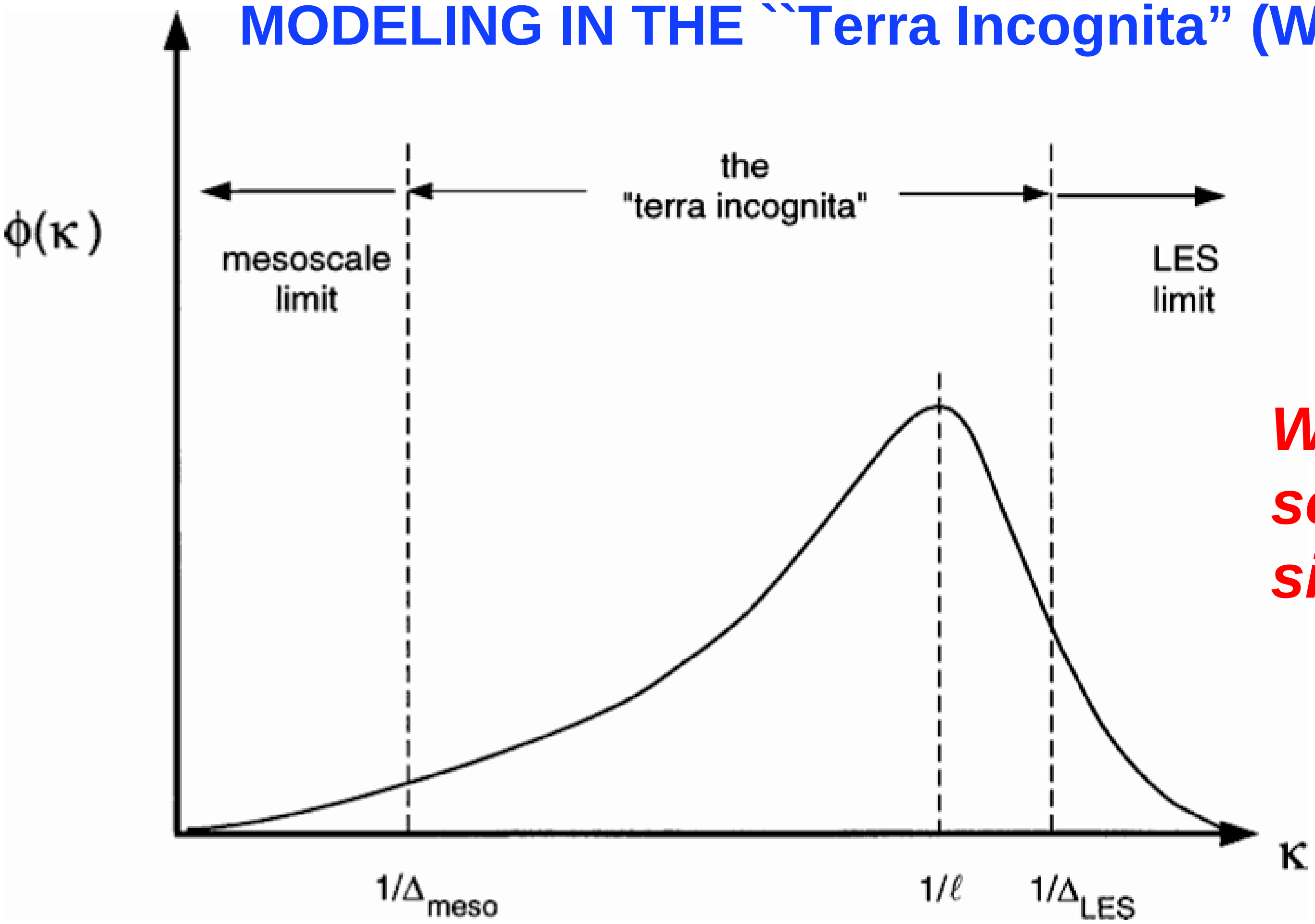
IMPACT OF GRID RESOLUTION ON SKEWNESS



No SGS skewness!

Where is the filter scale in your simulation?

MODELING IN THE ``Terra Incognita" (Wyngaard,2004)



Where is the filter scale in your simulation?

Deterministic

Stochastic

~ 1-D PBL

~ Smagorinsky

Scale sensitive parameterizations

See also Blair Perot, 2009

SCALAR FLUX

How does subgrid-scale scalar flux f_i vary across filter scale $\triangle_f$?

$$f_i \;=\; \overline{u_i\, c} \;-\; \overline{u_i}\,\overline{c}$$

Local Free Convection, Similarity, and the Budgets of Shear Stress and Heat Flux

J. C. Wyngaard, O. R. Coté and Y. Izumi

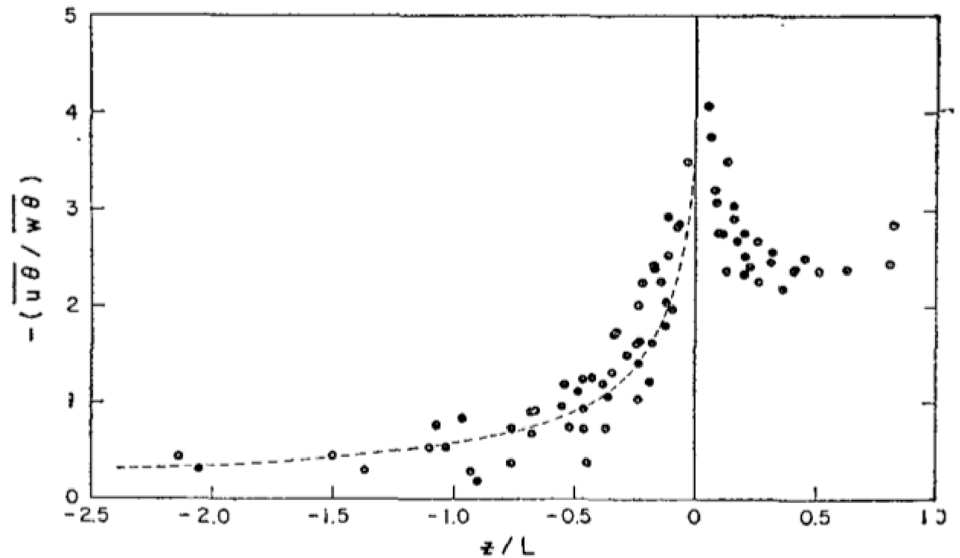


Fig. 4. Ratio of horizontal and vertical components of heat flux. The curve is the local free convection prediction.

Conditions in Kansas were adequately stationary, and although horizontal homogeneity was not measured, it appears to be a good approximation in view of the long (2400 m) uniform fetch, as discussed by Wyngaard and Coté (1971).

The same procedure gives the vertical heat flux ($\overline{w\theta}$) budget

$$\frac{\partial\overline{w\theta}}{\partial t}+\overline{w^2}\frac{\partial\Theta}{\partial z}-\frac{g}{T}\overline{\theta^2}+\frac{\partial\overline{w^2\theta}}{\partial z}+\frac{1}{\rho}\overline{\theta\frac{\partial p}{\partial z}}=0, \qquad (13)$$

and the horizontal heat flux ($\overline{u\theta}$) budget

$$\frac{\partial\overline{u\theta}}{\partial t}+\overline{w\theta}\frac{\partial U}{\partial z}+\overline{uw}\frac{\partial\Theta}{\partial z}+\frac{\partial\overline{uw\theta}}{\partial z}+\frac{1}{\rho}\overline{\theta\frac{\partial p}{\partial x}}=0. \qquad (14)$$

RATE EQUATIONS FOR SUBGRID SCALAR FLUX ACROSS SCALES: $\triangle_f < \ell$ and $\triangle_f > \ell$

- **How do we get to the LES approximation?**
  - Deardorff (1973), Wyngaard (2004), Hatlee & Wyngaard (2007)

$$f_i = \overline{u_i c} - \overline{u}_i \overline{c}$$

$$\frac{Df_i}{Dt} = -\frac{2}{3} e \frac{\partial \overline{c}}{\partial x_i} - f_j \frac{\partial \overline{u}_i}{\partial x_j} + \tau_{ij} \frac{\partial \overline{c}}{\partial x_j}$$
$$+\frac{1}{\rho}\left(\overline{p \frac{\partial c}{\partial x_i}} - \overline{p} \frac{\partial \overline{c}}{\partial x_i} \right)$$
$$+ \text{ transport } + \text{ buoyancy}$$

Isotropic production

Pressure destruction

Anisotropic production

RATE EQUATIONS FOR SUBGRID SCALAR FLUX ACROSS SCALES: $\triangle_f < \ell$ and $\triangle_f > \ell$

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$$+ \frac{1}{\rho} \left(\overline{p \frac{\partial c}{\partial x_i}} - \overline{p} \frac{\partial \overline{c}}{\partial x_i} \right)$$

$$+ \cancelto{0}{\text{transport}} + \cancelto{0}{\text{buoyancy}}$$

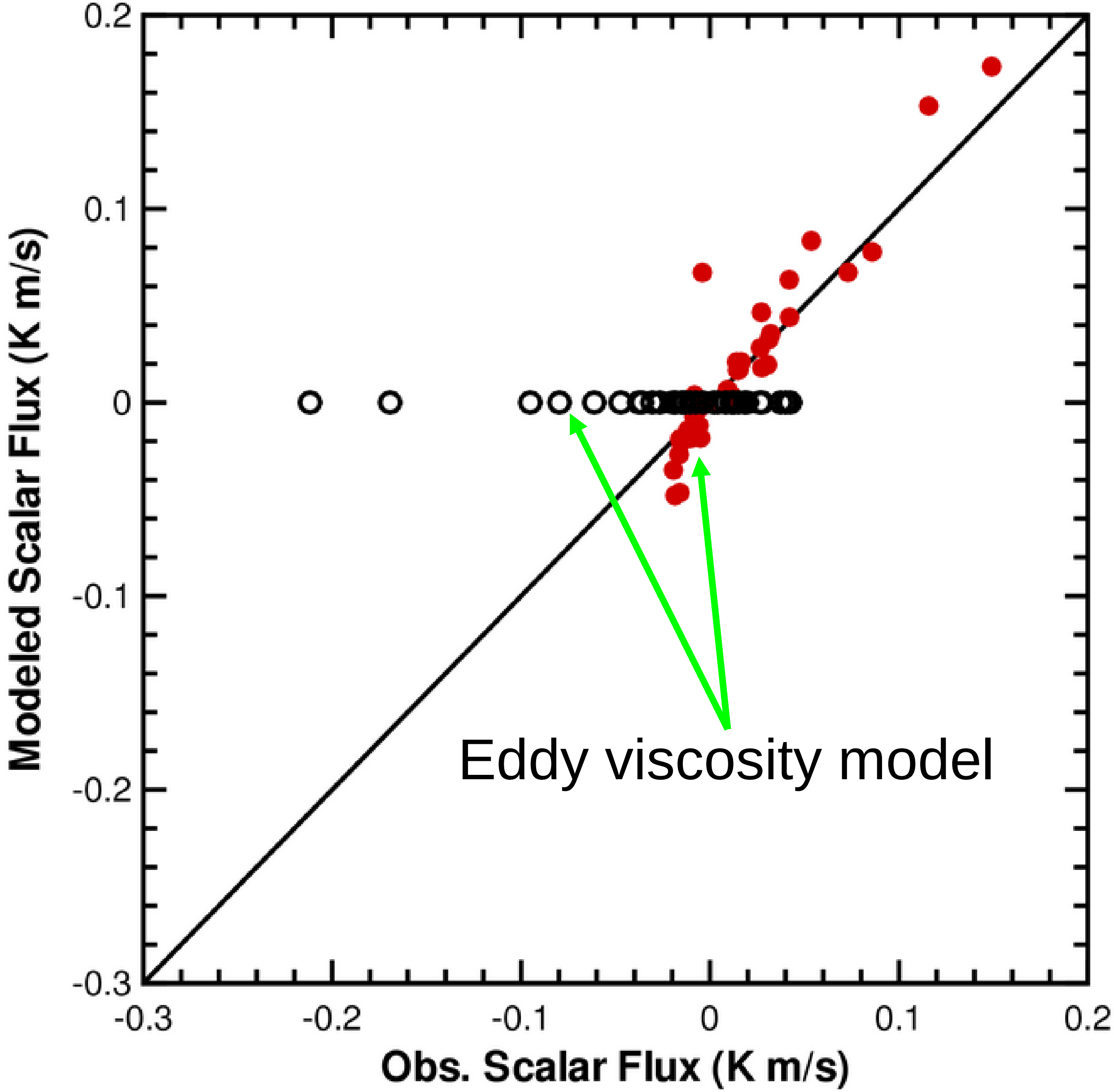
Rotta model

$$\frac{f_i}{T}$$

Eddy viscosity model

$$\boxed{f_i = -\nu_h \frac{\partial \overline{c}}{\partial x_i} \qquad \nu_h = \frac{2 c_h \triangle_f \sqrt{e}}{3}}$$

