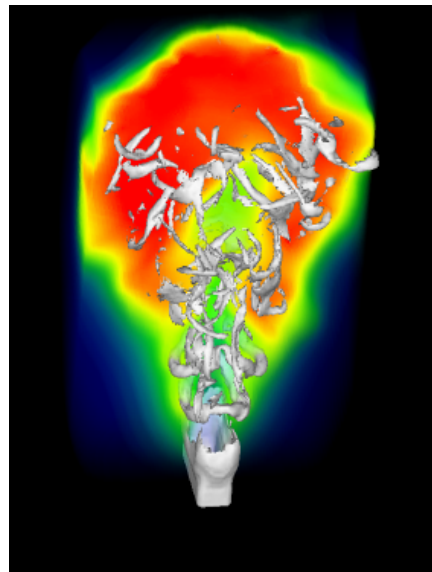
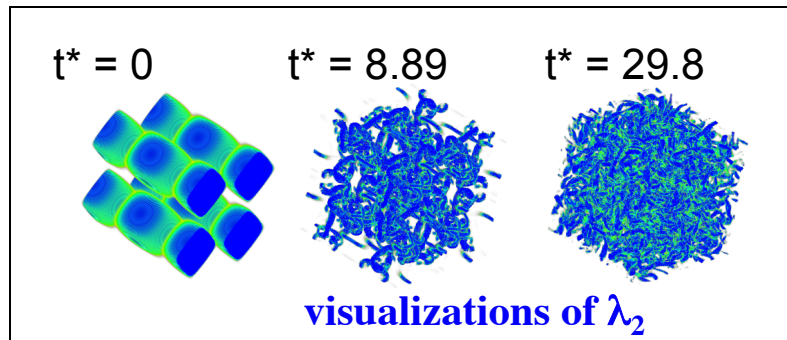
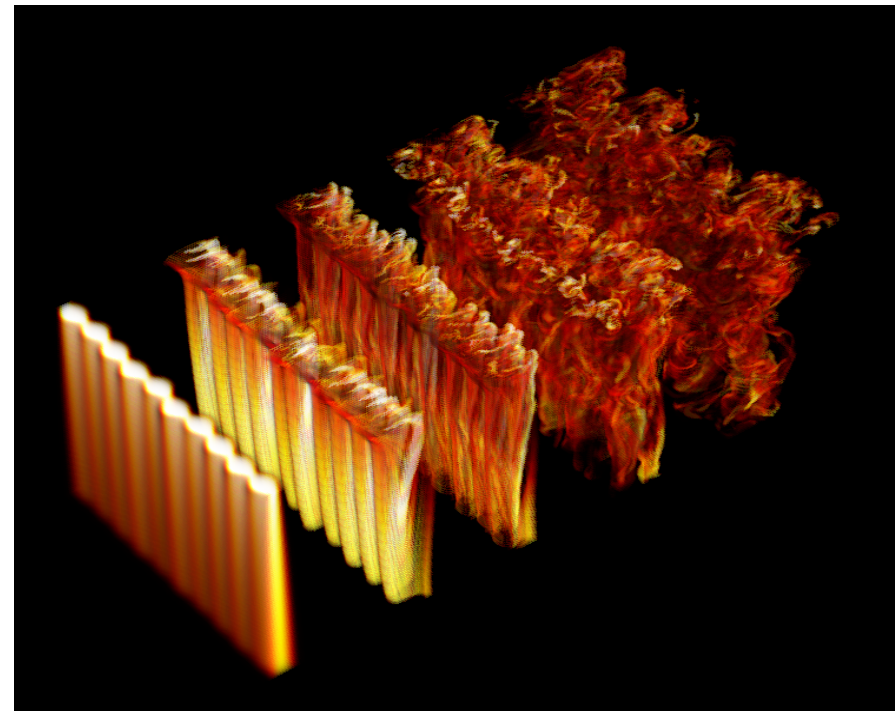


Simulations of Vortex Dynamics, Transition, and Material Mixing in Complex High-Re Flows



