# Real Flows Have Walls


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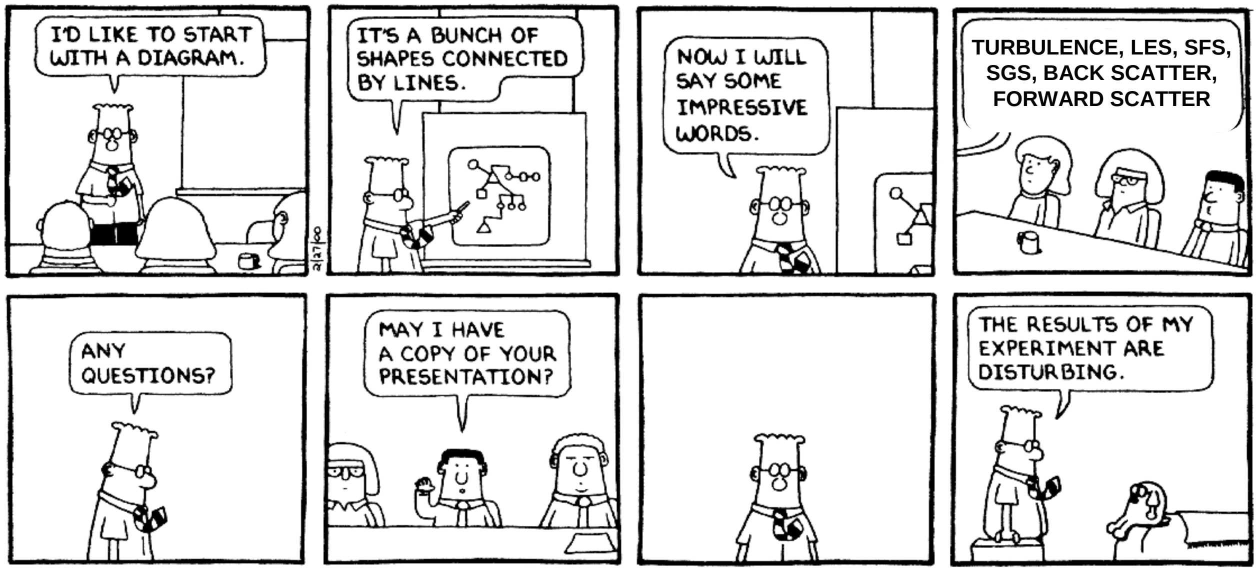


## The Wall:

Sullivan et al. (JFM, 2003) demonstrates clearly how, "in the atmospheric surface layer, the wavelength of the peak in the vertical velocity spectrum decreases with … proximity to the surface and this dependence constrains our ability to perform high-Reynolds-number large-eddy simulation (LES). Near the ground, the LES filter cutoff is comparable to or larger than spectral peak length and as a result the subfilter-scale (SFS) fluxes in LES are always significant."

Indeed, as we approach the surface [the wall] the turbulent energy becomes entirely subgrid scale. **Thus, special model treatment is needed there.**

# Themes and an outline.

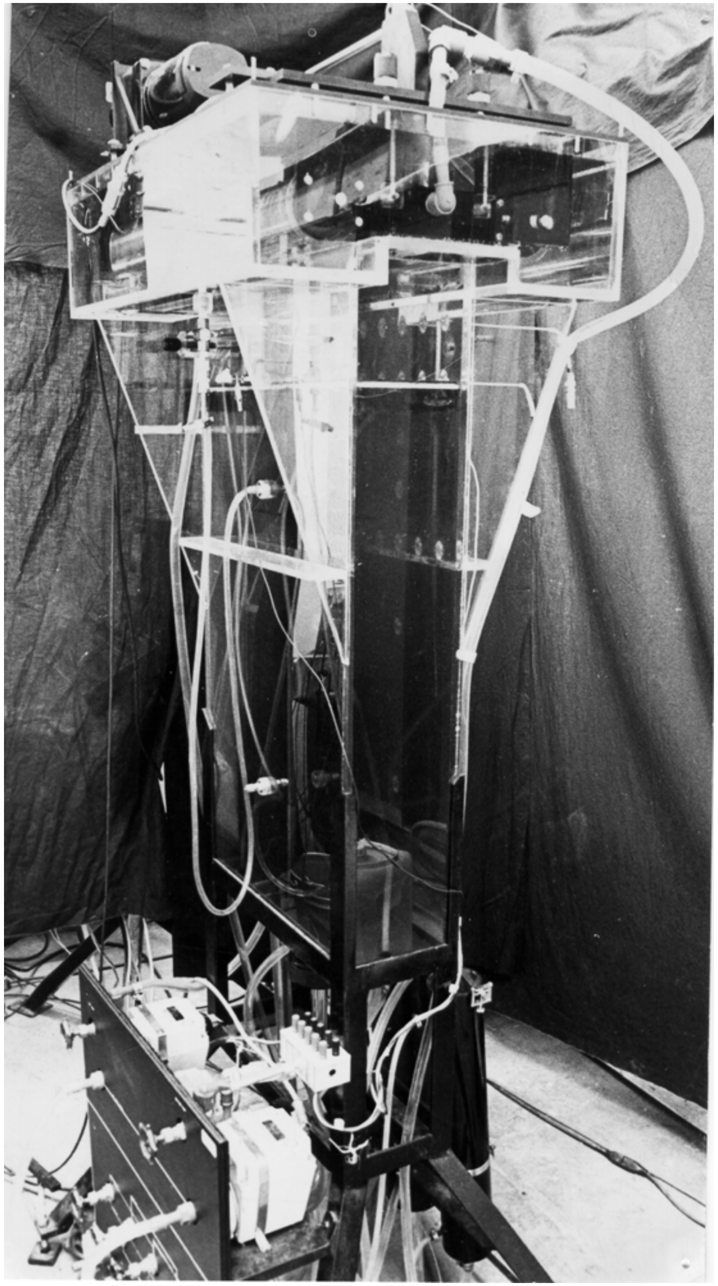
- ✓ A numerical model can succeed only if the algorithm (1) allows representation of the "real" flow physics and (2) does not suppress essential behavior. **Example:** the "classic" lid-driven cavity flow.
- ✓ A numerical simulation can accurately represent reality even when the elements of the code are not derived from first principles; both phenomenological and empirical models can suffice. **Example:** sediment transport and evolution of a sandy bed with turbulent flow beneath water waves [a coastal ocean problem].
- ✓ One can succeed in modeling "real" flows by (1) understanding the essential features of the flow and (2) specifically incorporating needed physics into numerical algorithms. **Example:** subfilter-scale and subgrid-scale turbulence models for large-eddy simulation [LES].

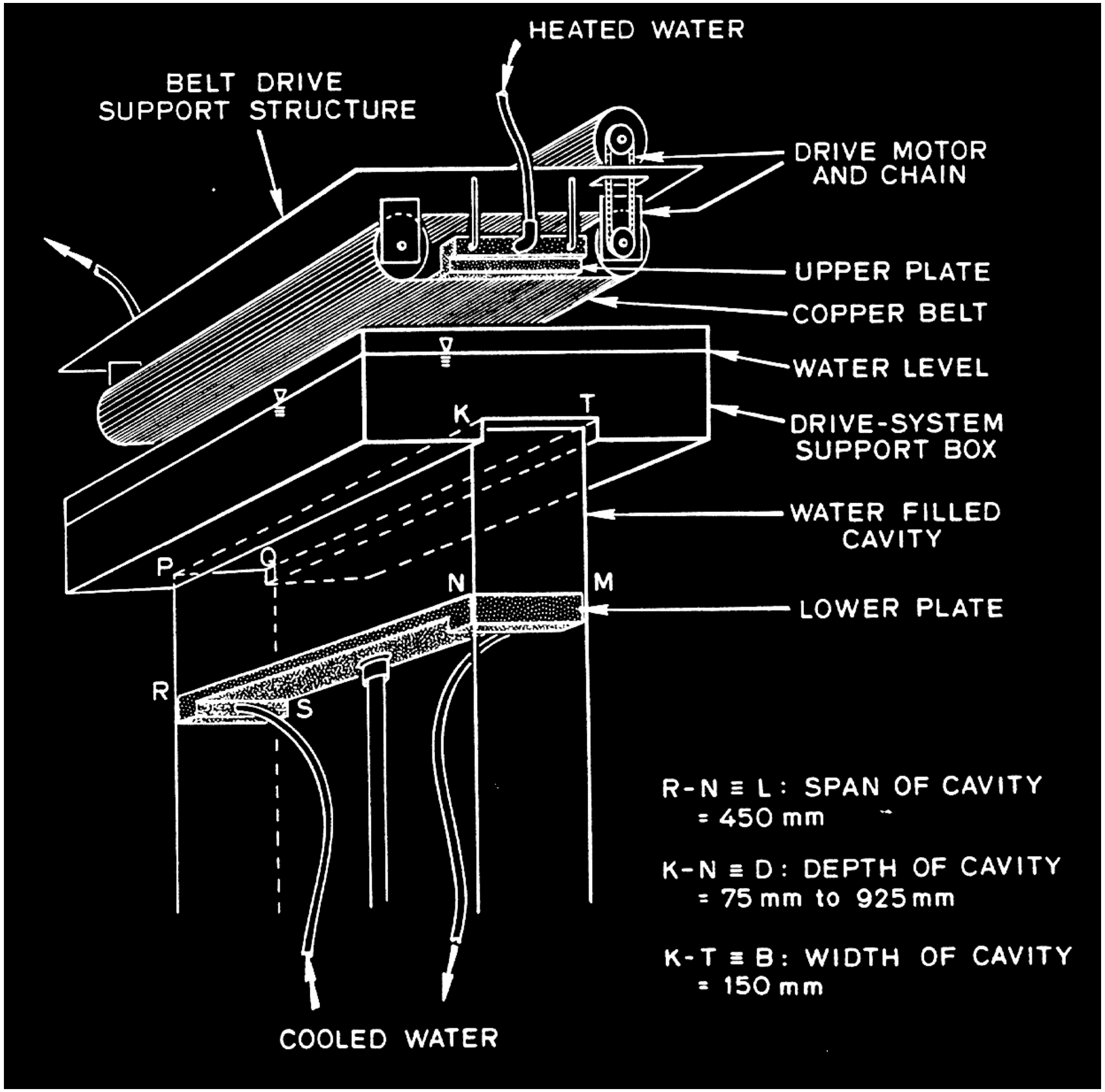
## **Example:** the "classic" lid-driven cavity flow.

**Context:** Mixing of stratified flows in large water bodies as well as heat transfers on ribs and cutouts in mechanical-engineering-scale flows led to a great interest in flow in and around cavities.

In the early days, the turbulent flow in cavities was not understood and simulators were focusing on two dimensional flows. Here we got involved with numerical codes that did not reproduce the flow physics and the influence of walls that totally changed the flow physics.

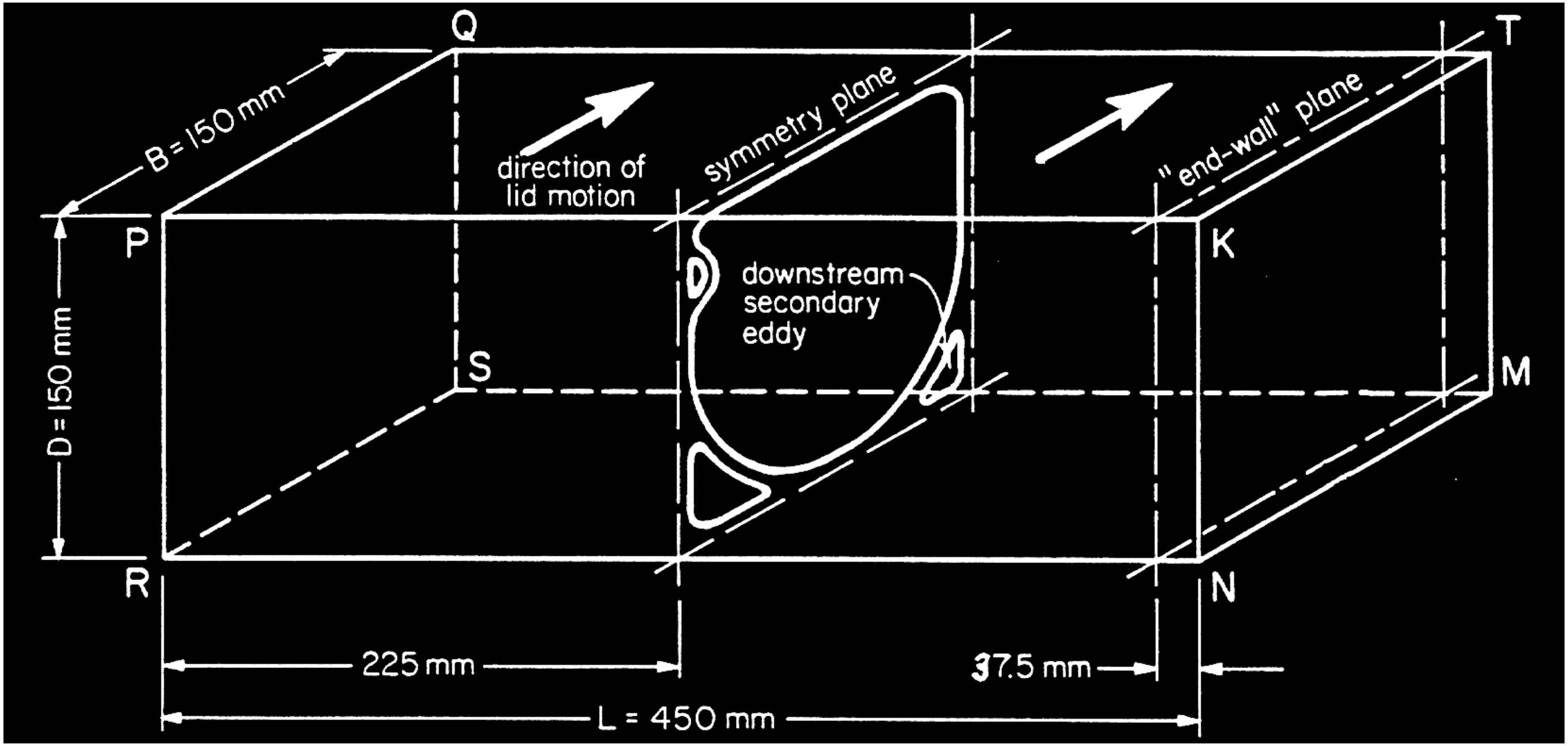
**The focus here is on the codes and physics; no models are involved beyond the implicit modeling of finite differences.**