SFS SCALAR FLUXES IN HATS



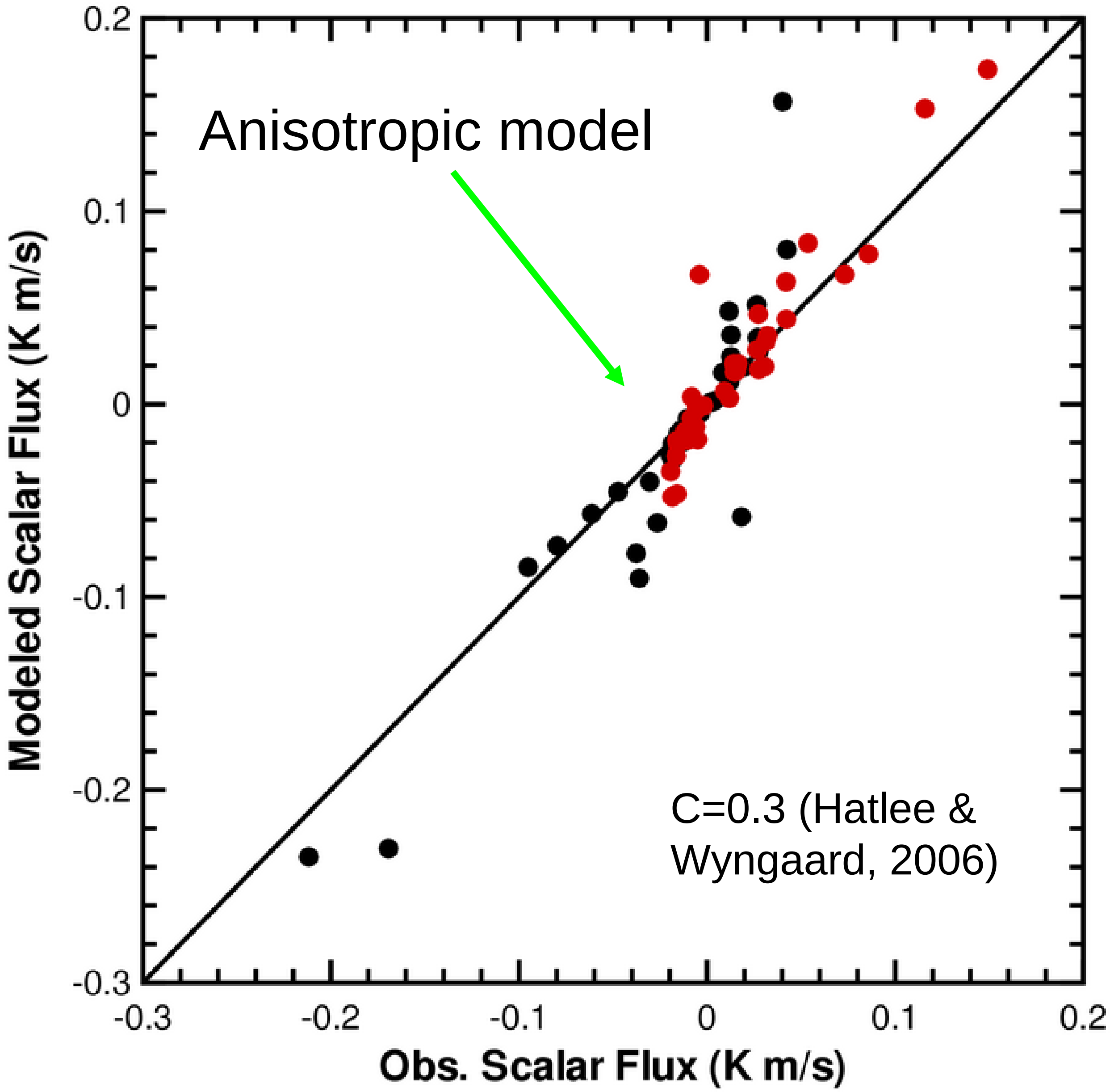
$f_1 = \langle \overline{uc} - \overline{u}\,\overline{c} \rangle$

● horizontal flux

● vertical flux

$f_3 = \langle \overline{wc} - \overline{w}\,\overline{c} \rangle$

SFS SCALAR FLUXES IN HATS



$$f_1 \sim f_3 \frac{\partial U}{\partial z}$$

$$f_1 = \langle \overline{uc} - \overline{u}\,\overline{c} \rangle$$

● horizontal flux

● vertical flux

$$f_3 = \langle \overline{wc} - \overline{w}\,\overline{c} \rangle$$

SUBGRID-SCALE SCALAR FLUX

Comments:

- Net horizontal scalar flux $f_1 = \langle \overline{uc} - \overline{u}\,\overline{c} \rangle \neq 0$ even horizontally homogeneous PBLs, $i.e.,$ $\dfrac{\partial}{\partial x}\langle C \rangle = 0$

- Tilting of vertical flux by vertical shear is important $f_1 \sim -f_3 \dfrac{\partial \overline{u}}{\partial z} T$

- No eddy viscosity model, including the "dynamic approach", can capture anisotropic production

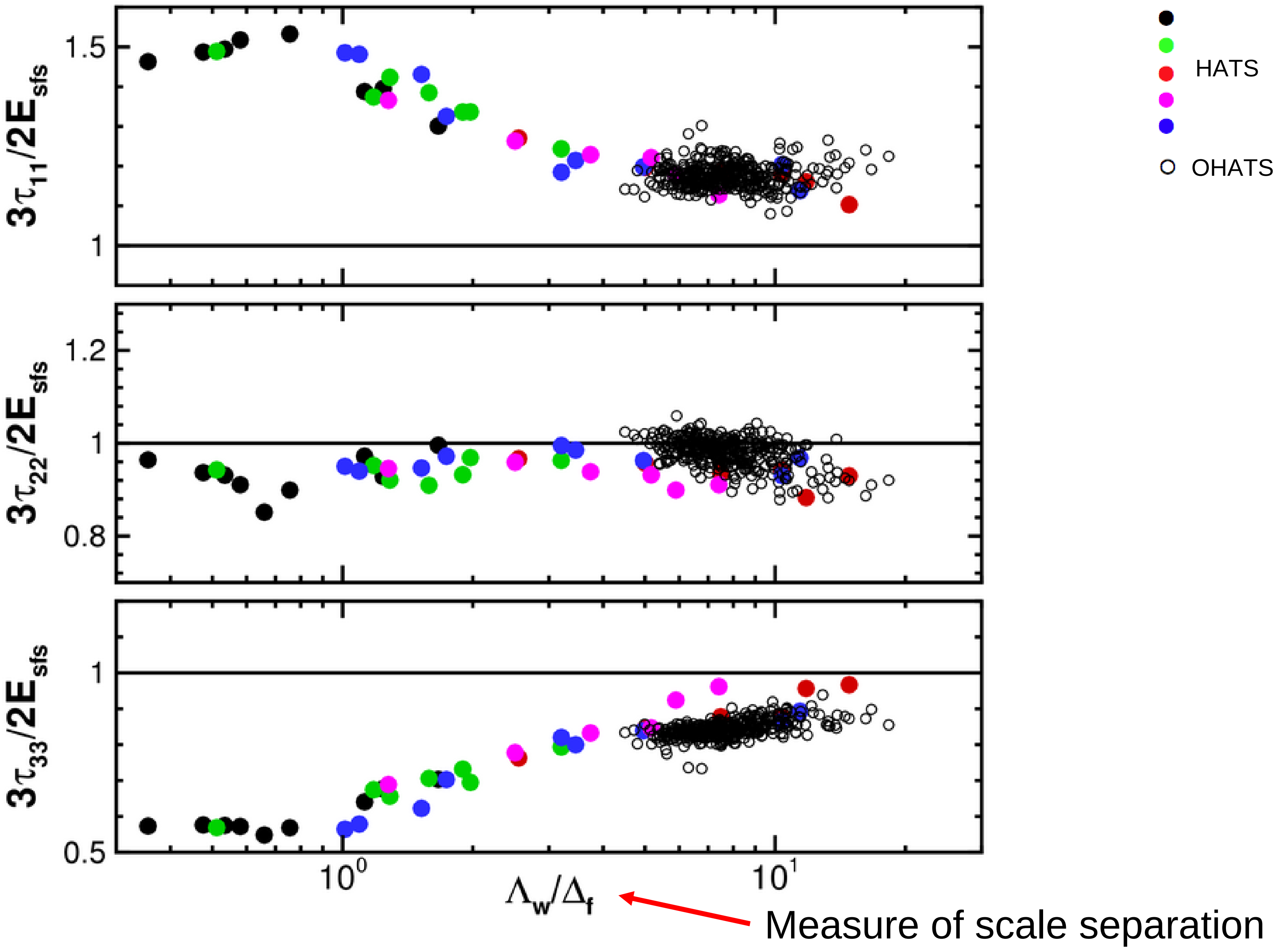
HORIZONTAL ARRAY TURBULENCE STUDY (HATS)

$\sim 36\ cases$

$-1.2 < z/L < 1.6$

$0.15 < \Lambda_w/\triangle_f < 15$

SFS VELOCITY VARIANCES



RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- **What are the parent equations for the Smagorinsky model?**
  - Lilly (1967), Deardorff (1973), Wyngaard (2004), Hatlee & Wyngaard (2007)

$$
\begin{aligned}
\frac{D\tau_{ij}}{Dt} \quad = \quad & \frac{2}{3} e \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \\
& - \left[\tau_{ik} \frac{\partial \overline{u}_j}{\partial x_k} + \tau_{jk} \frac{\partial \overline{u}_i}{\partial x_k} - \frac{1}{3} \delta_{ij} \tau_{kl} \left(\frac{\partial \overline{u}_k}{\partial x_l} + \frac{\partial \overline{u}_l}{\partial x_k} \right) \right] \\
& - \frac{1}{\rho} \left[\overline{p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} - \overline{p} \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \right] \\
& + \text{transport} \; + \; \text{buoyancy production}
\end{aligned}
$$

Isotropic production

Pressure destruction

Anisotropic deviatoric production

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& - \frac{1}{\rho} \left[\overline{p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} - \overline{p} \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \right] \\
& + \cancelto{0}{\text{transport}} \; + \; \cancelto{0}{\text{buoyancy production}}
\end{aligned}
$$