Fernando F. Grinstein
X-Computational Physics Div.
LANL, Los Alamos NM

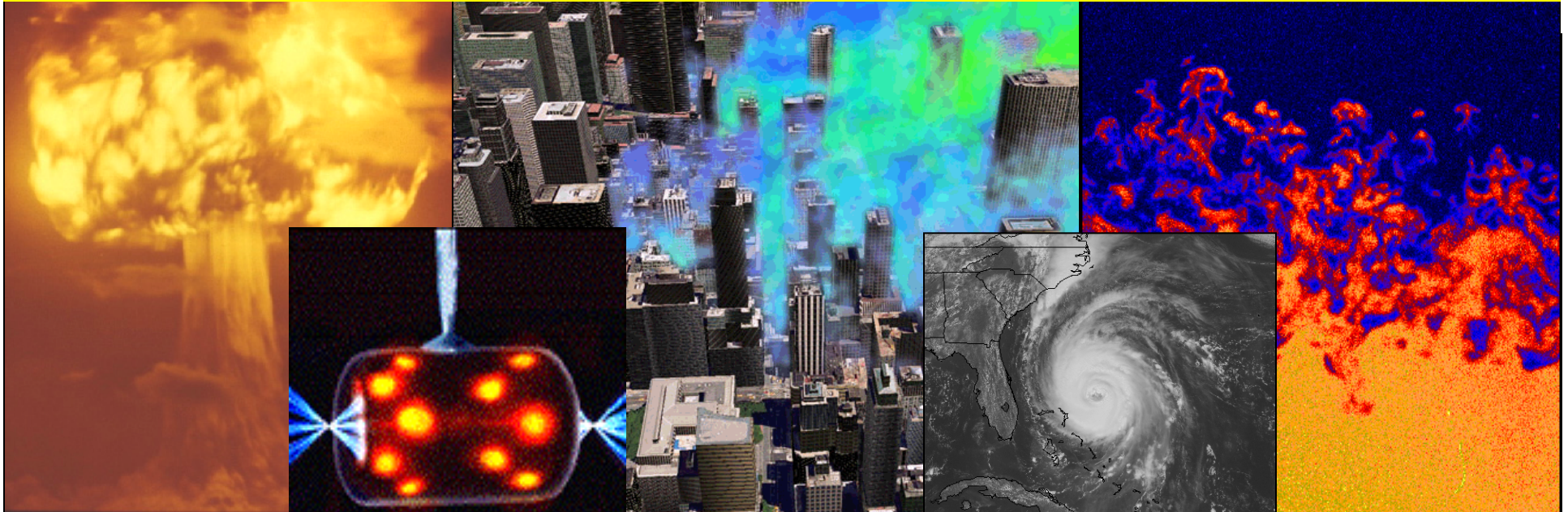


LANL LDRD-DR Program on "Turbulence by Design"

LANL LDRD-ER Program on "LES Modeling for Predictive Mixing"

Simulations Based Mixing Prediction in Extremes Conditions

threat reduction, geophysics, inertial confinement fusion, climate modeling, weapons science, astrophysics, ...



- **Unavoidably Under-Resolved** due to Inherent Complexities
- **Verification, Validation, and Uncertainty Quantification (VVUQ) Issues**