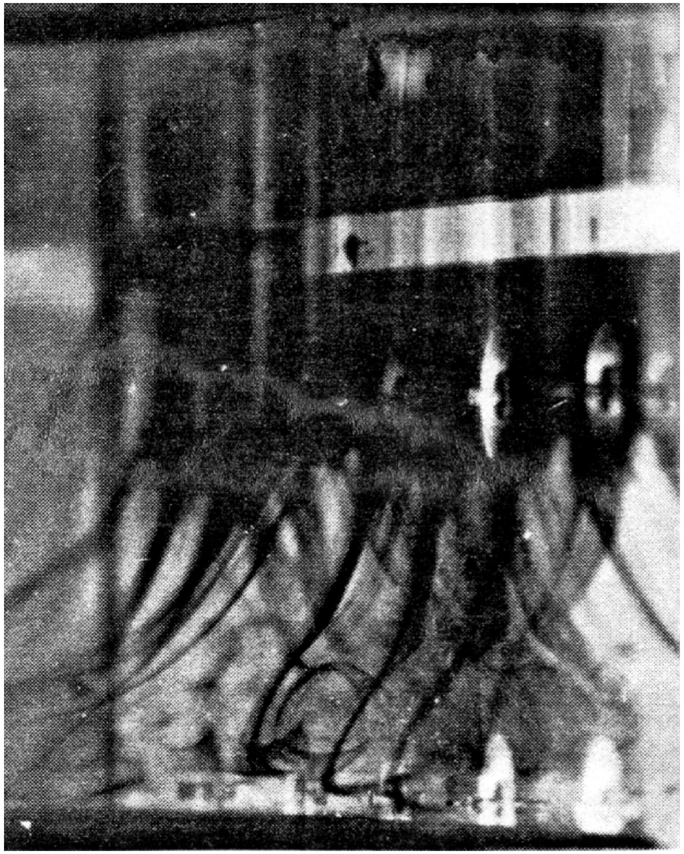




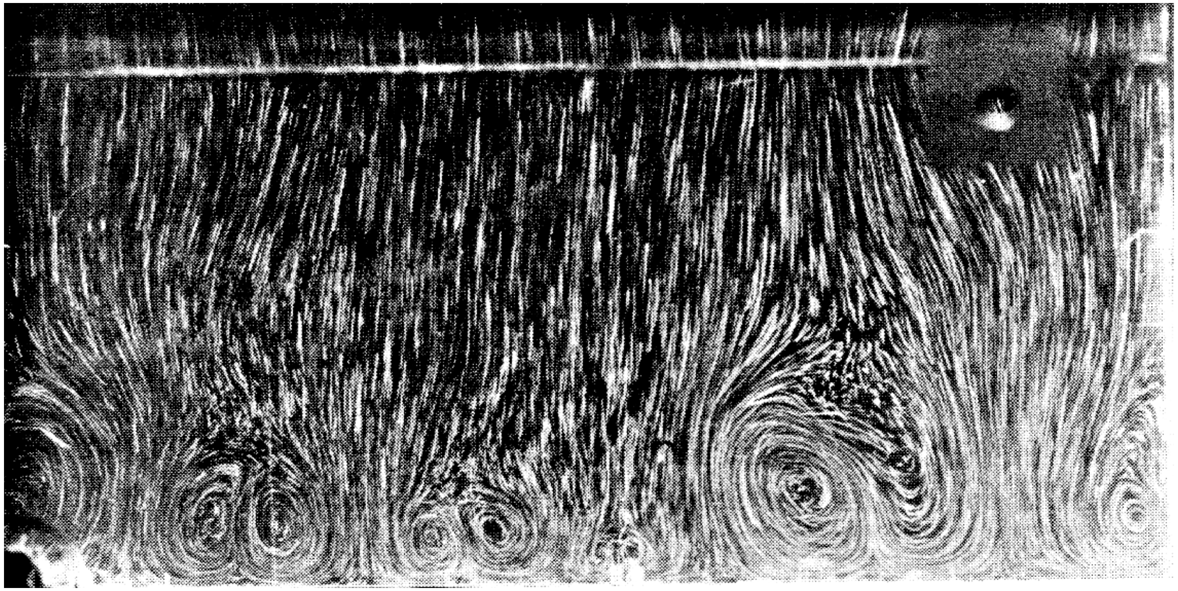

## The Turbulence Modeling Facility

## This schematic shows a layout of the test section of the facility and typical flow.





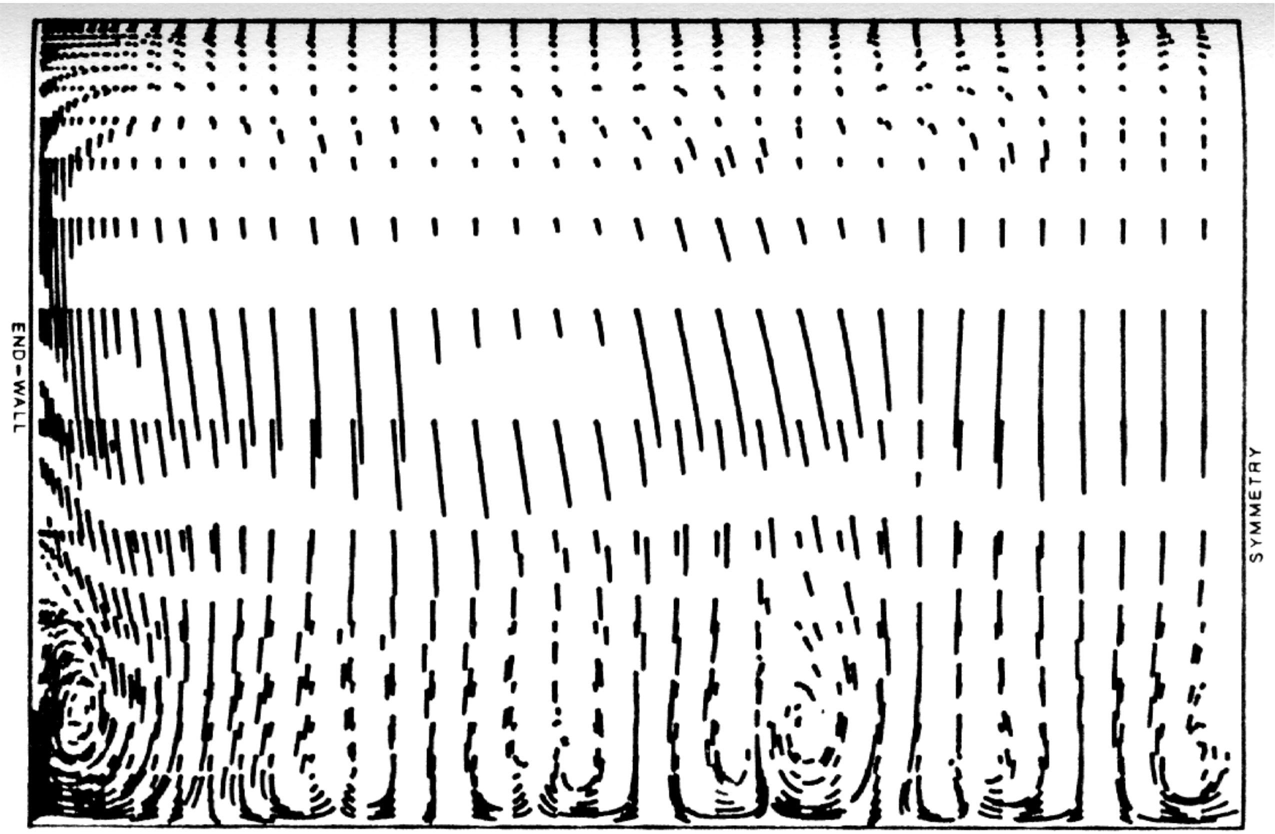
This view shows electrically-generated "thymol-blue" neutrally buoyant traces which outline the downstream eddy.



This plane is a few centimeters upstream of wall shown above; light-sheet visualization of particles in flow.

**2-D simulations do not capture the essential 3-D nature of the flow and its unsteadiness.**

**Circa 1986** Another student, Chris Freitas, created **REMIXS** [**RE**circulating **MIX**ed Convection **S**imulator]. Derived from REBUFFS [UCBerkeley] which used the SIMPLE algorithm which iteratively solves all the difference equations together. Key new feature here was use of QUICK upstream scheme to remove diffusion in first-order upwind schemes. Freitas was first to simulate these vortices. DNS.