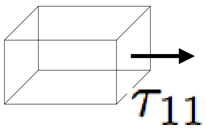
Rotta model

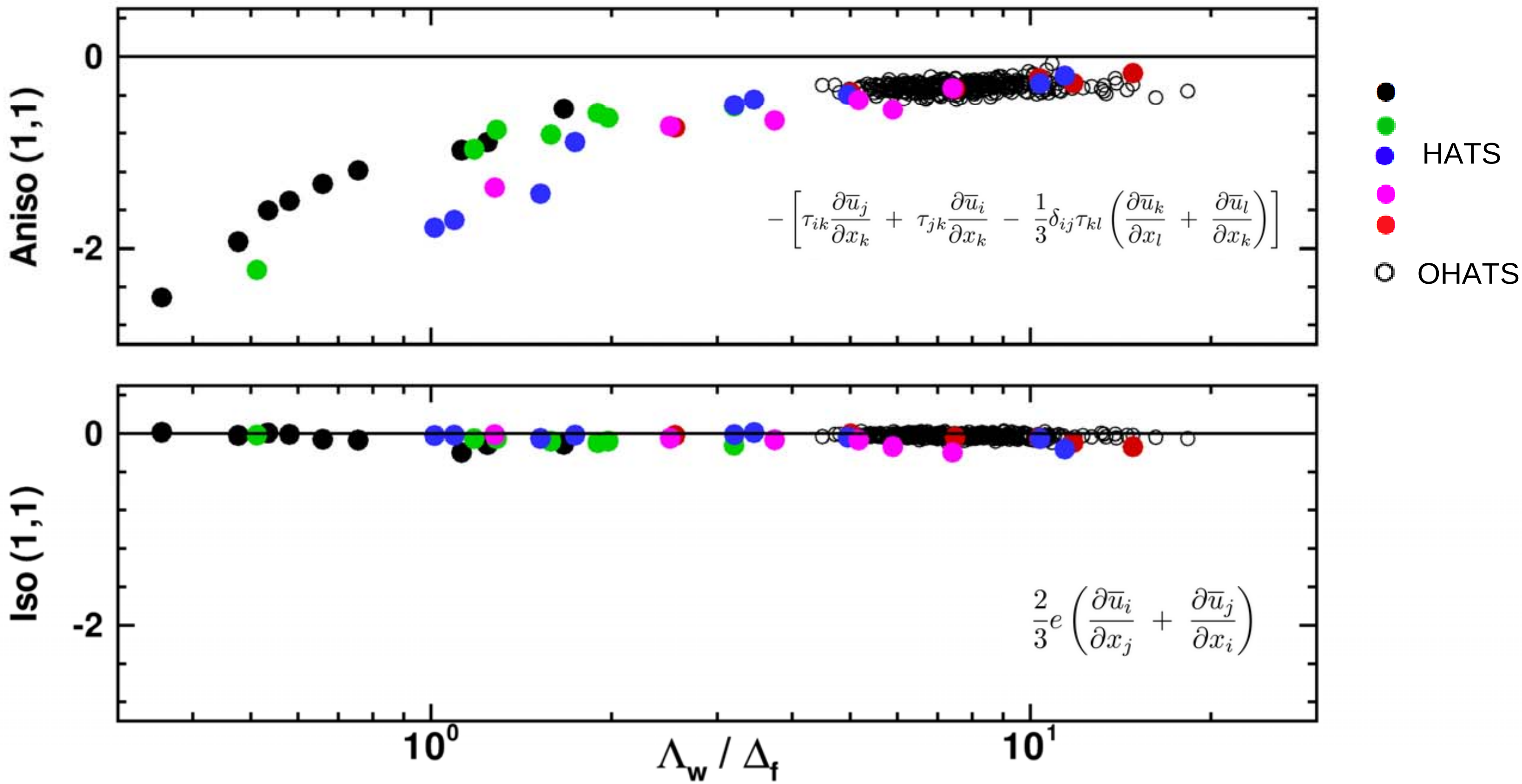
$$
\boxed{\frac{\tau_{ij}}{T} = \frac{2}{3} e \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right)}
$$

Time scale

$$
T = c \frac{\Delta_f}{\sqrt{e}}
$$

PRODUCTION OF SUBFILTER SCALE FLUX τ_{11}

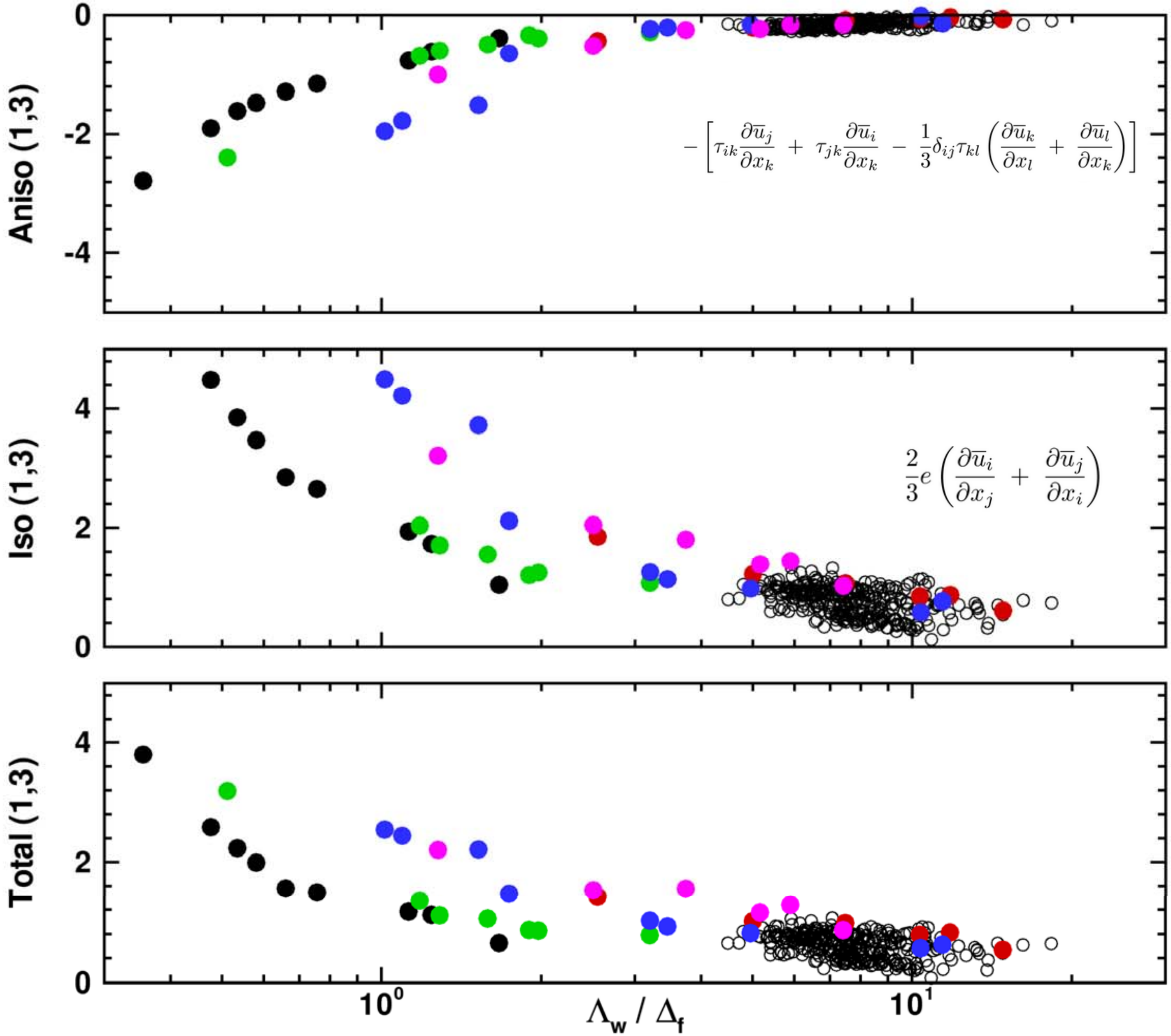


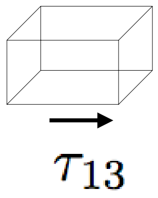


RANS ***Ideal LES***

Measure of scale separation

PRODUCTION OF SUBFILTER SCALE FLUX τ_{13}



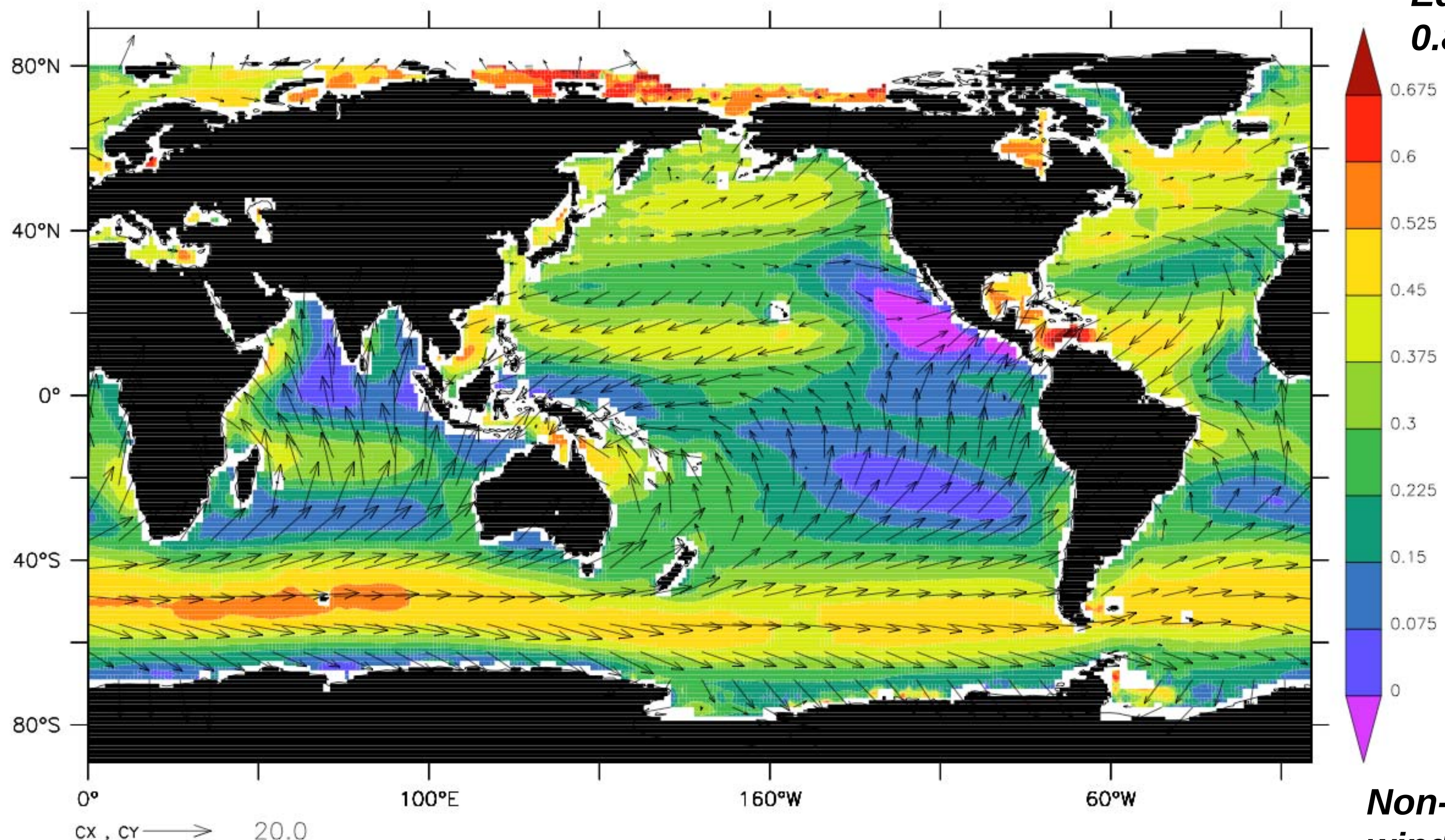


SUMMARY

- Atmospheric and oceanic boundary-layer dynamics are unique compared to flat-wall boundary layers because of surface waves
  - Winds, currents, drag, variances, dissipation, entrainment, ...
- Carefully crafted high Re LES neatly exposes the interactions between winds-waves, waves-currents
- LES solutions for means and second-order moments converge with mesh refinement provided $z_i/C_s\triangle_f > 300$ (for daytime convective BL)
  - Solutions exhibit approximate Reynolds-number similarity
  - Entrainment rate decreases with increasing mesh resolution
  - Vertical velocity skewness is an indicator of mesh sensitivity
- Measurements of subgrid-scale variables show SGS (eddy viscosity) parameterizations used in LES are inadequate when the ratio $\Lambda/\triangle_f \sim \mathcal{O}(1)$ or less
  - Anisotropic production of scalar and momentum flux in surface layers is important
- Yes! LES is exceedingly useful, but can be improved