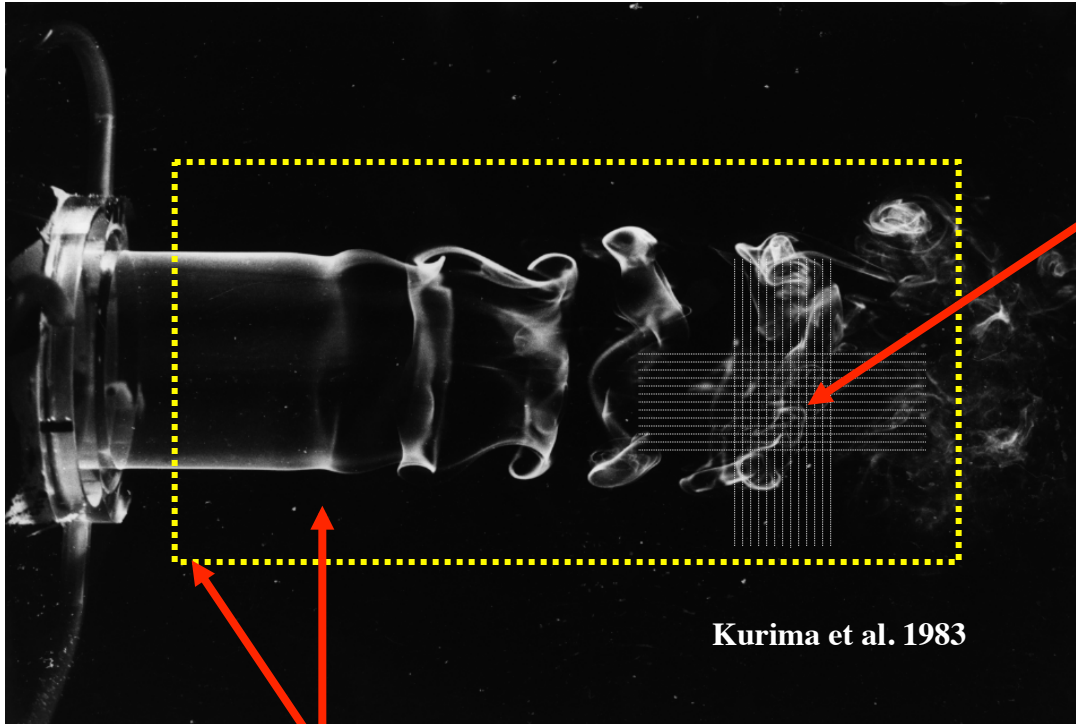
Simulations are based on augmented Navier-Stokes equations:

- 3) **Direct Numerical Simulation** (“*resolves all*” relevant space / time scales)
- 2) **Coarse Grained Simulation** (resolves *large eddies* + subgrid scale models)
- 1) **Moment-Closures / Reynolds Averaged NS** (resolve *mean quantities*)

↑
cost

Intrusiveness of Flow Experiments

- Characterization (and Modeling) of Flow Conditions
- Uncertainty Quantification (UQ) in V&V Metrics



Kurima et al. 1983

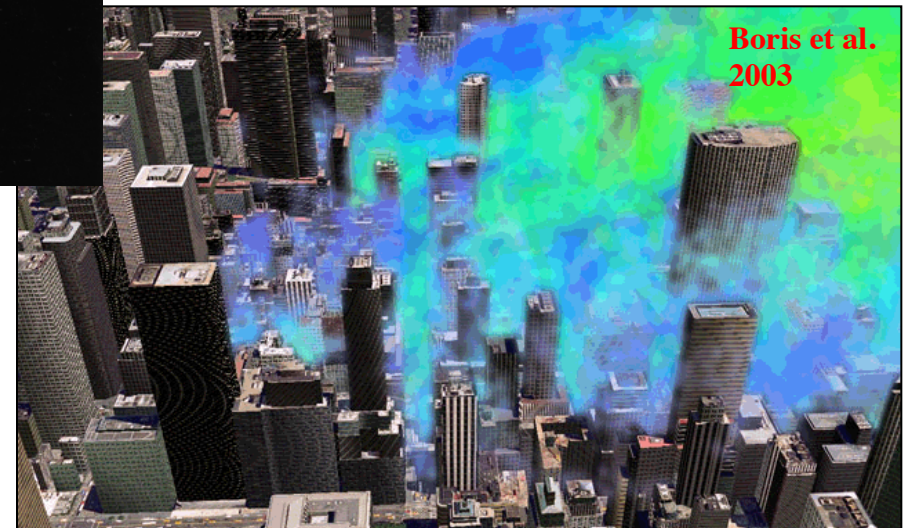
Subgrid

within a computational cell
(or CGS filter-length)
within instrumentation size
(e.g., hotwire cross-section)

Supergrid

Initial Conditions (ICs) and
Boundary Conditions (BCs)

turbulent flow remembers its ICs
(e.g., George & Davidson 2004)



Boris et al.
2003

intertwined subgrid & supergrid
issues at material interfaces

Coarse Grained Simulations (LES, ILES / MILES): why (and when) do they work ?