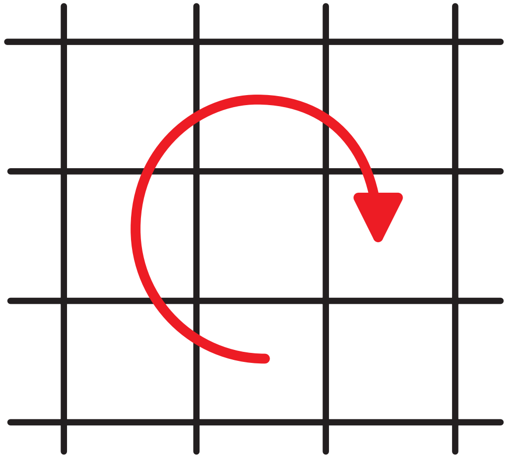


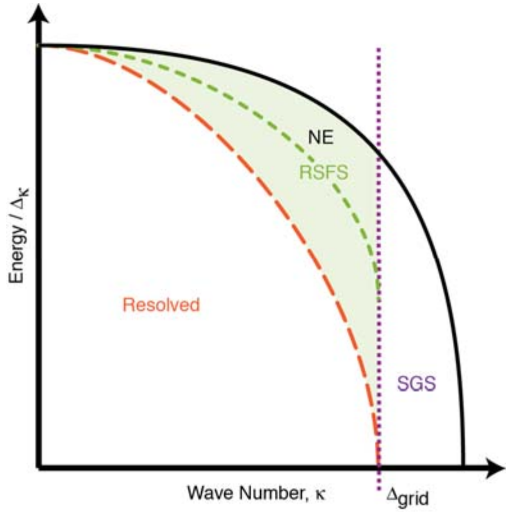

b. Particle Track Field

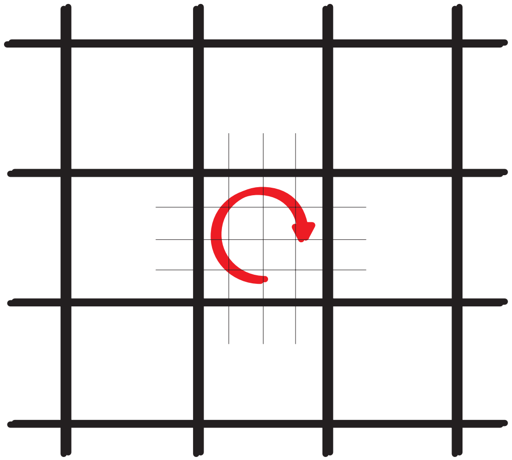
# TIME OUT

## Quick review of our Large-eddy simulation (LES) approaches

Spatial filtering





LES - Resolved

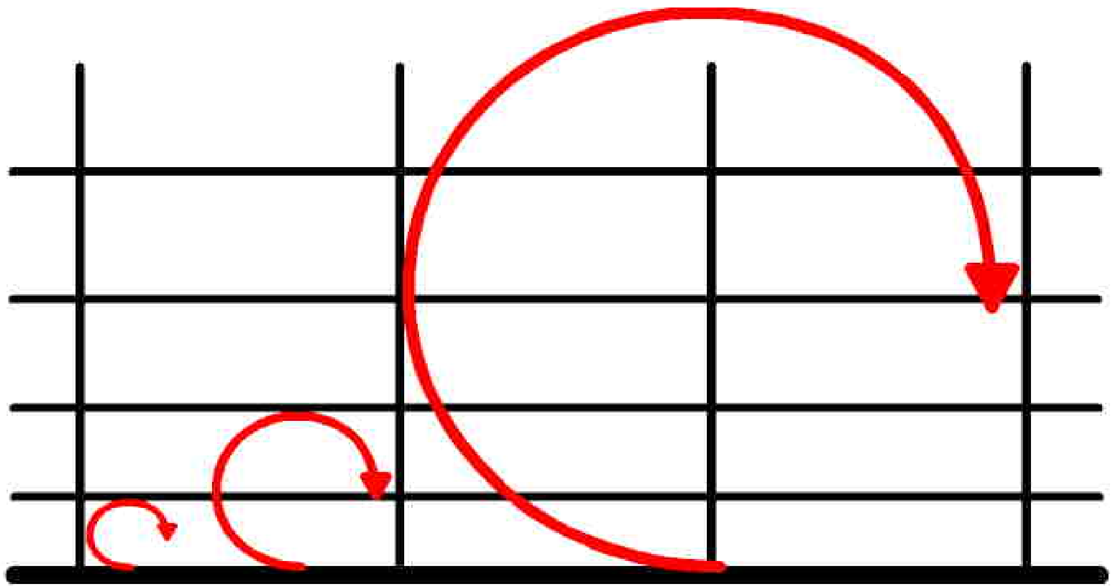
LES – Subfilter-scale

$$\tau_{ij} = \tau_{RSFS} + \tau_{SGS}$$

# Subfilter-scale (SFS) model importance

- SFS model critical for boundary layers
  - High Reynolds number and rough boundary
  - Near-wall energy-containing eddies not resolved
- Error at wall/surface affects entire boundary layer

Wall is a major influence



# Discretized LES Momentum Equation

cf., Carati, et al. (2001)

$$\underbrace{\frac{\partial \bar{\tilde{u}}_i}{\partial t}}_{\text{Local Acceleration}} + \underbrace{\bar{\tilde{u}}_k \frac{\partial \bar{\tilde{u}}_i}{\partial x_k}}_{\text{Advection}} = -\underbrace{\frac{\partial}{\partial x_i}\frac{\bar{\tilde{\rho}}}{\rho_o}}_{\text{Pressure Gradient}} + \underbrace{\nu \frac{\partial^2 \bar{\tilde{u}}_i}{\partial x_k^2}}_{\text{Viscous}} - \underbrace{\frac{\bar{\tilde{\rho}}}{\rho_o} g \delta_{i3}}_{\text{Buoyancy}} + \underbrace{\varepsilon_{lmn} f_n \bar{\tilde{u}}_m}_{\text{Coriolis}}$$

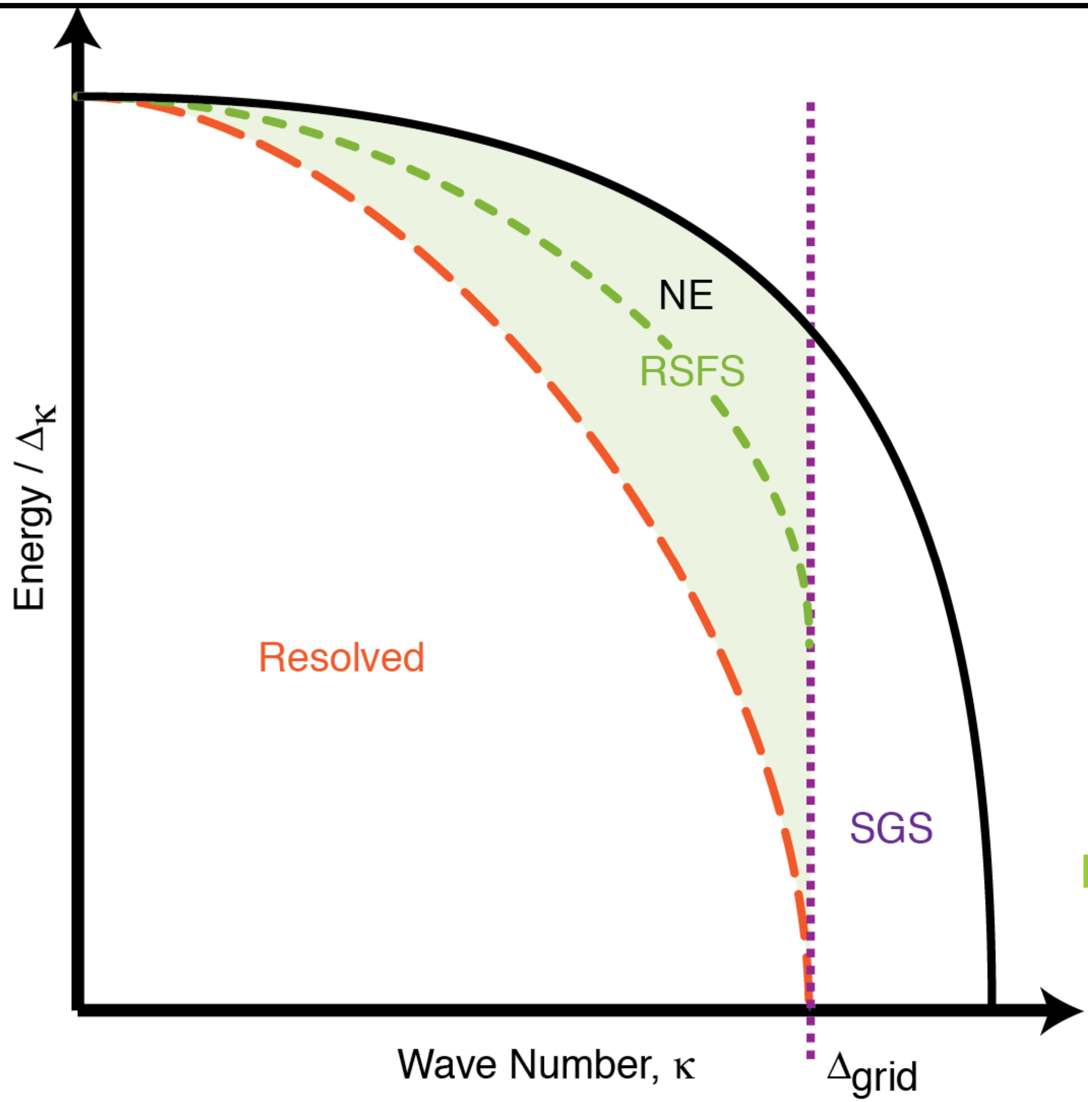
$$-\underbrace{\frac{\partial \Im_{ik}}{\partial x_k}}_{\text{Reynolds stress}}$$

$$\Im_{ij} = \overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$

$$= \underbrace{\overline{\widetilde{u_i u_j}} - \overline{\tilde{u}_i \tilde{u}_j}}_{\text{Subgrid-scale}} + \underbrace{\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j}_{\text{Subfilter-scale}}$$

$$\square_{ik} = A_{ik} + B_{ik}$$

# Splitting up the turbulence




$$\mathfrak{I}_{ij} = \overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$

$$= \underbrace{\overline{\widetilde{u_i u_j}} - \overline{\tilde{u}_i \tilde{u}_j}}_{\text{Subgrid-scale}} + \underbrace{\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j}_{\text{Subfilter-scale}}$$

$$\Box_{ij} = A_{ij} + B_{ij}$$

NE: Numerical error
RSFS: Resolved subfilter-scale
SGS: Subgrid-scale

# Dealing with the turbulence terms: Alternatives

1. Ignore the RSFS and just model the SGS : this assumes essentially that the filter and grid sizes are the same. Use Smagorinsky formulation or dynamic version:

$$\overline{A}_{ij} = -2\nu_T \overline{\tilde{S}}_{ij} = -\nu_T \left( \frac{\partial \overline{\tilde{u}_i}}{\partial x_j} + \frac{\partial \overline{\tilde{u}_j}}{\partial x_i} \right)$$

2. Reconstruct the RSFS $B_{ij} = \overline{\tilde{u}_i \tilde{u}_j} - \overline{\tilde{u}}_i \overline{\tilde{u}}_j$

and model SGS by simple or complex model

3. Add wall model: $\tau_{i,\text{near-wall}} = -\int C_c a(z) \left| \overline{\tilde{u}} \right| \overline{\tilde{u}}_i \, dz$

# Reconstruction of the subfilter-scale [SFS] stress

Recipe to get SFS stress

see Chow et al. JAS (2005)

1. Reconstruct estimated velocity, $\tilde{u}^*$, from the resolved velocity, $\bar{\tilde{u}}$

$$\tilde{u}^*_i = \bar{\tilde{u}}_i + (I - G) * \bar{\tilde{u}}_i + (I - G)^2 * \bar{\tilde{u}}_i + \ldots$$

$$\tilde{u}^*_i = 3\bar{\tilde{u}}_i - 3\bar{\bar{\tilde{u}}}_i + \bar{\bar{\bar{\tilde{u}}}}_i - \ldots$$

van Cittert (1931) iteration

2. Substitute $\tilde{u}_j^*$ into the SFS stress equation to obtain $B_{ij}$

Subfilter-scale stress = $\overline{\tilde{u}_i^* \tilde{u}_j^*} - \overline{\tilde{u}_i^*}\,\overline{\tilde{u}_j^*}$

0-th order: $\tilde{u}^*_i = \bar{\tilde{u}}_i$

# Gullbrand and Chow (2003) Turbulent channel flow – effects of reconstruction

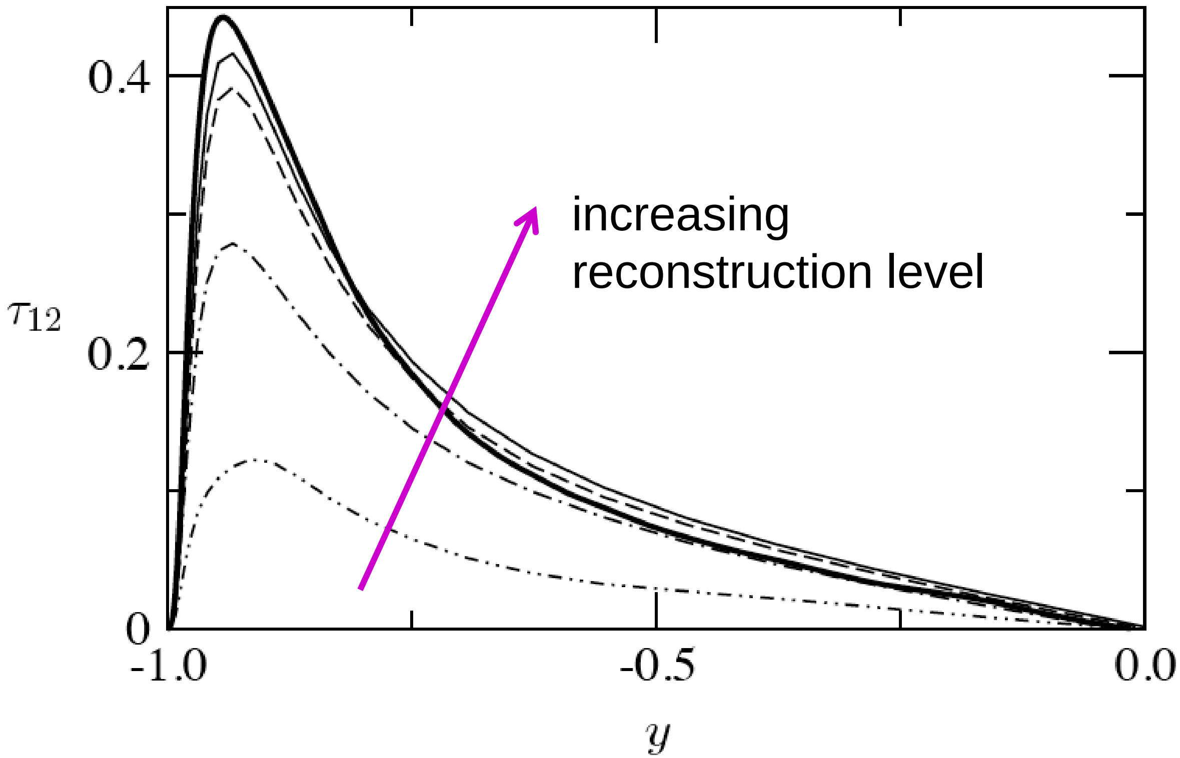



Figure 5.11: Profiles of the turbulent stress $\tilde{\tau}_{12}$ for the fourth-order (64,49,48) code with explicit filtering (tophat) and reconstruction. —··— : DSM, —·— : DMM, ---- : DRM5, —— : DRM10, and ━━ : DNS.