GLOBAL CLIMATOLOGY OF INVERSE WAVE AGE

$U_a \cos(\phi)/C_p$ AVERAGED OVER 1958 - 2001

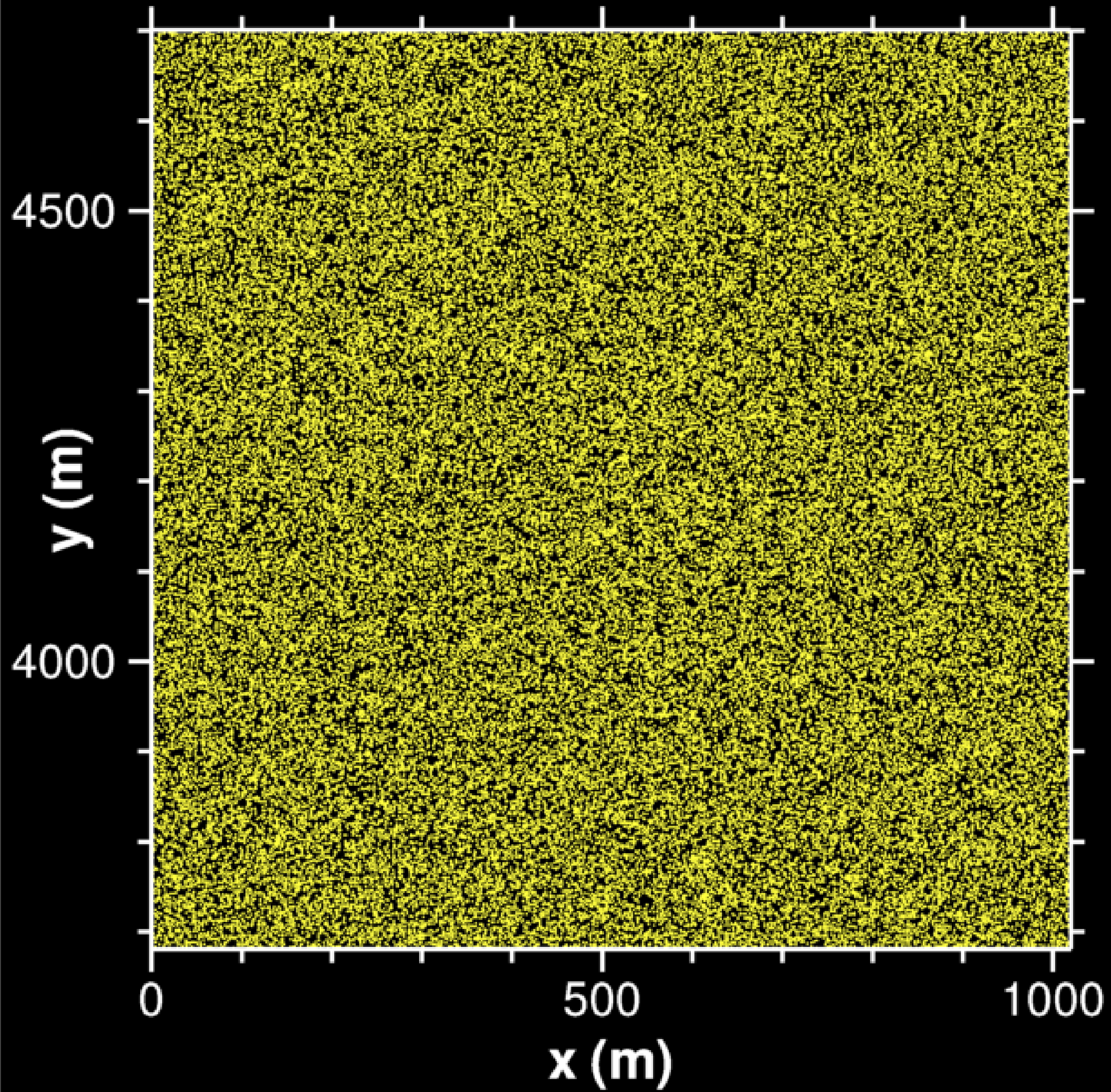


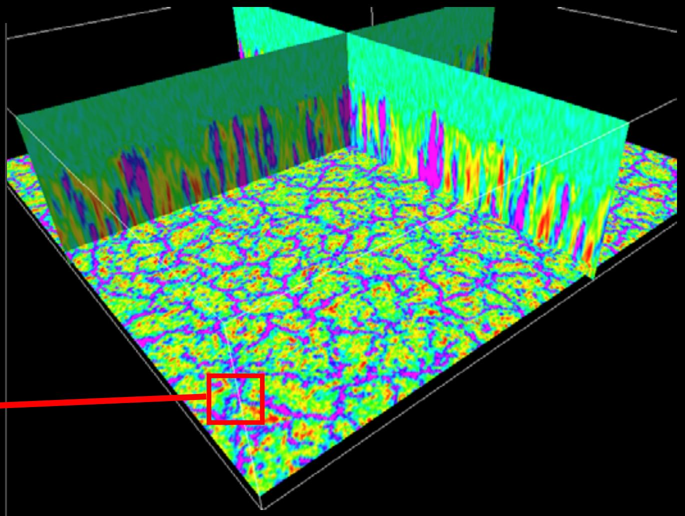
Equilibrium ~ 0.83

Non-equilibrium winds and waves

K. Hanley PhD thesis
2008, U. Reading

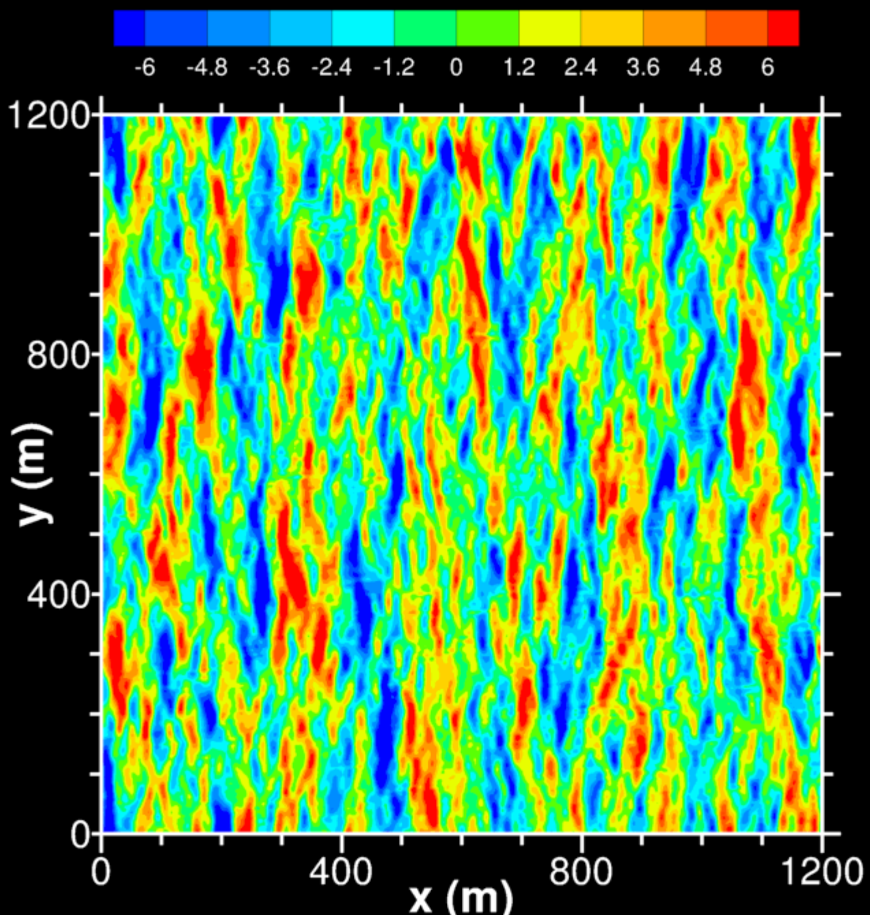
LES OF CONVECTIVE PBL, 4096 CPUS, 1024^3 GRIDPOINTS