(Level 0)

**Example:** sediment transport and evolution of a sandy bed with turbulent flow beneath water waves [a coastal ocean problem].

**Goal:** to demonstrate that a numerical simulation can accurately represent reality even when the elements of the code are not derived from first principles; both phenomenological and empirical models can suffice.

We look at two cases:

First, to get a feeling for the flow: oscillating flow over sinusoidal waves in a closed channel, called flow over, so-called, vortex ripples.

Second, the real-time evolution of ripples on a flat bed in a channel.

# Why use numerical modeling to study sediment transport over vortex ripples?

- *The model resolves suspended sediment transport which is dominant over vortex ripples.*
- *The model provides knowledge of the 3D, time-dependent dynamics which can explain why the direction and magnitude of vortex ripple transport may be predicted incorrectly from pointwise measurements in the field.*

# Numerical Methods

Large-eddy Simulation Code (Calhoun & Street, 2001)

-- solves spatially-filtered Navier Stokes equations for fluid using a second-order accurate projection method (Zang, et al., 1994)

-- solves a spatially-filtered advection-diffusion equation with a settling term for sediment (Zedler & Street, 2001)

-- models turbulence with Dynamic Mixed Model (DMM; Zang, et. al., 1993)

-- rough boundary & turbulent boundary layer

# Key Equations

1. Continuity

Stress = Smagorinsky eddy viscosity model
+ 0th order reconstruction model

2. Filtered Navier-Stokes

$$\boxed{\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j}$$

3. Sediment concentration (volume fraction)
and settling velocity

$$\frac{\partial \overline{C}}{\partial t} + \frac{\partial}{\partial x_j}\left( (\overline{u}_j - w_s)\overline{C} - \frac{\nu}{\sigma}\frac{\partial \overline{C}}{\partial x_j} + \chi_j \right) = 0$$

$$w_s = \frac{10\nu}{d}\left[ \left( 1 + \frac{0.01(s-1)gd^3}{\nu^2} \right)^{0.5} - 1 \right].$$

$$\boxed{\chi_j = \overline{u_j C} - \overline{u}_j \overline{C}}$$

Note that we handle turbulent part of the sediment motion with a mixed model of the same form as for momentum.

# Key Equations

At the bed we use the *empirical* pickup function suggested by van Rijn:

Boundary Condition: $$\frac{\partial C}{\partial n} = -\frac{P}{\nu_T}$$

$$P = 0.00033 \left(\frac{\theta - \theta_{cr}}{\theta_{cr}}\right)^{1.5} \frac{(s-1)^{0.6} g^{0.6} d^{0.8}}{\nu_T^{0.2}} \quad \text{for} \quad \theta > \theta_{cr} \qquad (5a)$$

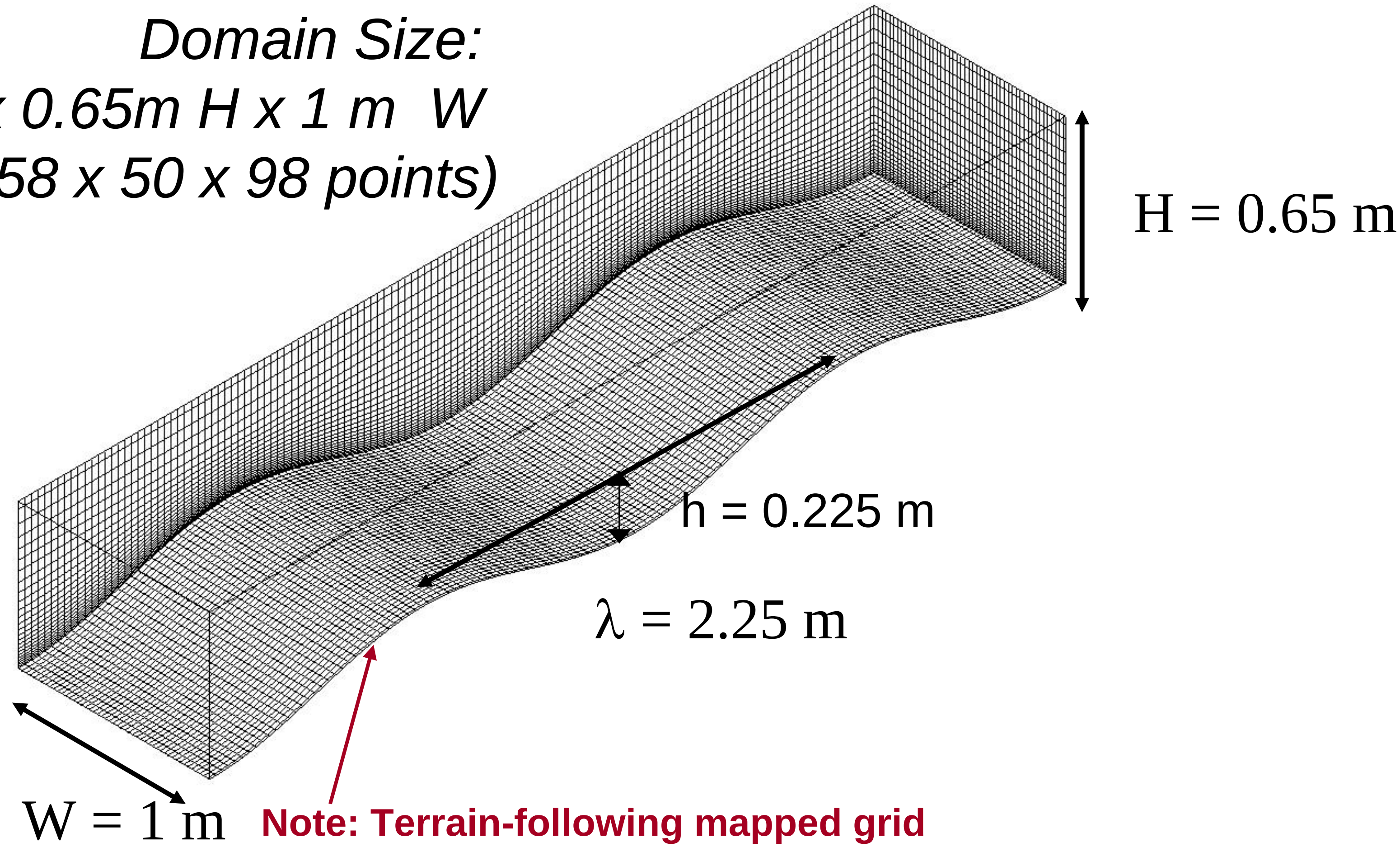
$$P = 0 \quad \text{for} \quad \theta < \theta_{cr} \qquad (5b)$$

where $\theta = \tau_b/(\rho_s - \rho)gd$ = Shields parameter, based on the bottom shear stress $\tau_b$; and $\theta_{cr}$ = critical Shields parameter.

At the bed we apply a boundary condition that forces a logarithmic velocity profile with drag based on the boundary roughness measure $z_0$. In addition we augment the near bed eddy viscosity to provide a wall model to account for the near bed influence of roughness on the stress [see Nakayama and Saiko, 2002 and Nakayama *et al*., 2004] and the grid anisotropy [high vertical resolution compared to horizontal] that does not allow small eddies to be properly represented.