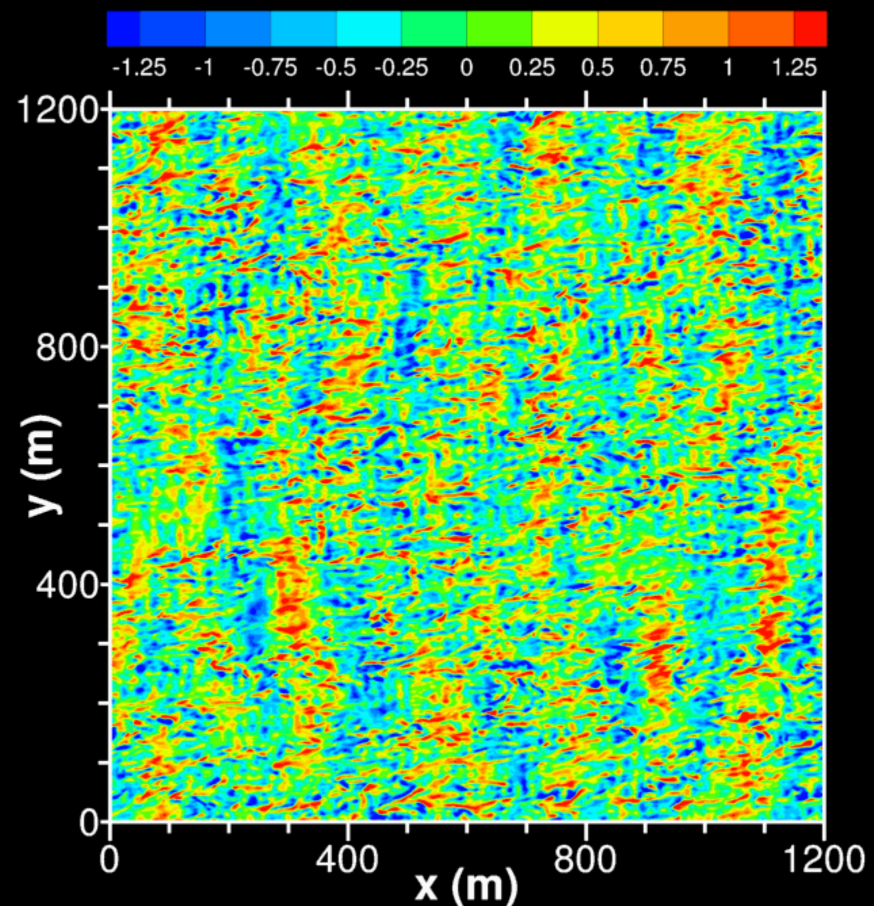


VERTICAL VELOCITY NEAR WATER SURFACE $\zeta = 2.5m$

$C_p/U_{10} \sim 4.8$
$u_* = 0.12\ m/s$

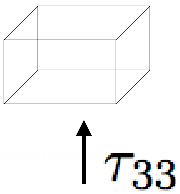


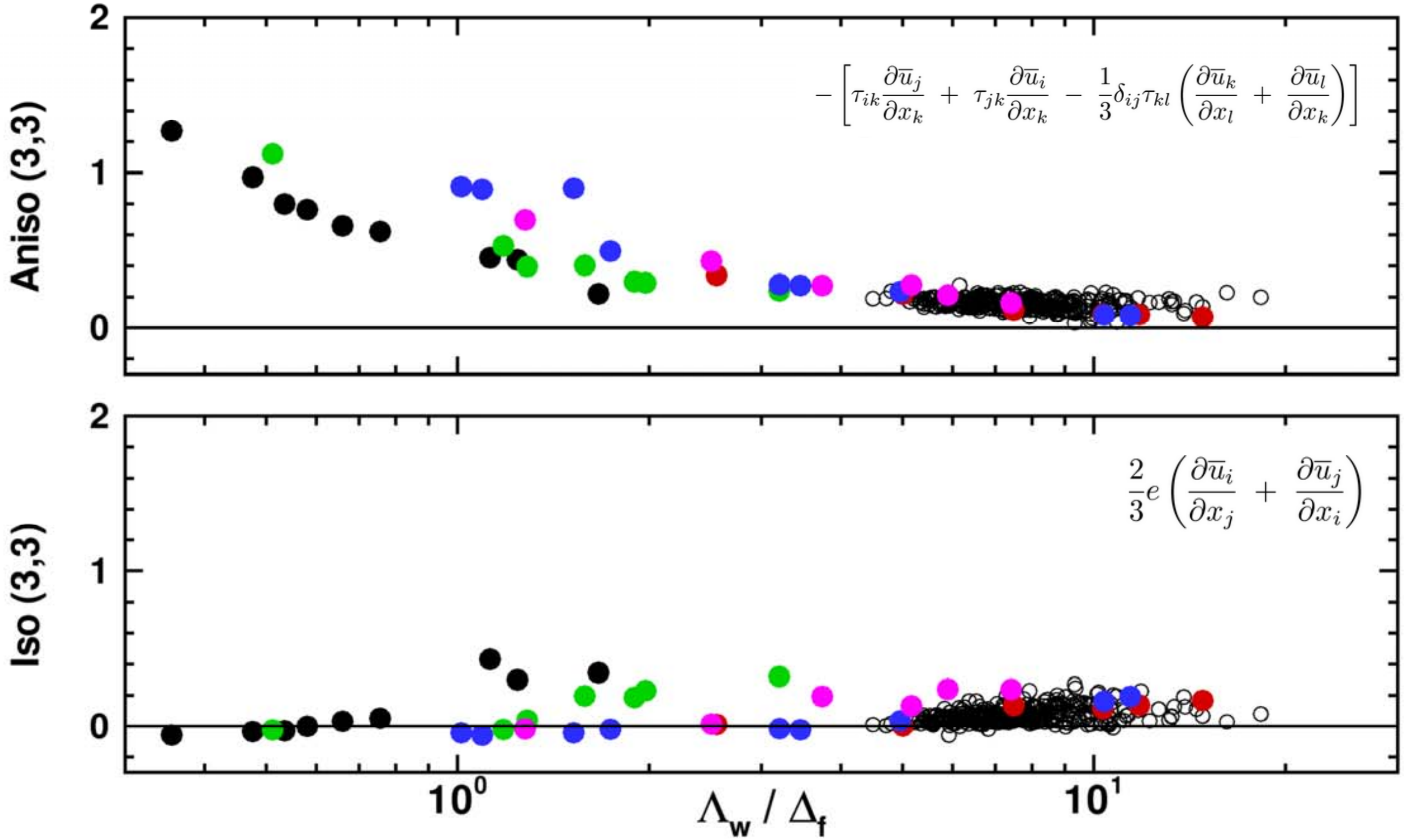
$C_p/U_{10} \sim 1.5$
$u_* = 0.45\ m/s$



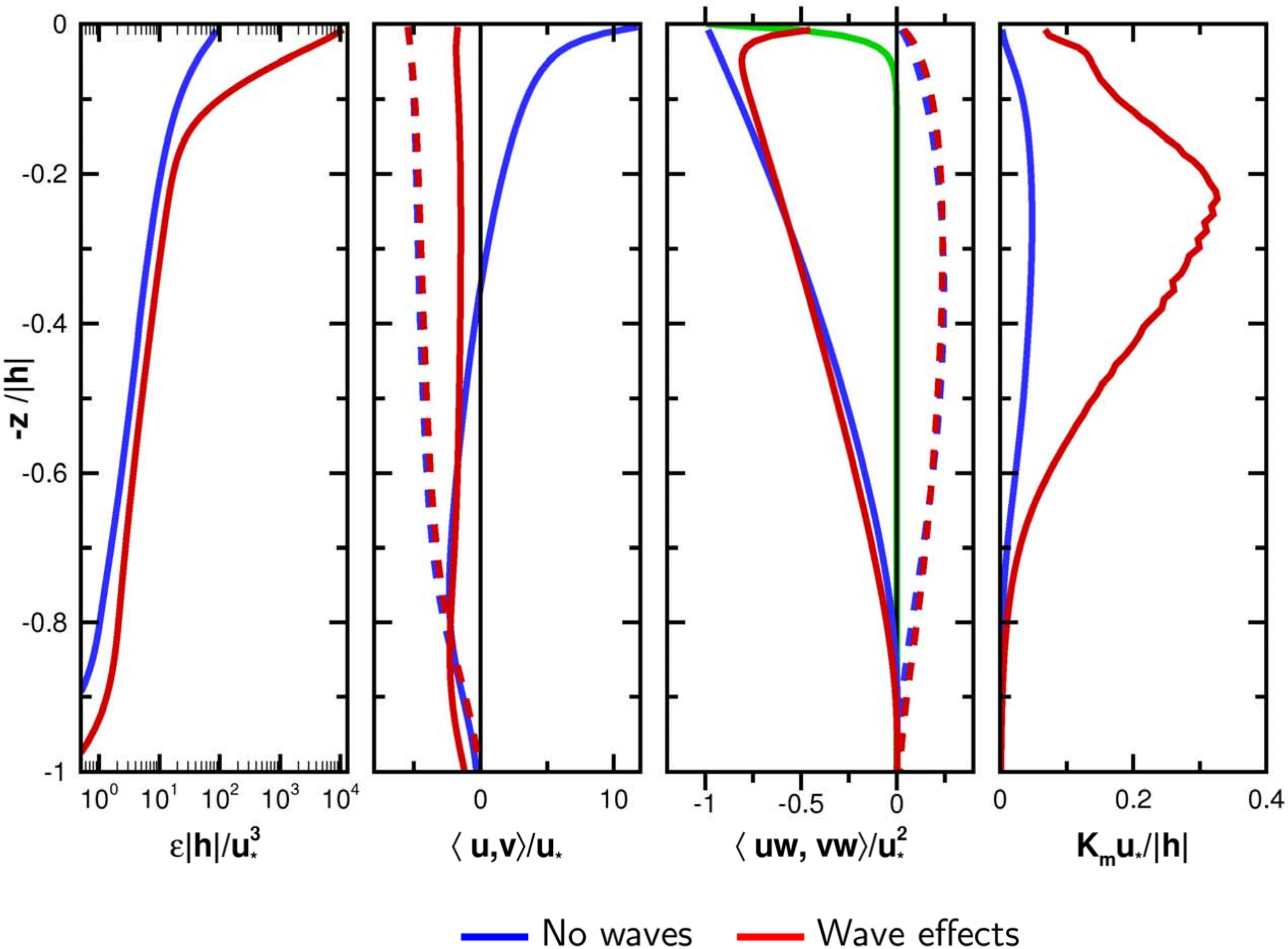
note different scales

PRODUCTION OF SUBFILTER SCALE FLUXτ_{33}





OBL LANGMUIR AND BREAKER TURBULENCE



LANGMUIR TURBULENCE

Langmuir turbulence $\Rightarrow$ the OBL regime where phase-averaged wave-current interactions are comparable to or greater than shear/buoyancy generated turbulence

Characteristics of Langmuir turbulence:

- Non-local vertical transport of momentum and scalars
- Near surface intensification of spanwise and vertical velocity variances
- Coherent structures
  - streamwise oriented Langmuir cells
  - downwelling jets induced by the CL2 instability and breaker vorticity

McWilliams *et al.*(1997) argue that the high-Reynolds number parameter measuring the competition between shear instability and vortex force is the turbulent Langmuir number:

$$\boxed{La_t = \sqrt{\frac{u_{*w}}{u^{St}}}}$$

OBLs WITH WAVE EFFECTS?

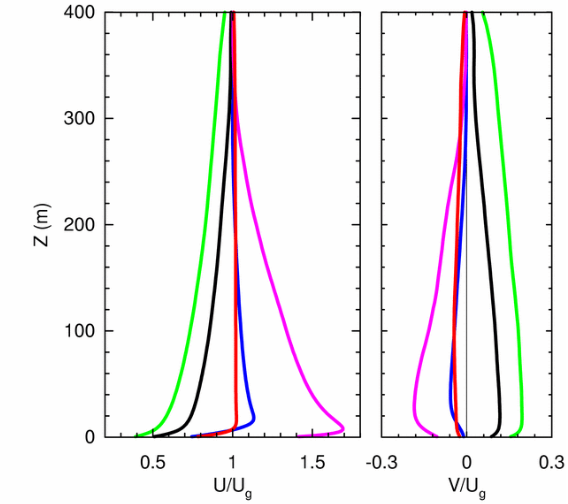
- Turbulence simulation and observational results of wind-wave driven OBLs:
  - Homogenize the vertical structure of the currents
  - Alter momentum and scalar fluxes and velocity variances
  - Energize the near surface TKE and elevate the dissipation
  - Enhance mixing at the thermocline
  - Generate depth filling coherent structures ($e.g.$, Langmuir cells and downwelling jets)

- Phase-averaged wave-current interactions and wave breaking invalidate Monin-Obukhov wall scaling to varying degrees

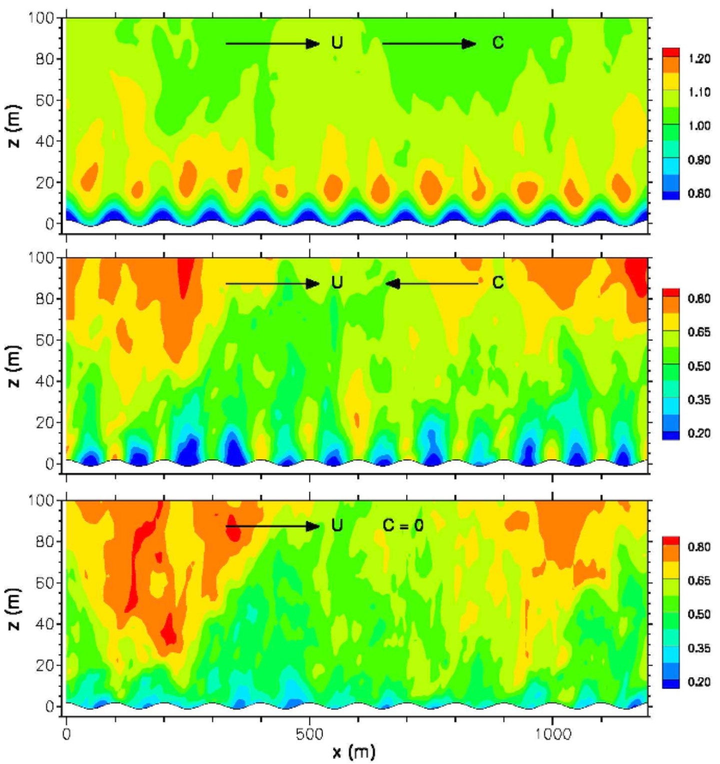
- Incomplete validation of simulation results by observations