# *Numerical Domain*

*Domain Size:*
*4.5m L x 0.65m H x 1 m W*
*(258 x 50 x 98 points)*




**Note: Terrain-following mapped grid at bottom.**

# *Wave & Sediment Parameters*

*Wave Parameters:*

$$T = 10\ s$$

$$Umax = 1.5\ m/s$$

$$Orb.\ Amp. \sim 2\text{-}3\ m$$

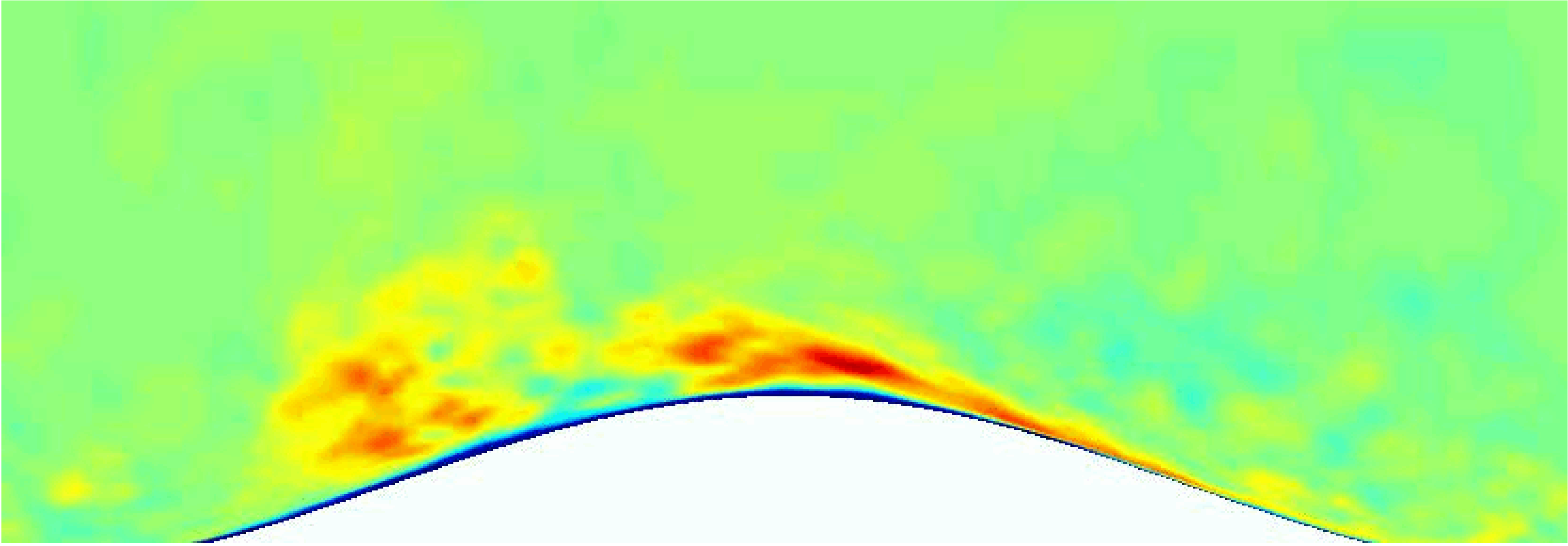
*Sediment Parameters: [sand]*

$$d = 200\ \mu m$$

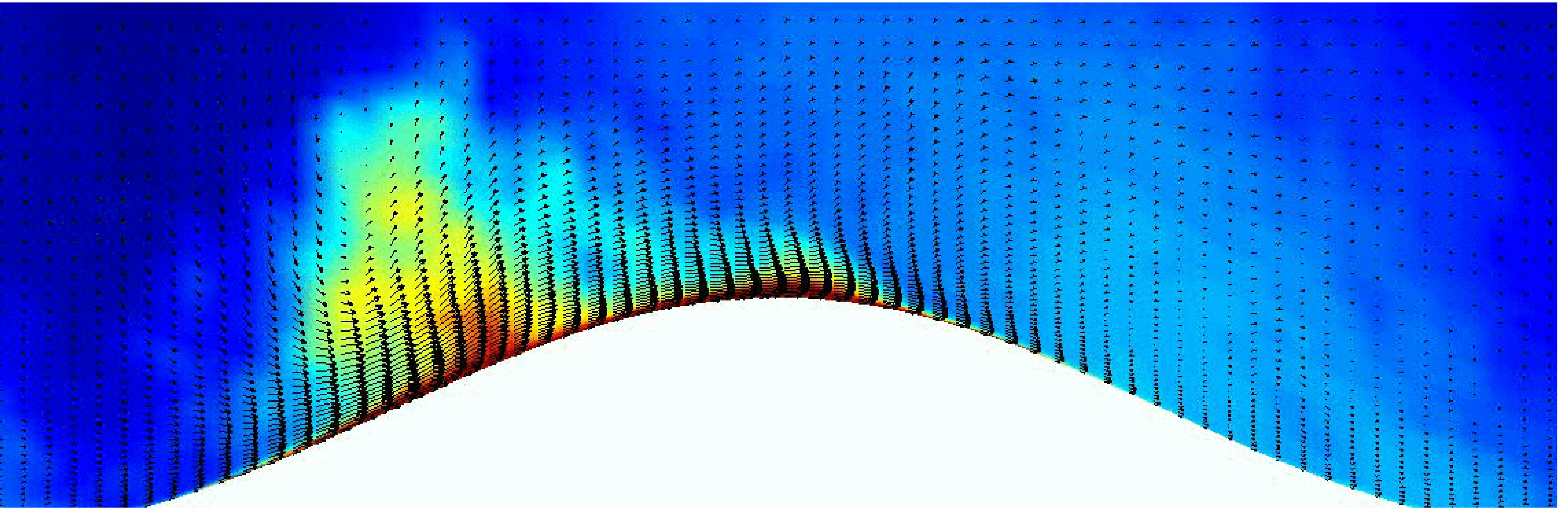
$$\rho = 2650\ kg/m^3$$

$$w_s = 2.4\ cm/s$$

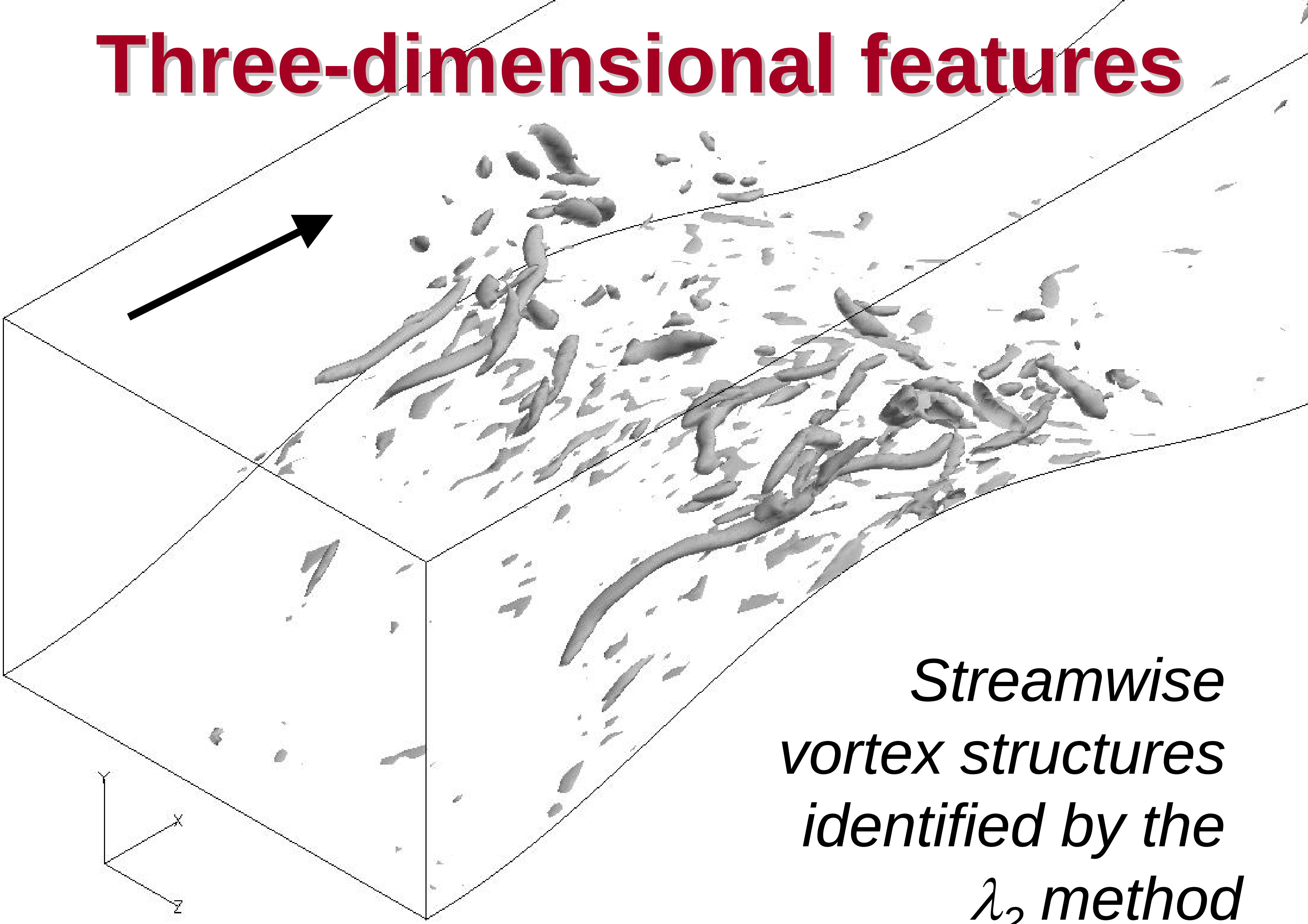
# Spanwise Vorticity Visualization



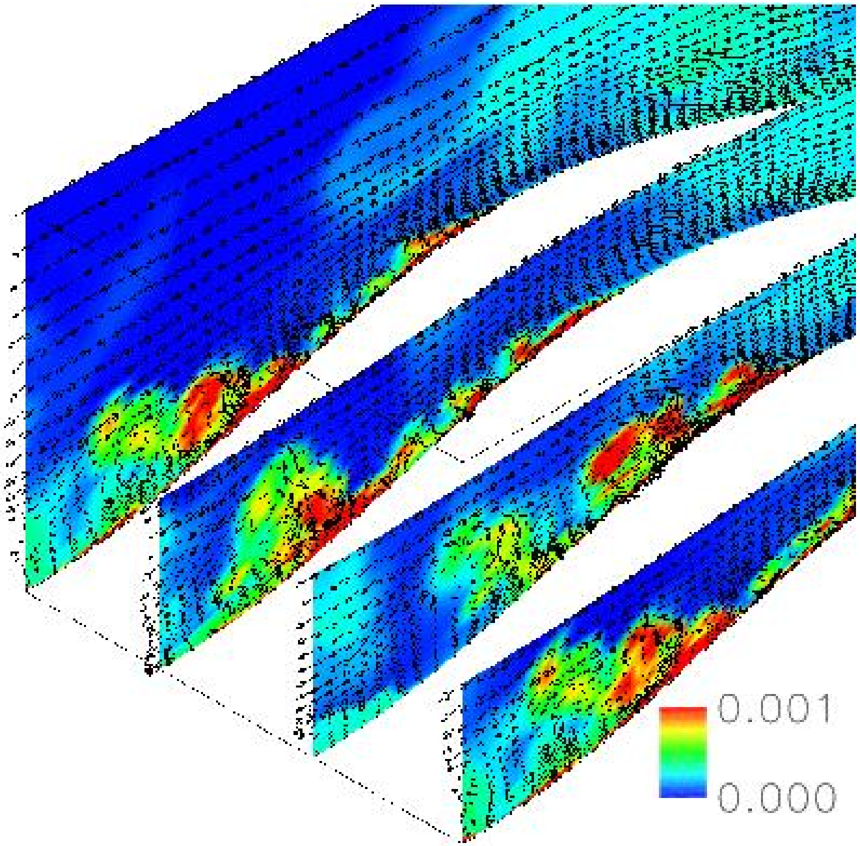
# Vortex Suspension of Sediment

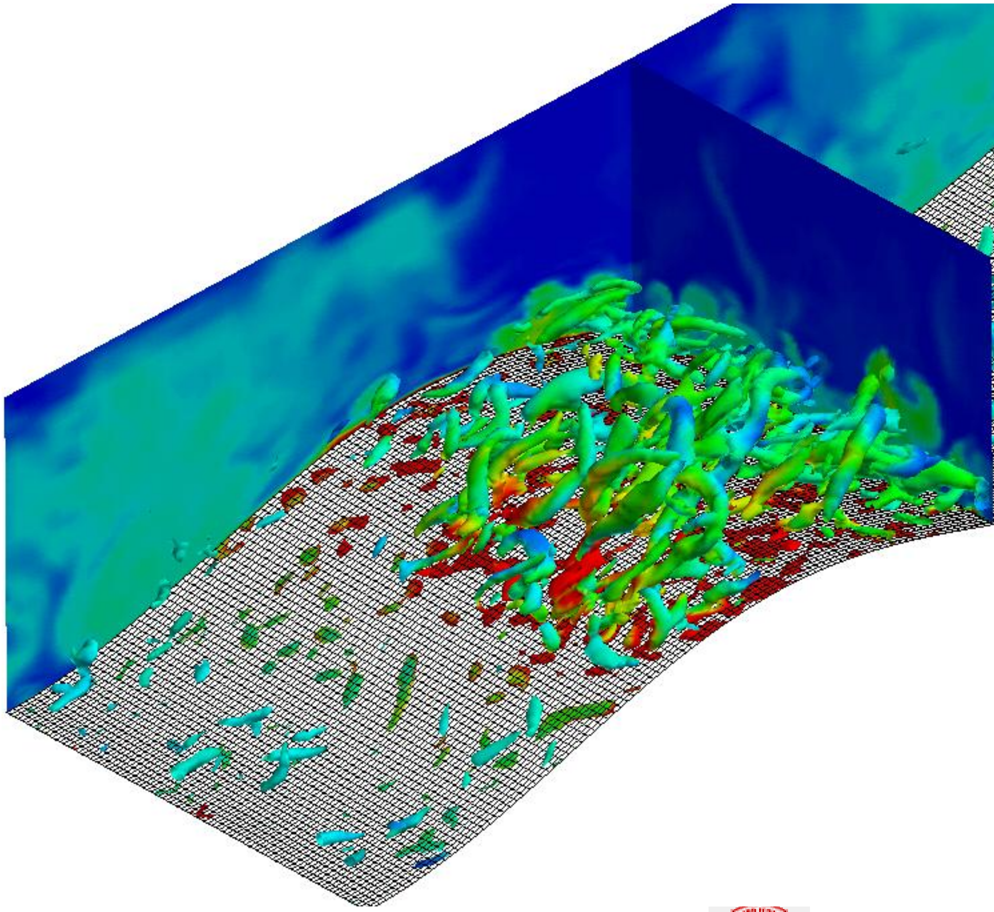


# Three-dimensional features



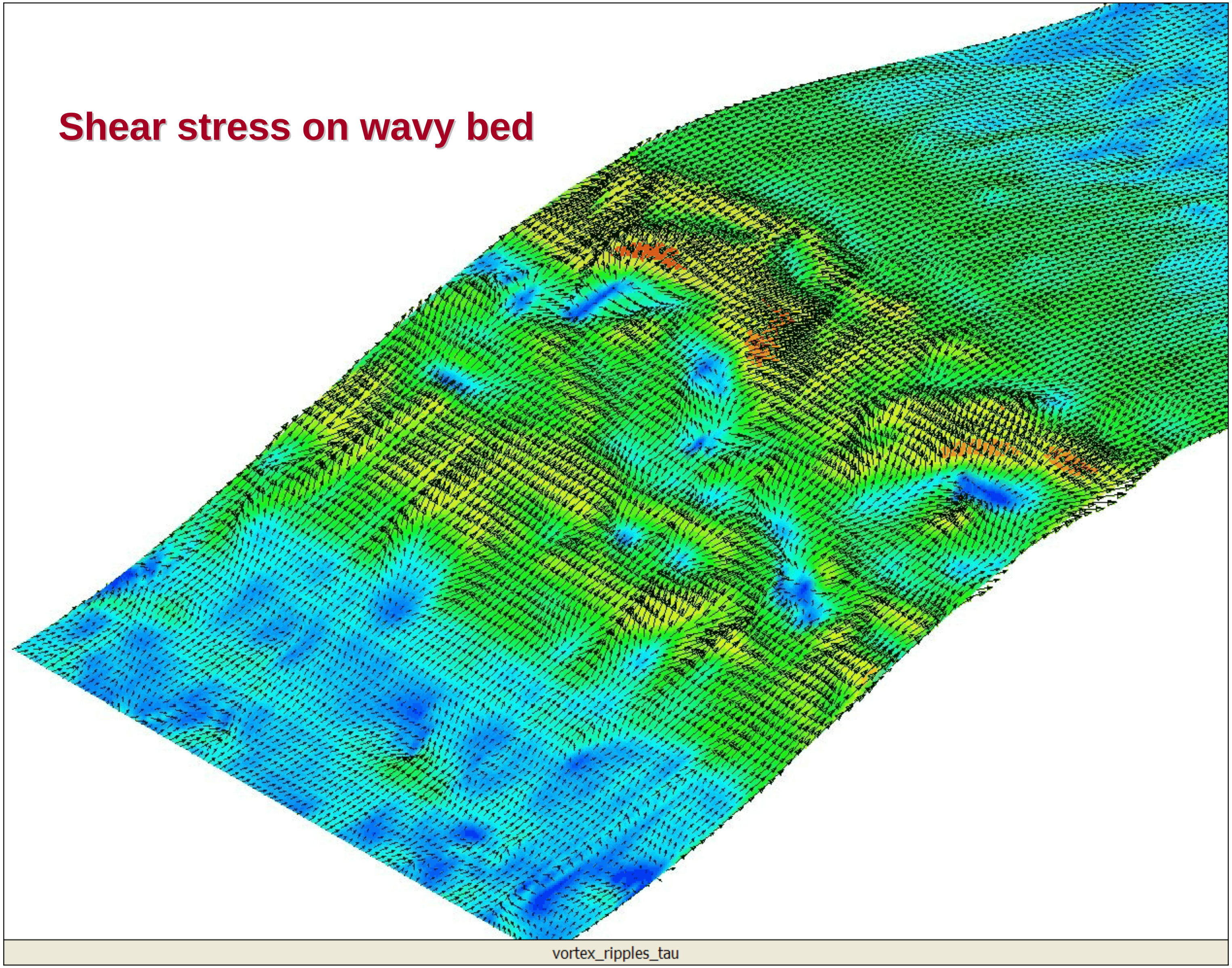

*Streamwise vortex structures identified by the $\lambda_2$ method*

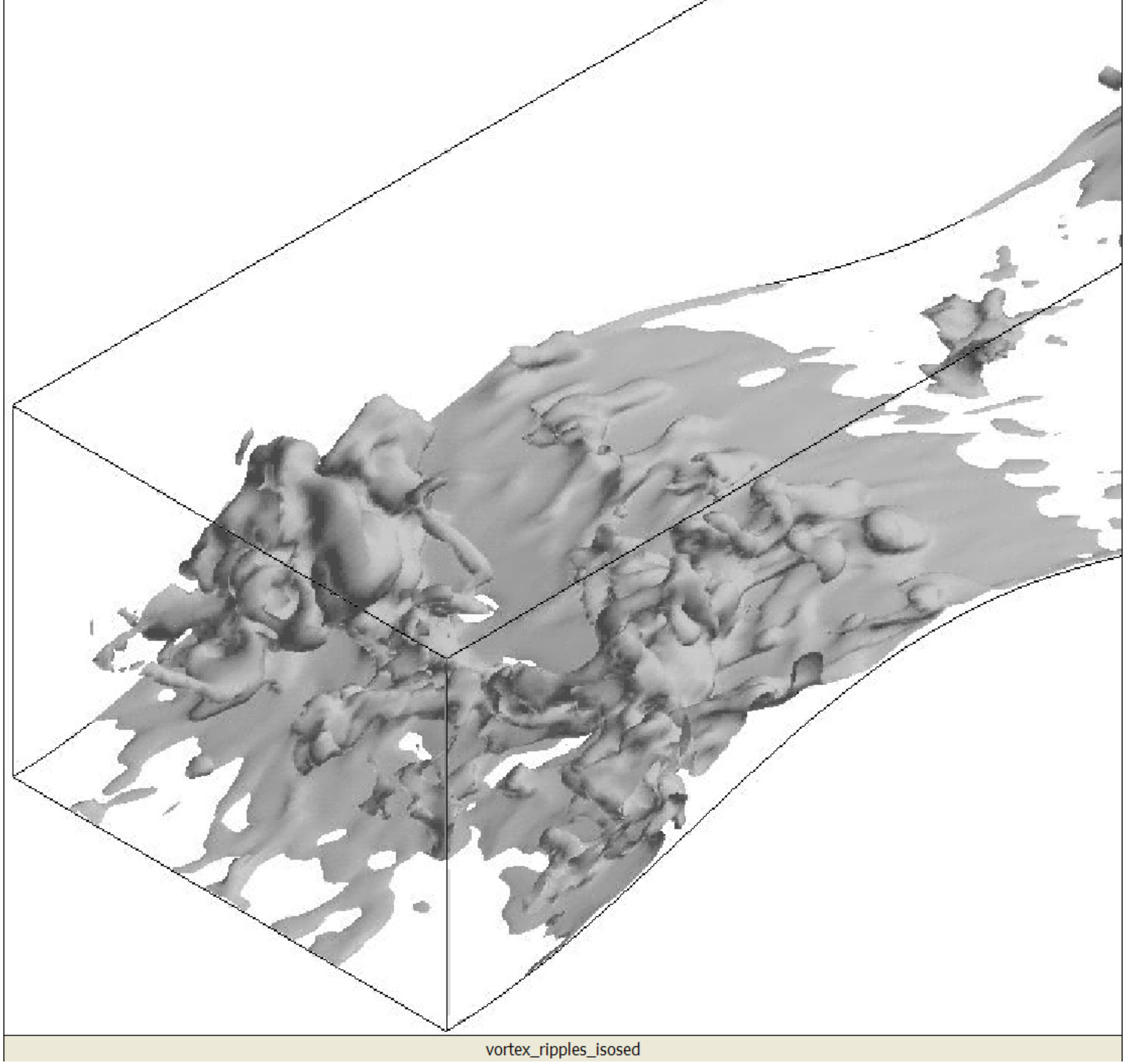


**Sediment concentrations**

**Vortices colored by concentration**


Shear stress on wavy bed
vortex_ripples_tau




**Iso-contours of sediment concentration**

# Overview of set up for evolution of ripples from a flat bed

## *Hydrodynamics (e.g. Zang et al., 1993, 1994; Cui & Street, 2001, 2004)*

$$\frac{\partial \overline{U}_m}{\partial \xi_m} = 0\,,$$

$$\frac{\partial (J^{-1}\overline{u}_i)}{\partial t} + \frac{\partial (\overline{U}_m \overline{u}_i)}{\partial \xi_m} = -\frac{\partial}{\partial \xi_m}\left(J^{-1}\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial \xi_m}\delta_{ij}\right) + J^{-1}\frac{\overline{\rho}-\rho_b}{\rho_0} g\delta_{i3} + \frac{\partial}{\partial \xi_m}\left(\nu G^{mn}\frac{\partial \overline{u}_i}{\partial \xi_n} - \mathcal{T}_{i,m}\right),$$