BACKGROUND

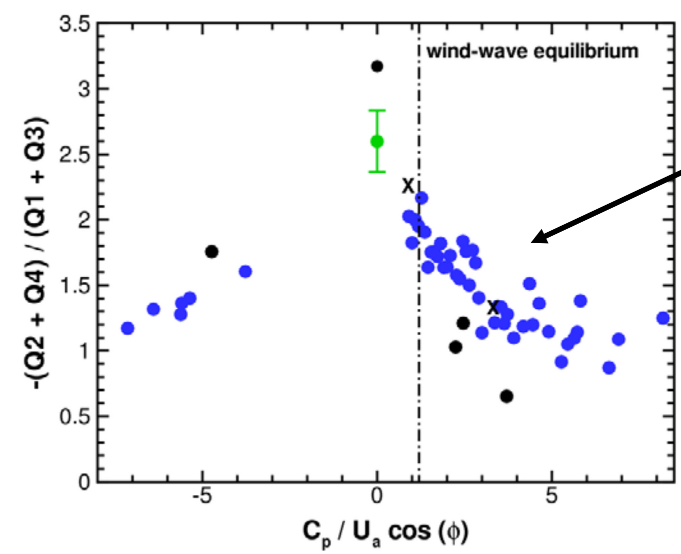
MEAN VELOCITY PROFILES



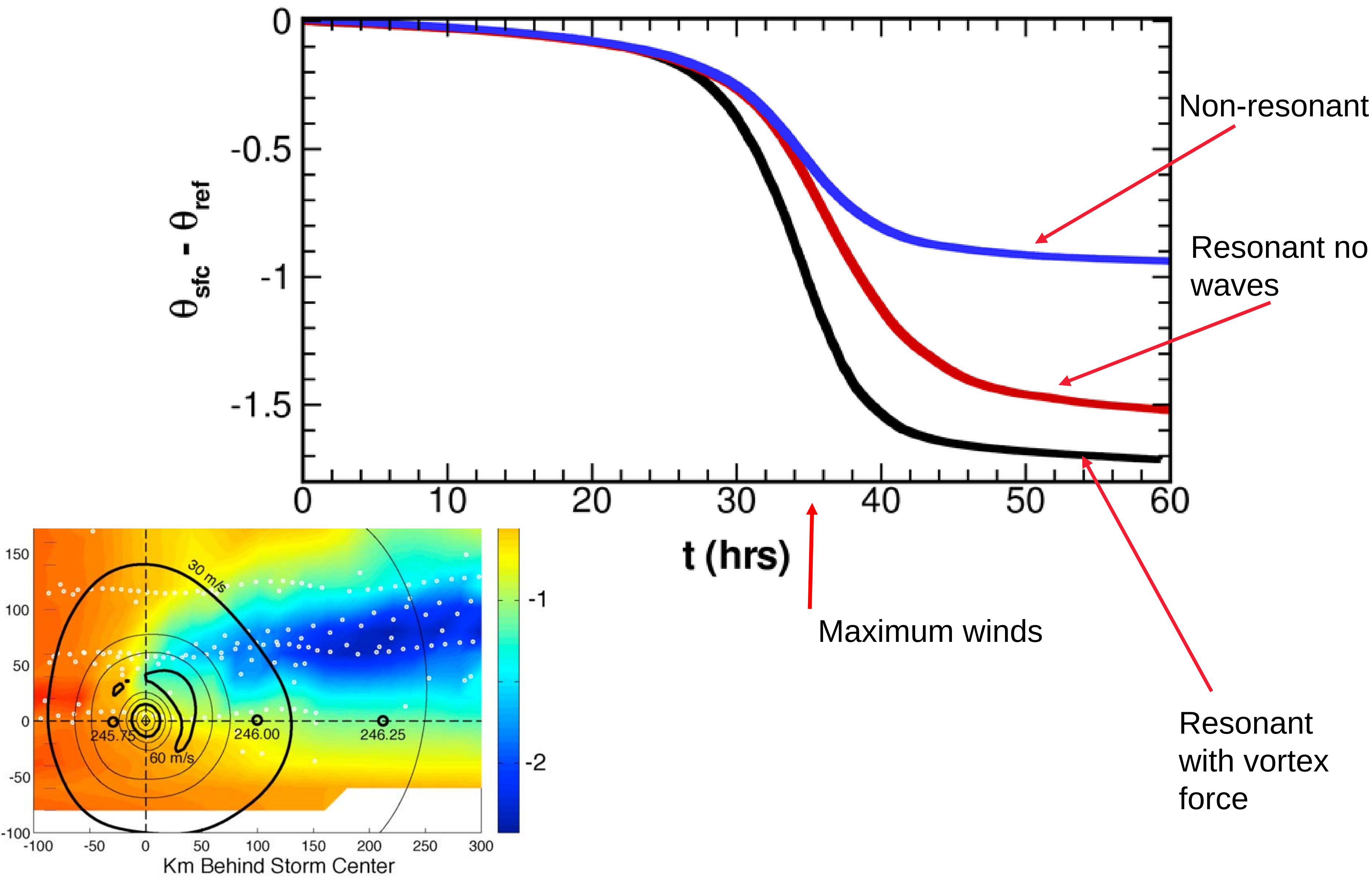
- **Waves and the marine ABL**
  - Turbulent flow *idealized* resolved waves (LES and DNS)
  - Interpretation of low-wind CBLAST observations



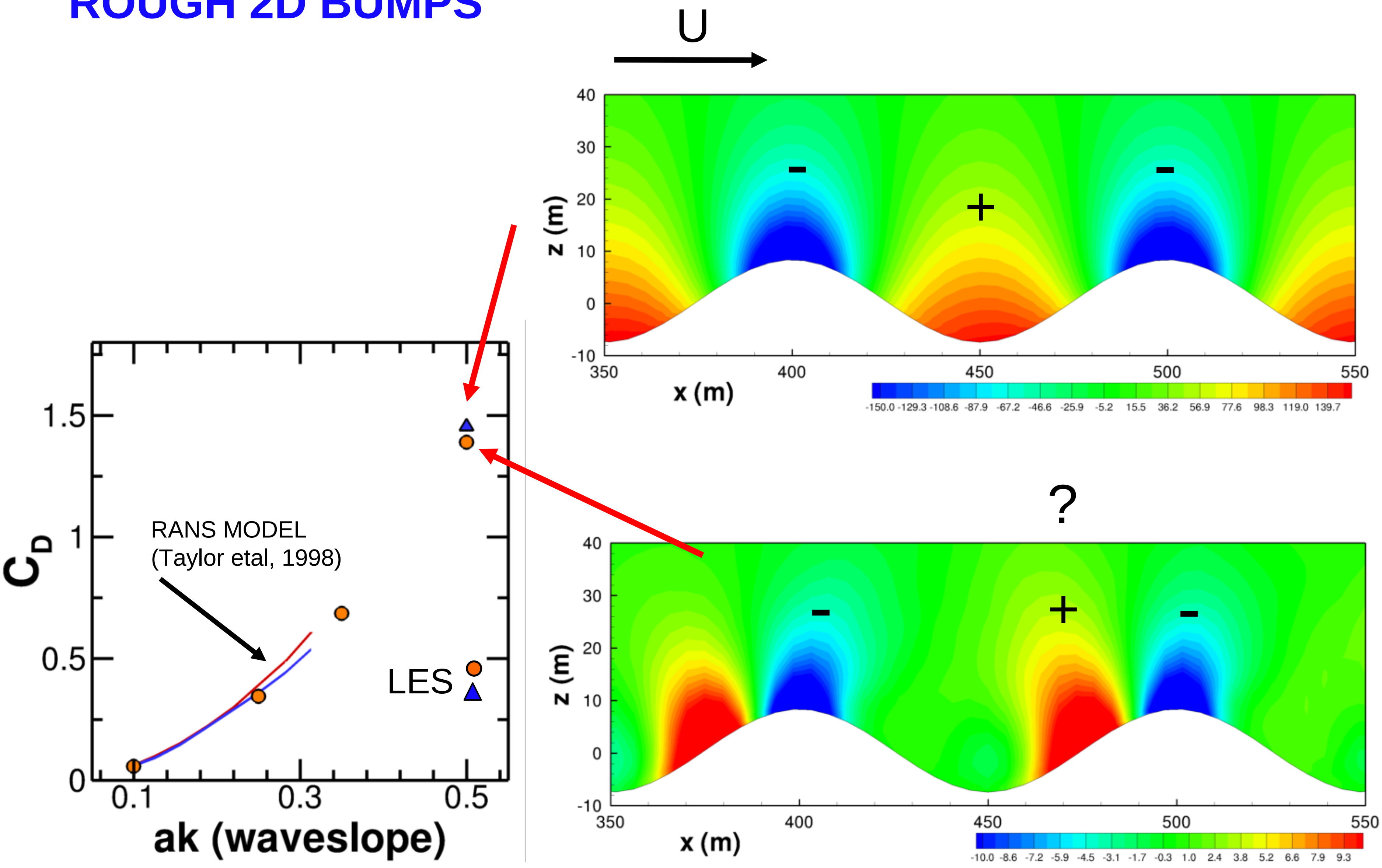
QUADRANT ANALYSIS OF $U'W'$ FROM CBLAST-LOW