**Density stratification of the sediment-water mixture**

**Sub-grid scale stress and flux in LES**

## *Sediment transport (e.g. Zedler & Street, 2001, 2006; Chou & Fringer, 2008)*

$$\frac{\partial \overline{C}}{\partial t} + \frac{\partial \overline{C}(\overline{U}_m - W_m\delta_{i3})}{\partial \xi_m} = -\frac{\partial \mathcal{F}}{\partial \xi_m}$$

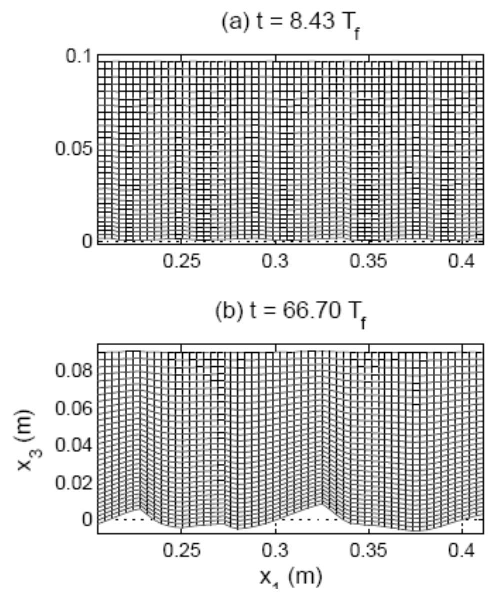
## *Bed elevation model (Chou & Fringer, 2009)*

$$(1-p')J_b^{-1}\frac{\partial h}{\partial t} = \frac{\partial}{\partial \xi_m}\left(kG_b^{mn}\frac{\partial h}{\partial \xi_n}\right)\Big|_{m,n=1,2} + J_b^{-1}(w_s\overline{C_b} - P_k)$$

**Diffusion term to model gravity-induced avalanche flow**

**Sediment deposition and erosion**

*Requires modeling sediment pick-up !!*

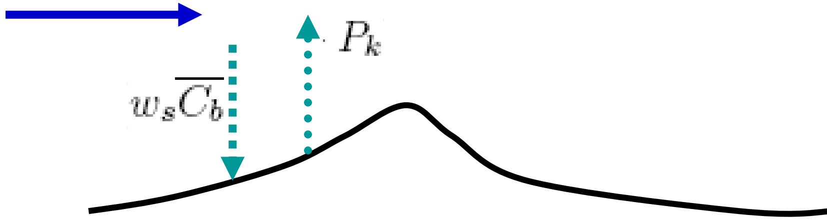



Chou & Fringer, 2009

# Bed elevation model

*Sediment pick-up*

$$\boxed{\frac{P_k}{\sqrt{(s-1)gd_0}} = \begin{cases} \alpha D^{*\beta} T^{*\gamma} & \theta > \theta_c\,, \\ 0 & \text{otherwise} \end{cases}}$$




$\alpha = 0.00033,\ \beta = 0.3$ and $\gamma = 1.5$ *van Rijn* [1993]

*where*

$$D^* = d_0\left[(s-1)g/\nu^2\right]^{1/3}$$

$$T^* = (\theta - \theta_c)/\theta_c$$

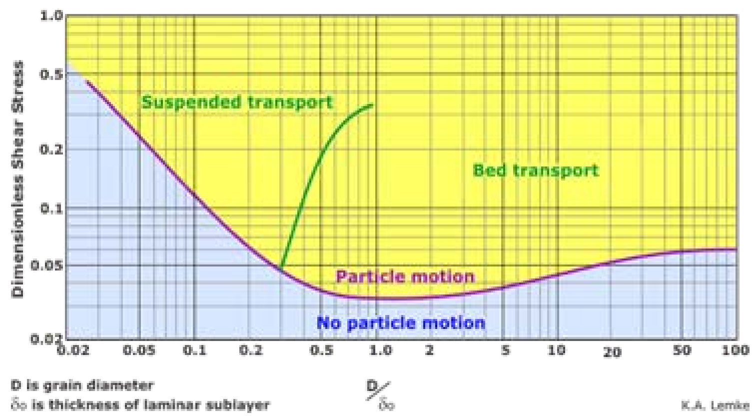
$$\theta = \frac{\tau_b}{(s-1)\,\rho g d_0}$$

***Non-dimensional parameters***

$$\frac{\tau_b}{\rho} = C_D U_{tan}^2$$

$$C_D = \left[\frac{1}{\kappa}\ln\left(\frac{z_1+z_0}{z_0}\right)\right]^{-2}$$

***Bed shear stress***




$$\theta_{cr,0} = \frac{0.3}{1+1.2D_*} + 0.055[1 - \exp(-0.02D_*)]$$

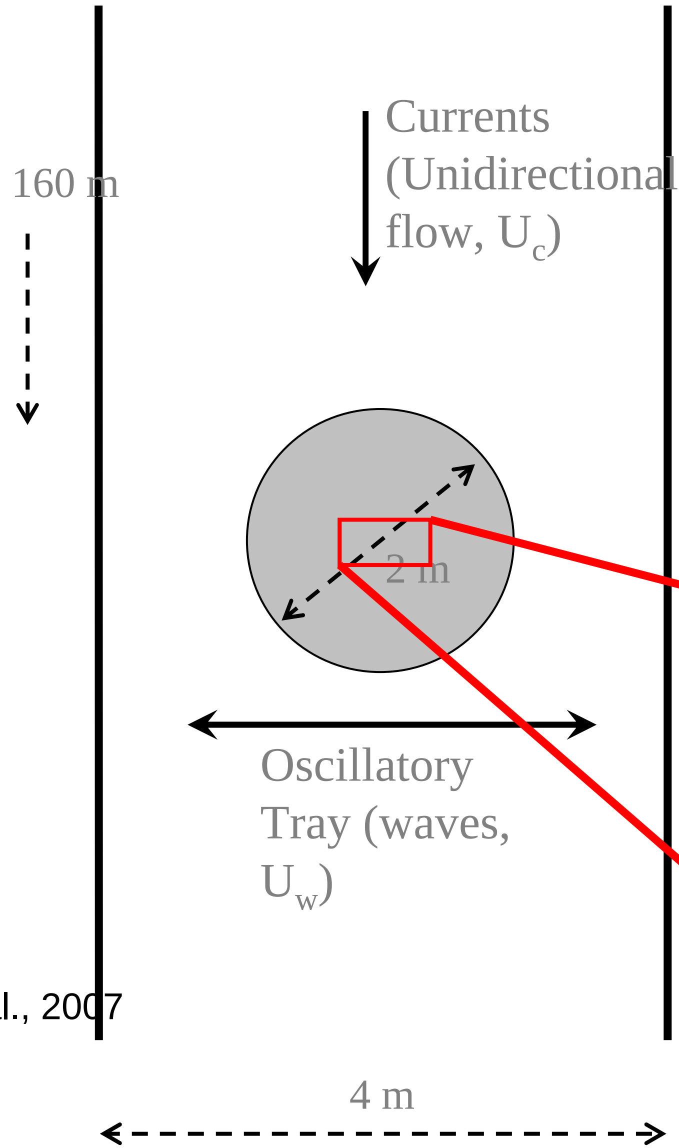
***Flat-bed critical non-dim. shear stress***

$$\frac{\theta_c}{\theta_{c,0}} = \frac{\sin(\phi_{rp}+\phi)}{\sin(\phi_{rp})}$$

$$\sin(\phi) = \frac{\overline{u}_{1,tan}\sin\phi_1 + \overline{u}_{2,tan}\sin\phi_2}{\sqrt{\overline{u}_{1,tan}^2 + \overline{u}_{2,tan}^2}}$$

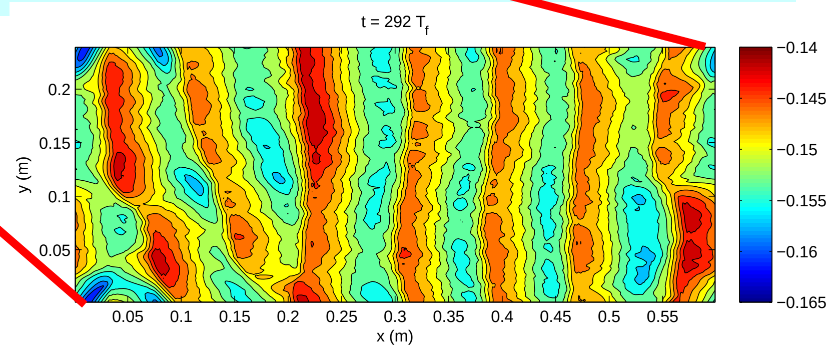
***Local geometric effects***

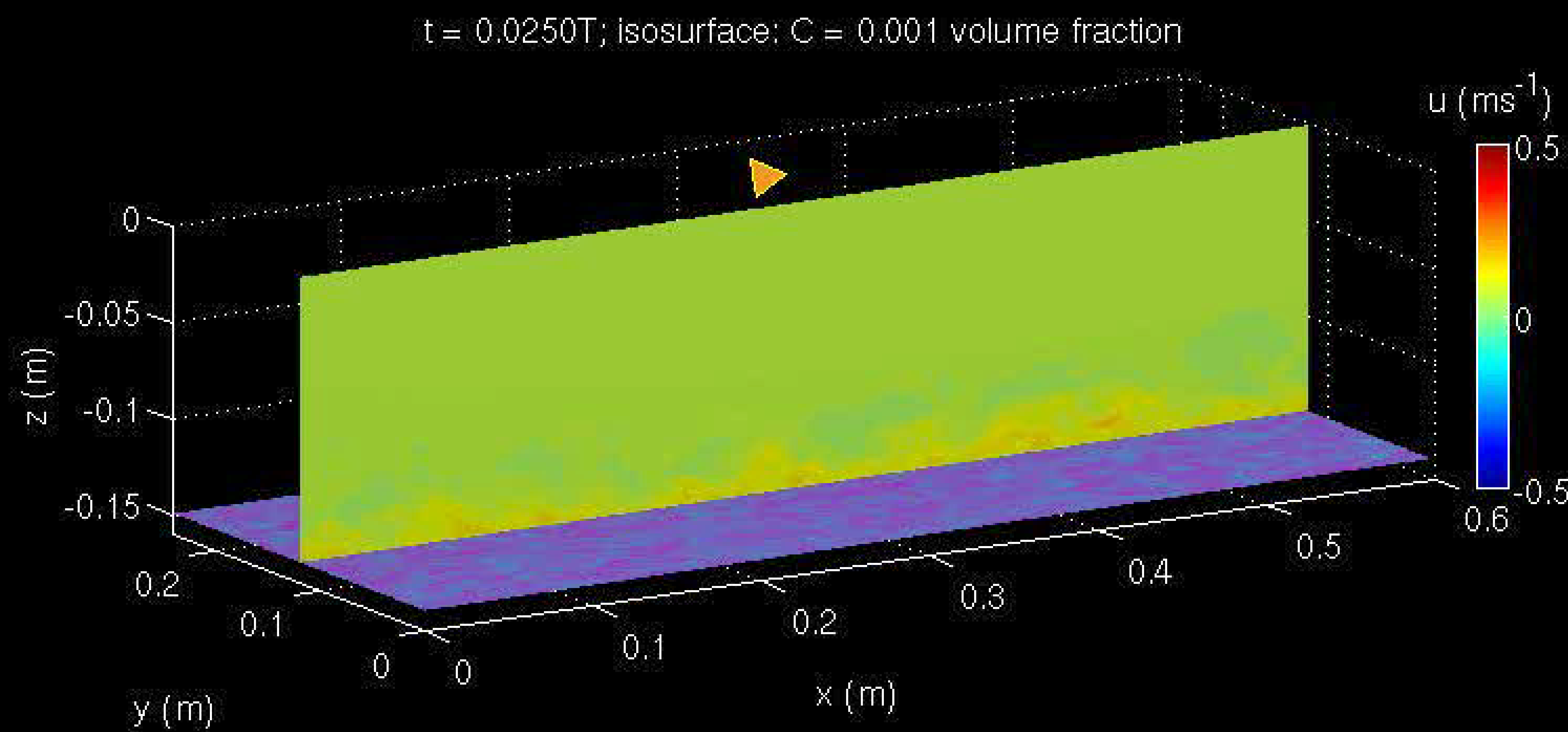
# Simulation Setup




Lacy. et al., 2007

- L = 0.6 m
- W = 0.24 m
- H = 0.15 m
- $U_w$ ~ 0.42 $ms^{-1}$
- T = 8 s
- $U_c$ ~ 0.0 $ms^{-1}$, 0.08 $ms^{-1}$, 0.30 $ms^{-1}$
- $Re_H$ ~ 40,000-50,000
- Periodic BC's in horizontal
- Resolution: 320 x 128 x 96




t = 0.0250T; isosurface: C = 0.001 volume fraction
u (ms$^{-1}$)
0.5
0
-0.5
z (m)
0
-0.05
-0.1
-0.15
y (m)
0.2
0.1
0
x (m)
0
0.1
0.2
0.3
0.4
0.5
0.6

**Example:** subfilter-scale and subgrid-scale turbulence models for large-eddy simulation [LES].

One can succeed in modeling "real" flows by (1) understanding the essential features of the flow and (2) specifically incorporating needed physics into numerical algorithms.

**Examination of the linear algebraic subgrid-scale stress [LASS] model, combined with reconstruction of the subfilter-scale stress, for large-eddy simulation of the neutral atmospheric boundary layer.**

**ARPS mesoscale nonhydrostatic code run in LES incompressible flow mode.**

# Mixed model strategy

Replace SGS eddy viscosity model to allow:

Near-wall SGS anisotropy

By adding more physics and merging this with reconstruction! Recall:

$$\Im_{ij} = \overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$

$$= \underbrace{\overline{\widetilde{u_i u_j}} - \overline{\tilde{u}_i \tilde{u}_j}}_{\text{Subgrid-scale}} + \underbrace{\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j}_{\text{Subfilter-scale}}$$

$$\square_{ij} = A_{ij} + B_{ij}$$

Focus on $A_{ij}$

# Subgrid-scale stress equation $A_{ij}$

This is easy to generate in usual way [e.g., as done with RANS] by adding and subtracting momentum equations multiplied by the appropriate velocity.

$$\frac{\partial}{\partial t}(\text{SGS Stress}) = \text{Advection} + \text{Transport \& diffusion}$$
$$+ \text{Production} + \text{Pressure redistribution}$$
$$+ \text{Dissipation} + \text{Buoyancy Generation}$$
$$+ \text{Coriolis}$$

$$\cancel{\frac{\partial}{\partial t}(\text{SGS Stress})} = \cancel{\text{Advection}} + \cancel{\text{Transport \& diffusion}}$$
$$+ \text{Production} + \text{Pressure redistribution}$$
$$+ \text{Dissipation} + \cancel{\text{Buoyancy Generation}}$$
$$+ \cancel{\text{Coriolis}}$$

# The Linear Algebraic Subgrid Scale Model

## $A_{ij}$: SGS Stress

Models

$$0 = \underbrace{-\bar{A}_{jk}\frac{\partial \bar{\tilde{u}}_i}{\partial x_k} - \bar{A}_{ik}\frac{\partial \bar{\tilde{u}}_j}{\partial x_k}}_{\text{Production}} \boxed{\underbrace{-\frac{2}{3}\bar{\varepsilon}\delta_{ij}}_{\text{Dissipation}} \quad \underbrace{+\Pi_{ij}}_{\text{Pressure Redistribution}}}$$

$$\Pi_{ij} = \underbrace{-c_1\frac{\bar{\varepsilon}}{\bar{e}}\left(\bar{A}_{ij} - \frac{2}{3}\bar{e}\delta_{ij}\right)}_{\text{Slow Pressure Strain}}$$

$$\underbrace{-c_2\left(P_{ij} - \frac{2}{3}P\delta_{ij}\right) - c_3\bar{e}\bar{\tilde{S}}_{ij} - c_4\left(D_{ij} - \frac{2}{3}P\delta_{ij}\right)}_{\text{Rapid Pressure Strain}}$$

$$\underbrace{+\left(c_5\frac{\bar{\varepsilon}}{\bar{e}}(\bar{A}_{ij} - \frac{2}{3}\bar{e}\delta_{ij}) + c_6 P_{ij} - c_7 D_{ij} + c_8\bar{e}\bar{\tilde{S}}_{ij}\right) f(z)}_{\text{Wall Effects}}$$

$$P_{ij} = -\left(\bar{A}_{ik}\frac{\partial \bar{\tilde{u}}_j}{\partial x_k} + \bar{A}_{jk}\frac{\partial \bar{\tilde{u}}_i}{\partial x_k}\right)$$

$$P = \frac{1}{2}P_{ii}$$

$$\bar{\tilde{S}}_{ij} = \left(\frac{\partial \bar{\tilde{u}}_i}{\partial x_k} + \frac{\partial \bar{\tilde{u}}_k}{\partial x_i}\right)$$

$$D_{ij} = -\left(\bar{A}_{ik}\frac{\partial \bar{\tilde{u}}_k}{\partial x_j} + \bar{A}_{jk}\frac{\partial \bar{\tilde{u}}_k}{\partial x_i}\right)$$

$$f(h) = \begin{cases} 0.2\dfrac{\Delta_g}{z} & \text{if } z < z_c, \\ 0 & \text{if } z \geq z_c. \end{cases}$$

$$\bar{\varepsilon} = 1.12\bar{e}^{1.5}\Delta_g^{-1}$$

$$z_c = 4\Delta x$$

Pressure Redistribution modeled by use of Launder, Reece, Rodi (1975) equations.

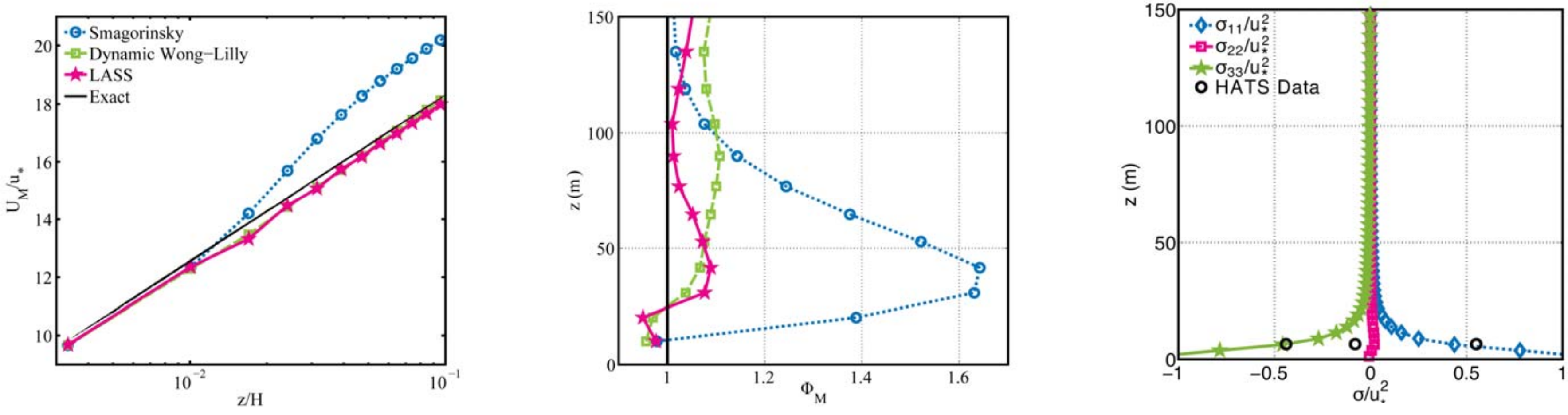
# Implementation

- Use Advanced Regional Prediction System [ARPS]
- Simulation is of neutral boundary flow; similar to several others for comparison [e.g., Andren et al (1994), Chow et al (2005), Ludwig et al (2009), Porté-Agel et al (2000), Sullivan et al (1994), etc.]
- 32 m x 32 m x 37.5 m [avg,10 m in vertical at ground] grid
- 1.3 km x1.3 km x 1.5 km domain