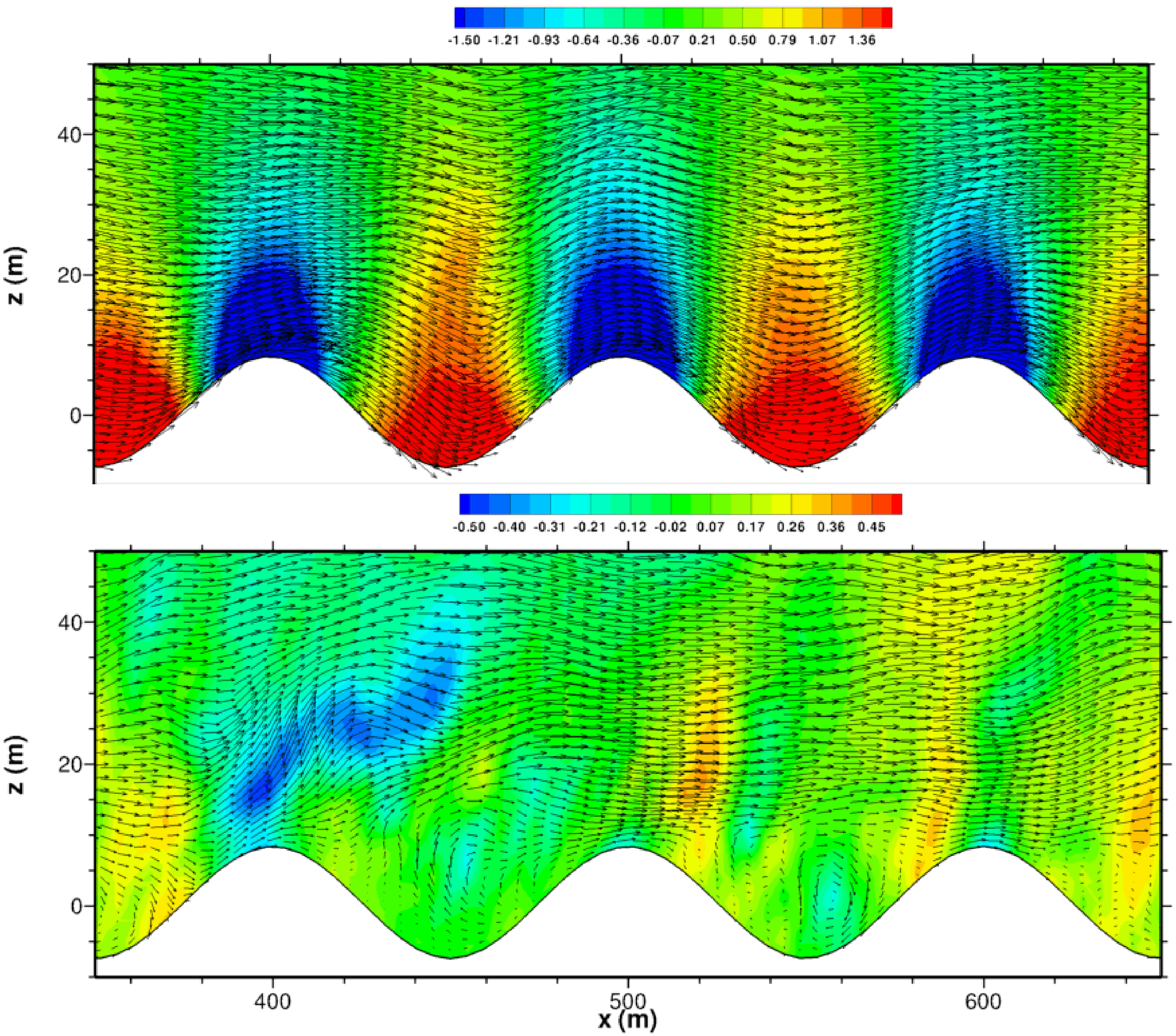
SST CHANGE AT RESONANT AND NON-RESONANT TRACK POSITIONS



PRESSURE FIELD IN TURBULENT FLOW OVER ROUGH 2D BUMPS



PRESSURE CONTOURS AND FLOW VECTORS



$$ak = 0.5$$
$$\lambda/z_o = 5 \times 10^5$$

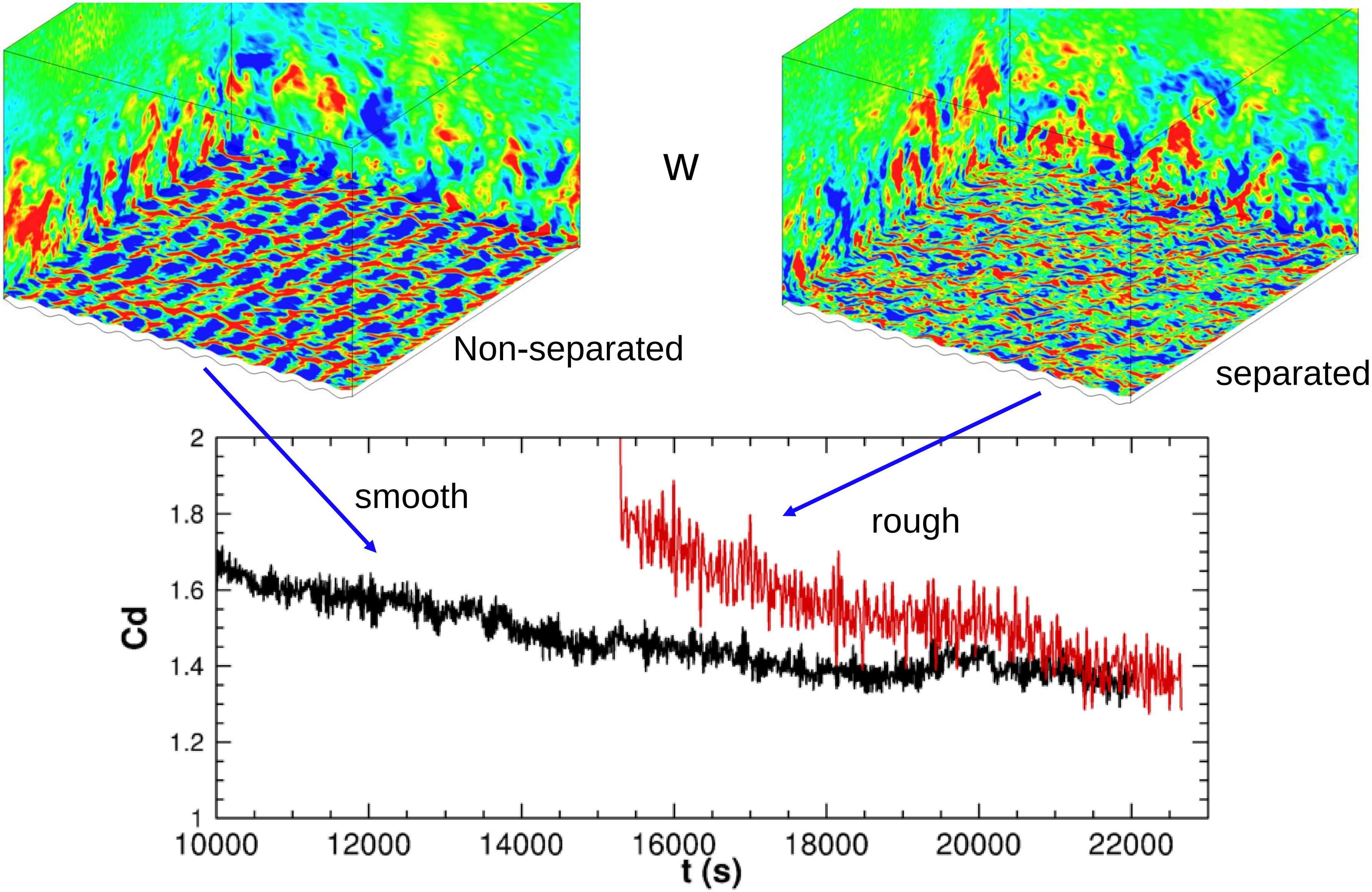
smooth

$$ak = 0.5$$
$$\lambda/z_o = 1 \times 10^3$$

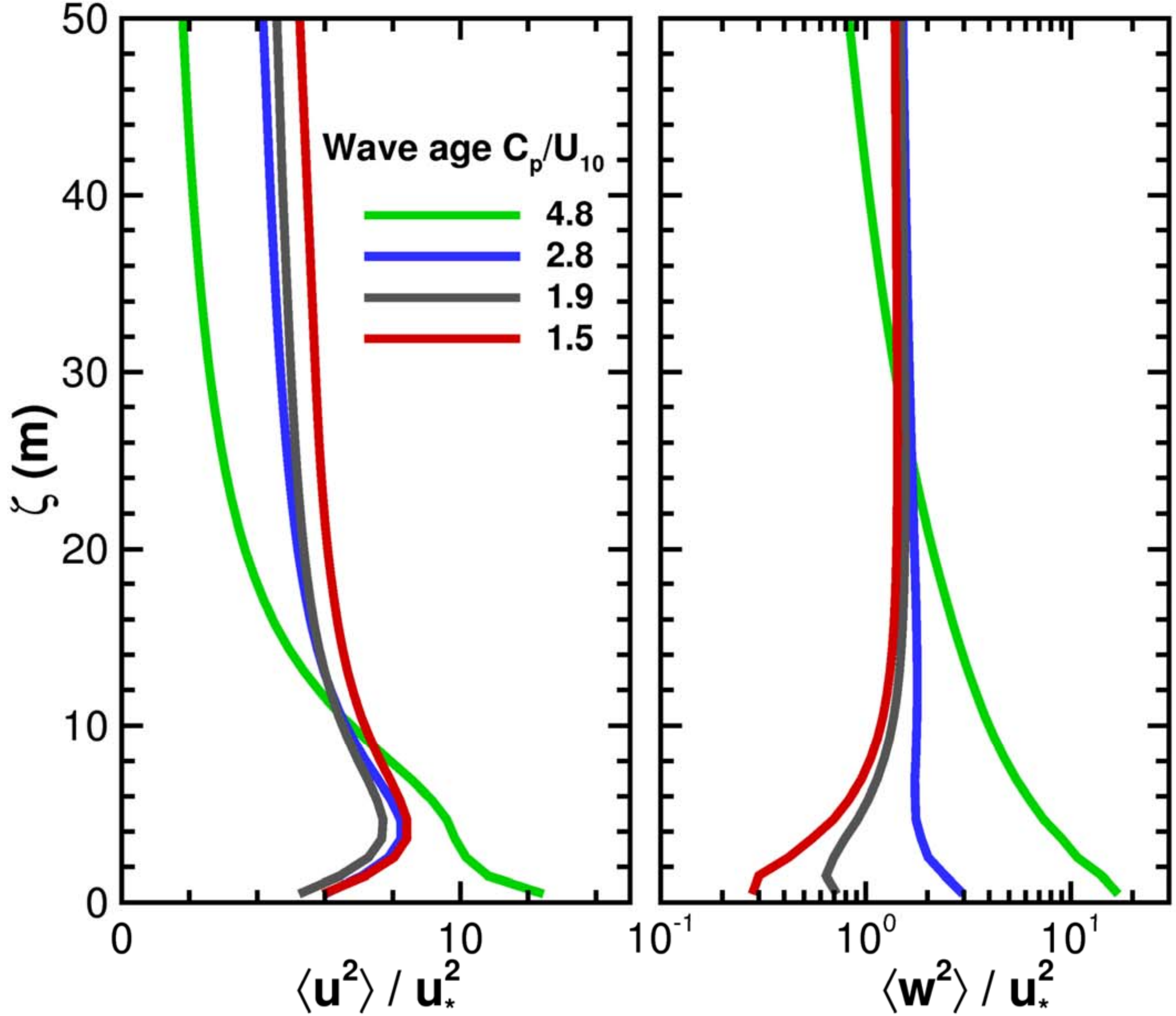
rough

Flow separation

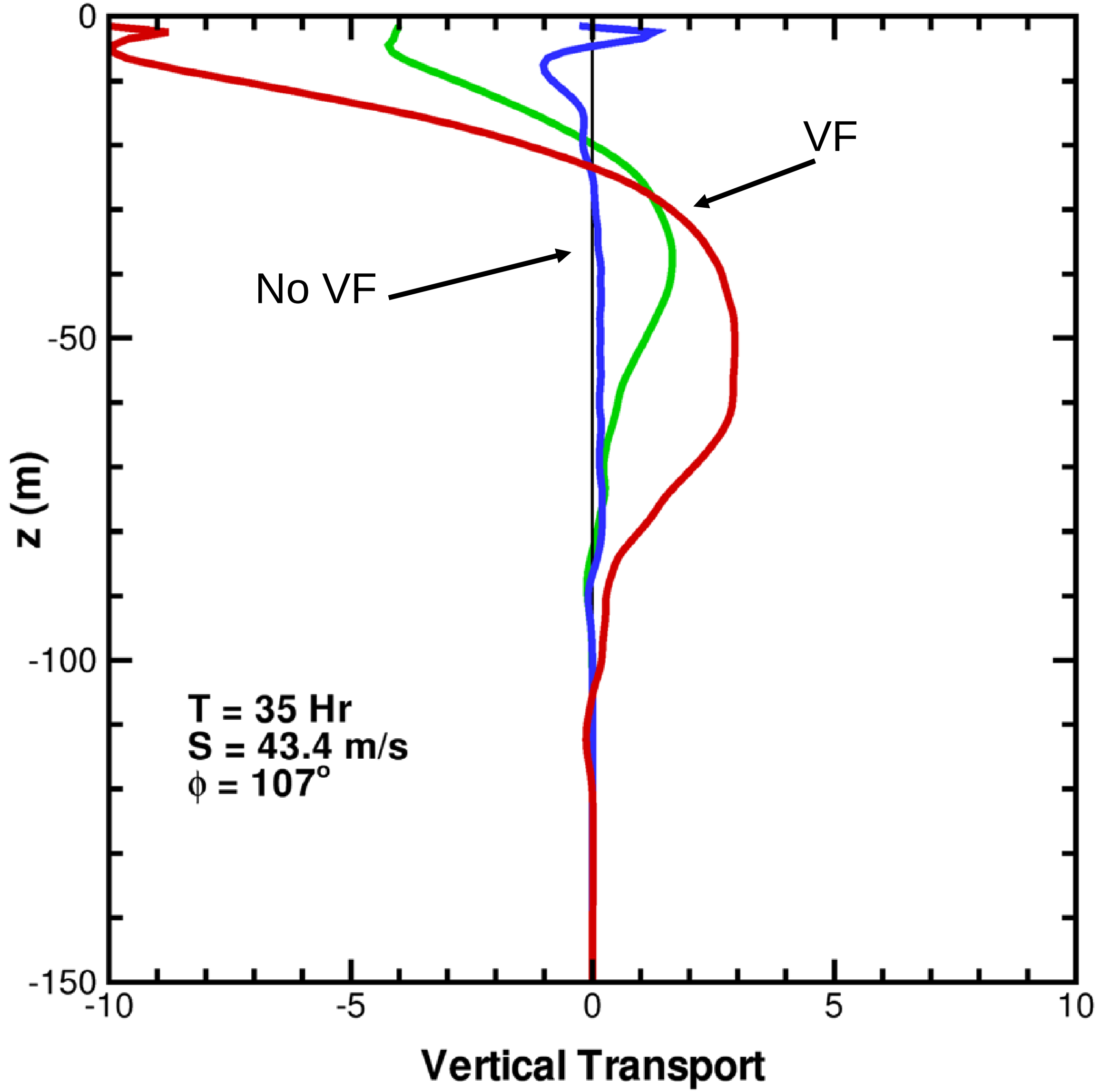
FORM DRAG FOR SMOOTH AND ROUGH BUMPS



VERTICAL PROFILE OF VARIANCES



TURBULENT TRANSPORT IN THE OBL

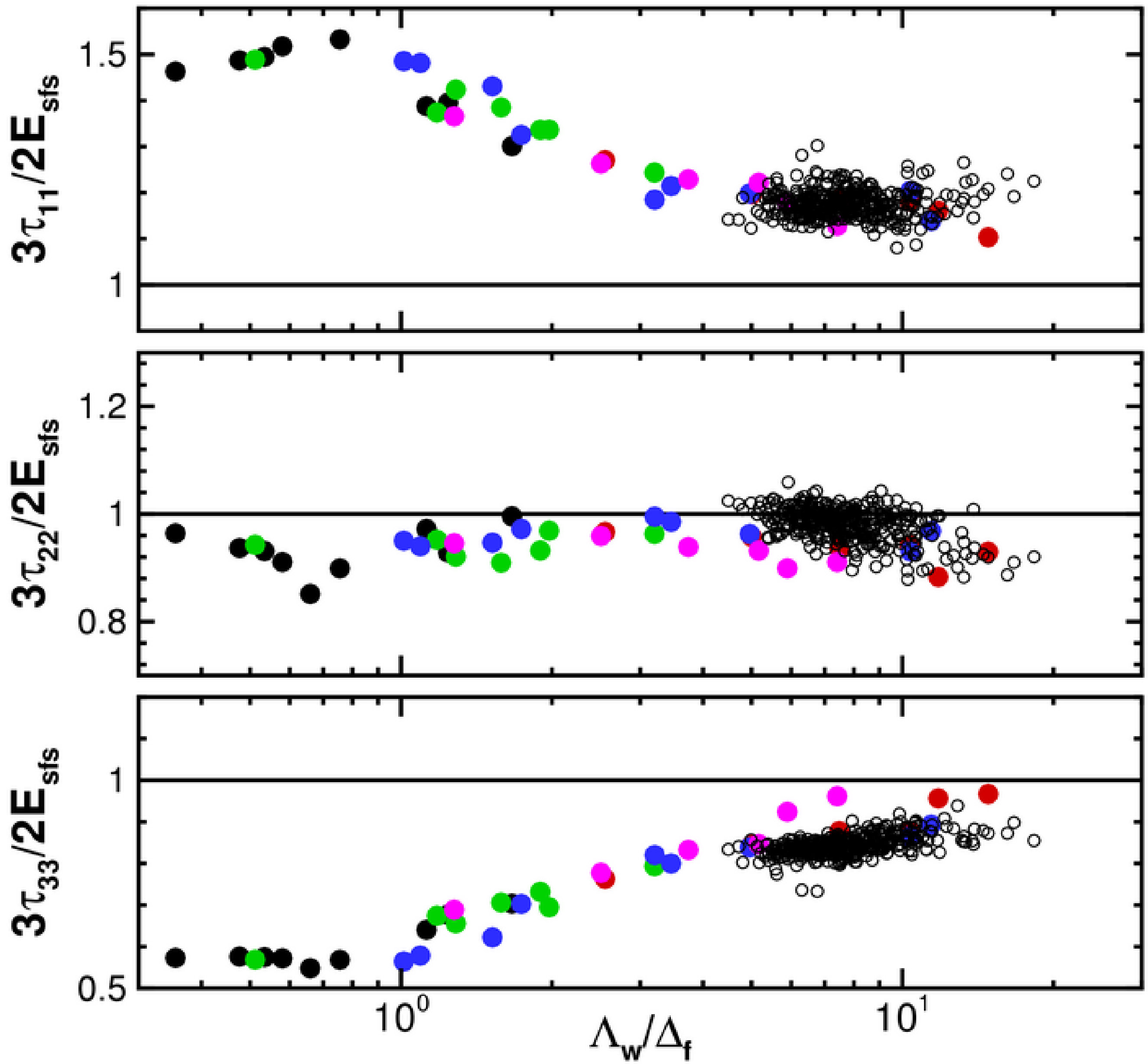


TKE budget term

$$TT = -\frac{\partial}{\partial z}\frac{\langle w w^2\rangle}{2}$$

Grant (2009)

SFS VELOCITY VARIANCES



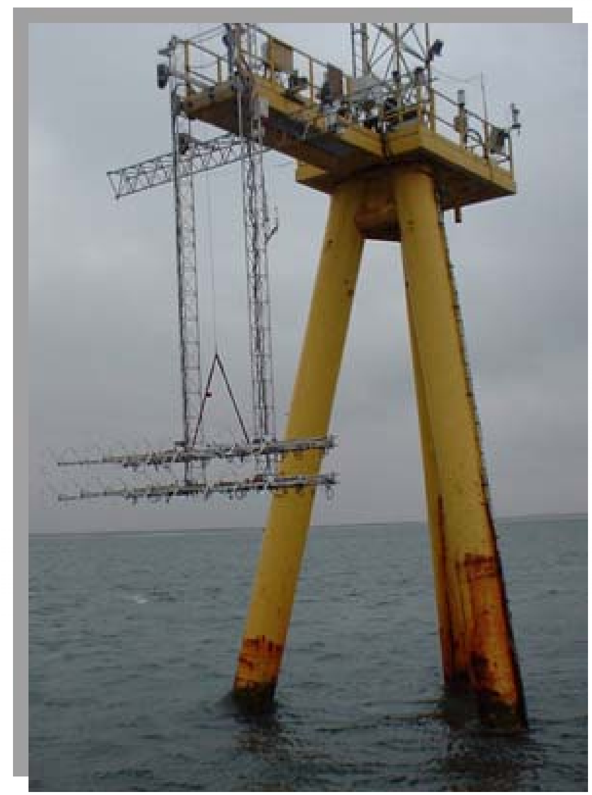
RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- What are the parent equations for the Smagorinsky model?

HORIZONTAL ARRAY TURBULENCE STUDY

HATS

OHATS



HORIZONTAL ARRAY TURBULENCE STUDY

HATS

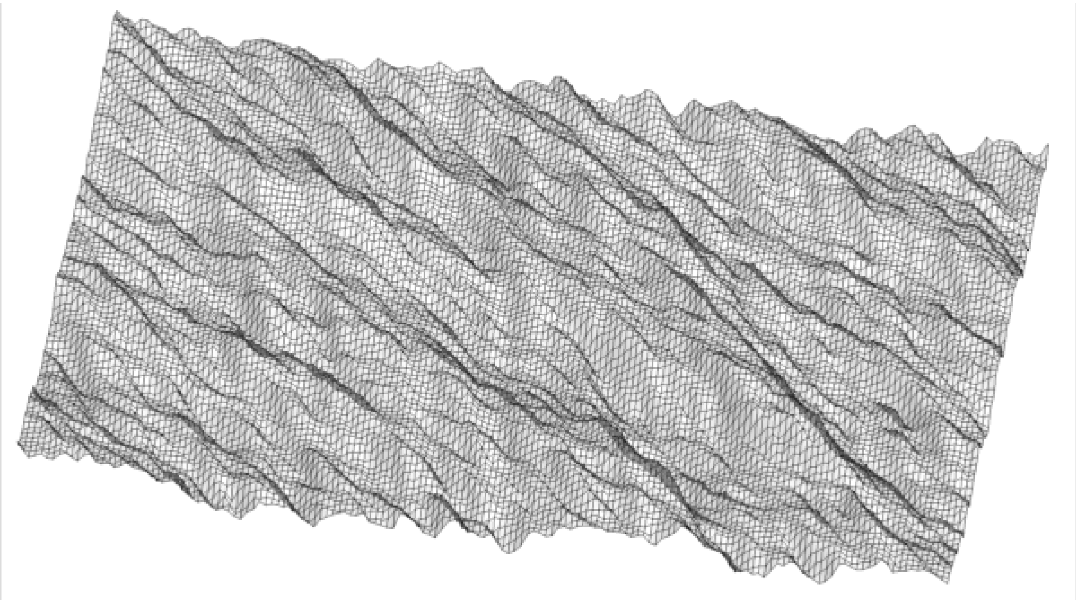
OHATS

LOW LEVEL FLIGHT IN HURRICANE ISABEL Courtesy M. Black

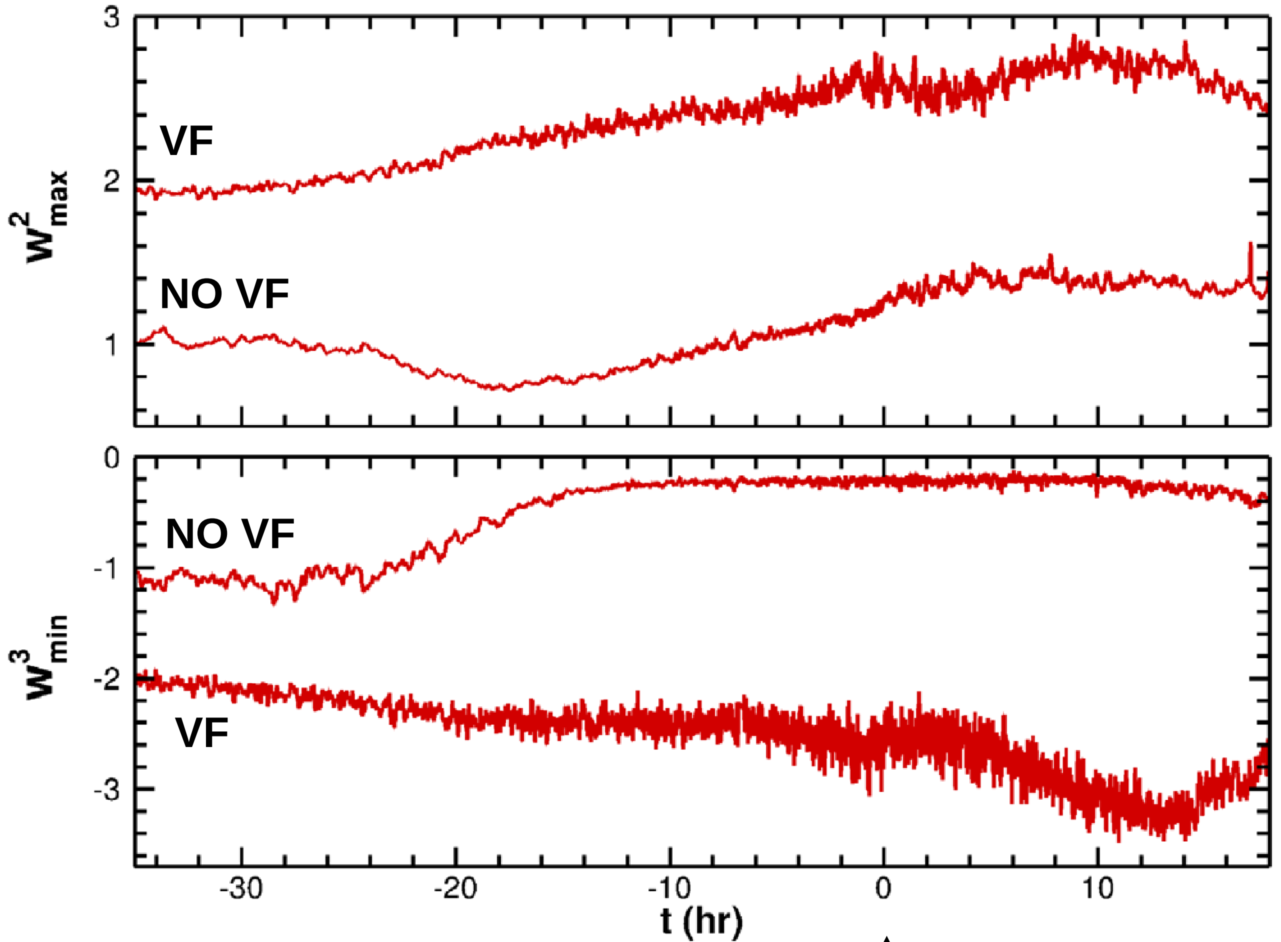
LES FOR AN ABL ABOVE SPECTRUM OF 3-D MOVING WAVES: SURFACE FITTED CO-LOCATED METHOD

Approach:

- Cast equations in surface fitted *moving* coordinates $x_i \Rightarrow \xi_i$
- Use contra-variant "flux" velocities U_i in formulating the LES equations
- Trick is to use "momentum-interpolation" of the right-hand sides Sullivan $et\ al.$(2008)
- Satisfy the grid conservation law (determines grid speeds)
- Initially, prescibed wave field



RESOLVED VERTICAL VELOCITY MOMENTS



Max Winds

Resonant