# LASS assessment in neutral BL simula.

Enriquez et al. (2010)

- Follows log law
  - much better than Smagorinsky
  - about the same as DWL
- Provides proper SGS anisotropy near the wall



# A new mixed model

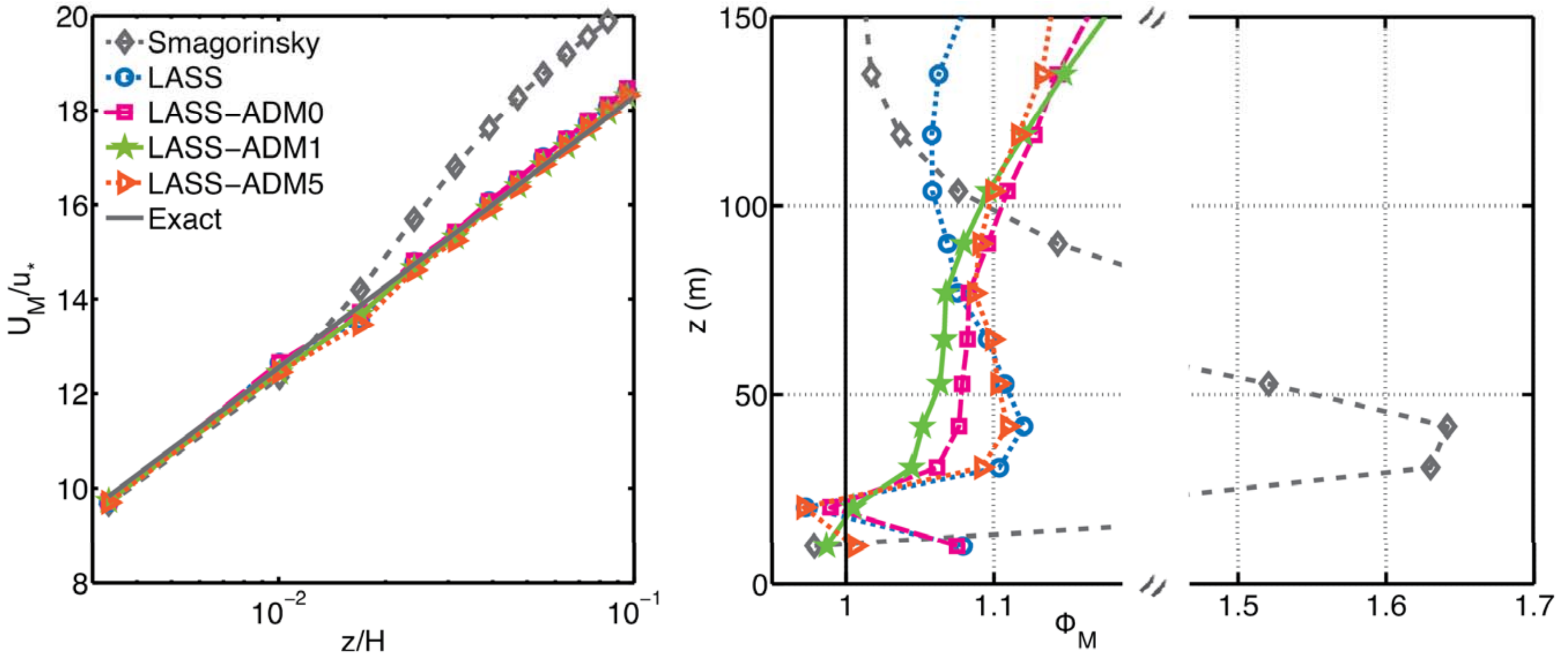
## LASS + Reconstruction

LASS alone does better than eddy viscosity models;
provides near-wall SGS anisotropies

↓

LASS with Reconstruction: Provides backscatter of energy; will it improve compliance to the log law and yield better high-wavenumber velocity components?

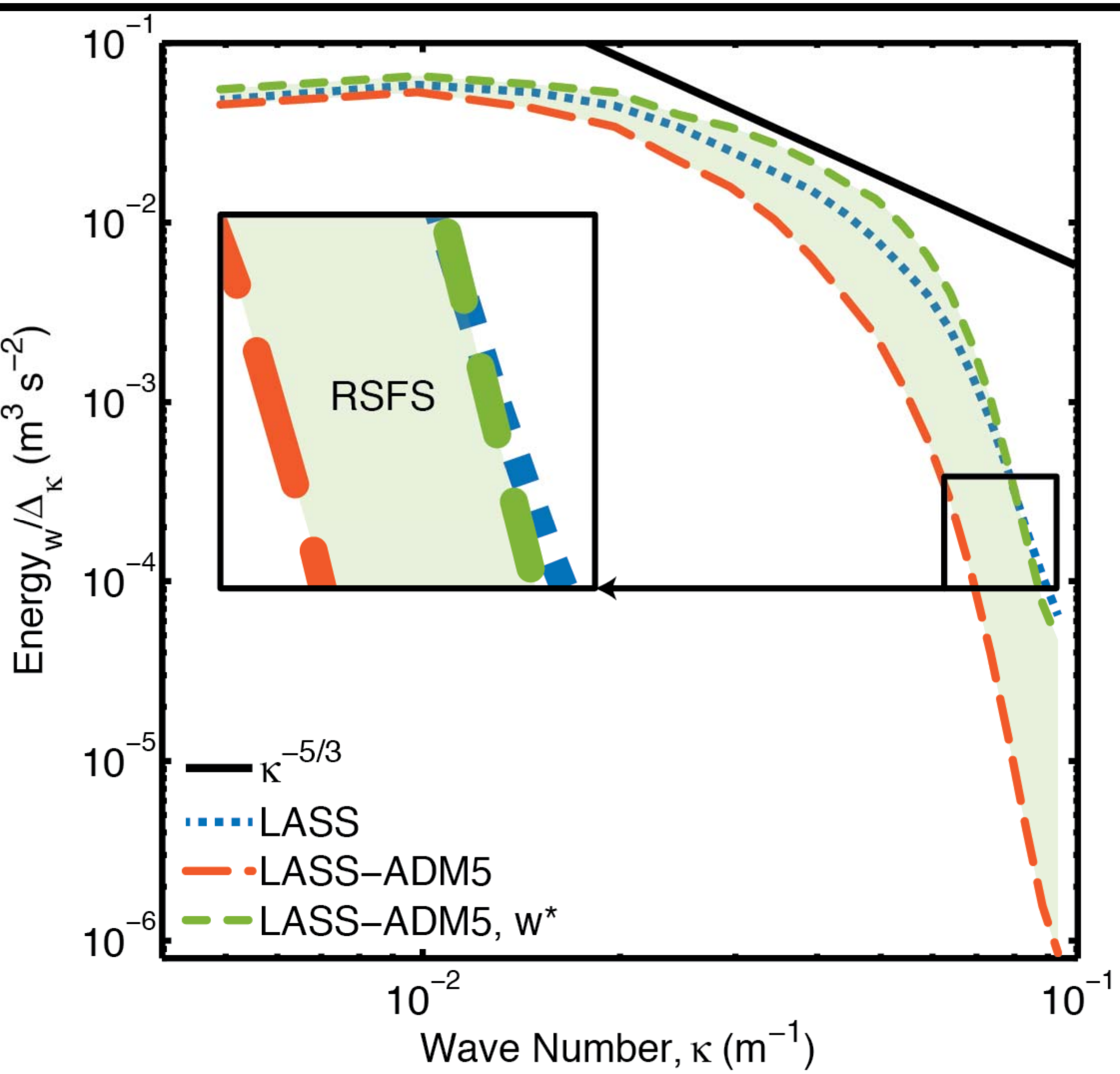
# Log Law Assessment






Models versus physical laws/first principles, or why models work?

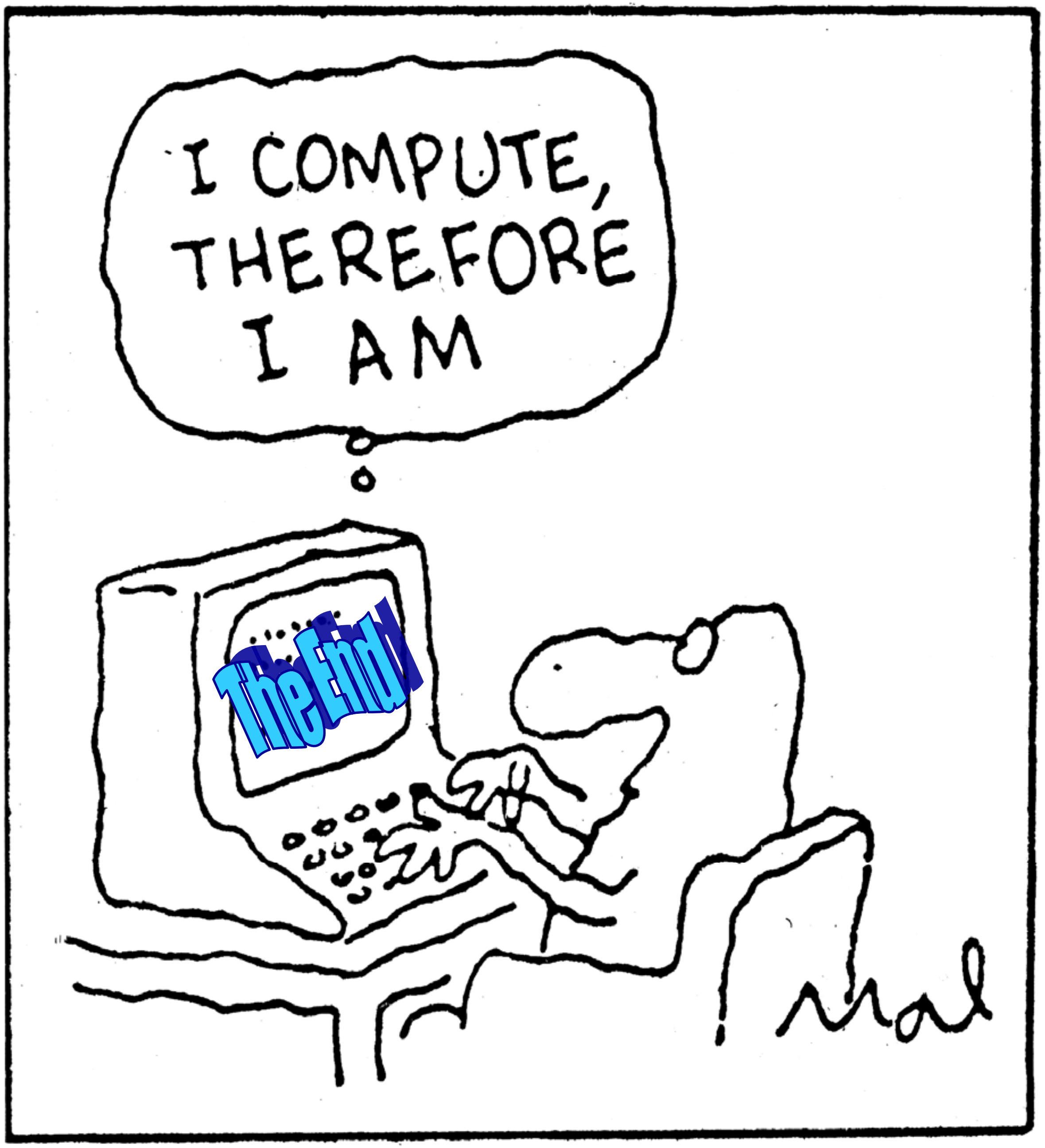
# Energy Spectra




**Spectrum of the vertical velocity**

# Conclusions.

- ✓ A numerical model can succeed if the algorithm (1) allows representation of the "real" flow physics and (2) does not suppress essential behavior. **Example:** the "classic" lid-driven cavity flow.
- ✓ A numerical simulation can accurately represent reality even when the elements of the code are not derived from first principles; both phenomenological and empirical models can suffice. **Example:** sediment transport and evolution of a sandy bed with turbulent flow beneath water waves [a coastal ocean problem].
- ✓ One can succeed in modeling "real" flows by (1) understanding the essential features of the flow and (2) specifically incorporating needed physics into numerical algorithms. **Example:** subfilter-scale and subgrid-scale turbulence models for large-eddy simulation [LES].
- ✓ **Models can clarify and elucidate the physics and produce accurate predictions.**


I COMPUTE, THEREFORE I AM
The End!
Mal

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