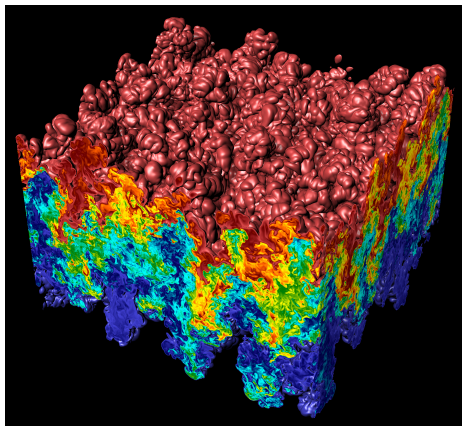
CGS: Background & Basis

- **Vortex dynamics:** Inhomogeneous flows
jets and channel flow
- **Transition and Decay:** Taylor-Green vortex
- **Material mixing**
material interface characterization & modeling
shock-driven turbulence

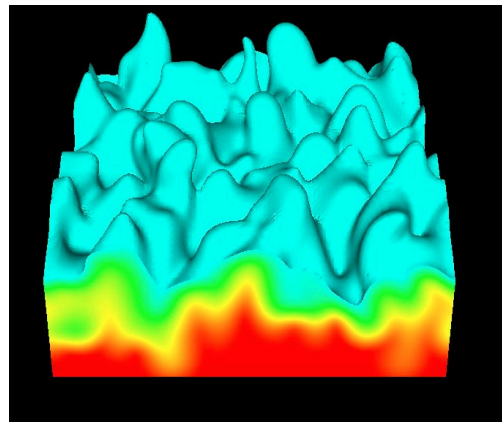
Outlook

Instantaneous, Filtered, Ensemble Averaged, DNS, Coarse Grained Solutions, Moment closures

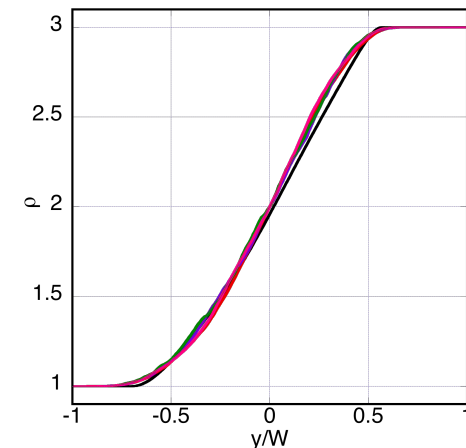
- **Direct Numerical Simulation:** resolve all *relevant* space / time scales
- **Moments methods:** ensemble - averages over many realizations within some constraint on initial and boundary conditions.
- **CGS (LES, ILES / MILES):** spatial filtering or averaging with either closure for effects of small scales or designed numerical dissipation.
depends on explicit or implicit filter-length (typically grid size)
- **COST:** DNS > CGS >> moment closures



DNS : 3D



CGS (LES, ILES) : 3D



Moment closures for applications (e.g., BHR): 1D

CGS (LES) Ingredients

- Low-pass filter
- Discretization (Finite Volume preferred ...)

$$\bar{f}(\mathbf{x}_P) = \frac{1}{\delta V_P} \int_{\Omega_P} f(\mathbf{x}') G(\mathbf{x}' - \mathbf{x}_P, \Delta) dV'$$

Modified Equation Analysis (satisfied by computed solutions)

$$\partial_t(\bar{\mathbf{v}}) + \nabla \cdot (\bar{\mathbf{v}} \otimes \bar{\mathbf{v}}) + \nabla \bar{p} - \nu \nabla \cdot \bar{\mathbf{S}} = -\nabla \cdot \mathbf{T}_v + \nabla \cdot \boldsymbol{\tau}_v + \mathbf{m}_v$$

$Sc = \nu / \kappa$

$$\partial_t(\bar{\theta}) + \nabla \cdot (\bar{\theta} \bar{\mathbf{v}}) - \kappa \nabla^2 \bar{\theta} = -\nabla \cdot \mathbf{T}_\theta + \nabla \cdot \boldsymbol{\tau}_\theta + \mathbf{m}_\theta$$

truncation and commutation "error" terms

explicitly modeled in LES

"well resolved" LES requires : $\nabla \cdot \boldsymbol{\tau} \ll \nabla \cdot \mathbf{T} + \mathbf{m}$

• Ghosal '96 -- Models and "errors" are comparable in typical LES --> motivates Implicit LES (ILES, MILES)

Coarse Grained Simulation

based on Euler, N-S, or augmented N-S, ...

- Resolve energy containing large scale physics
- Model subgrid scales
 - no universal theory
 - no exact solutions
 - pragmatic practice

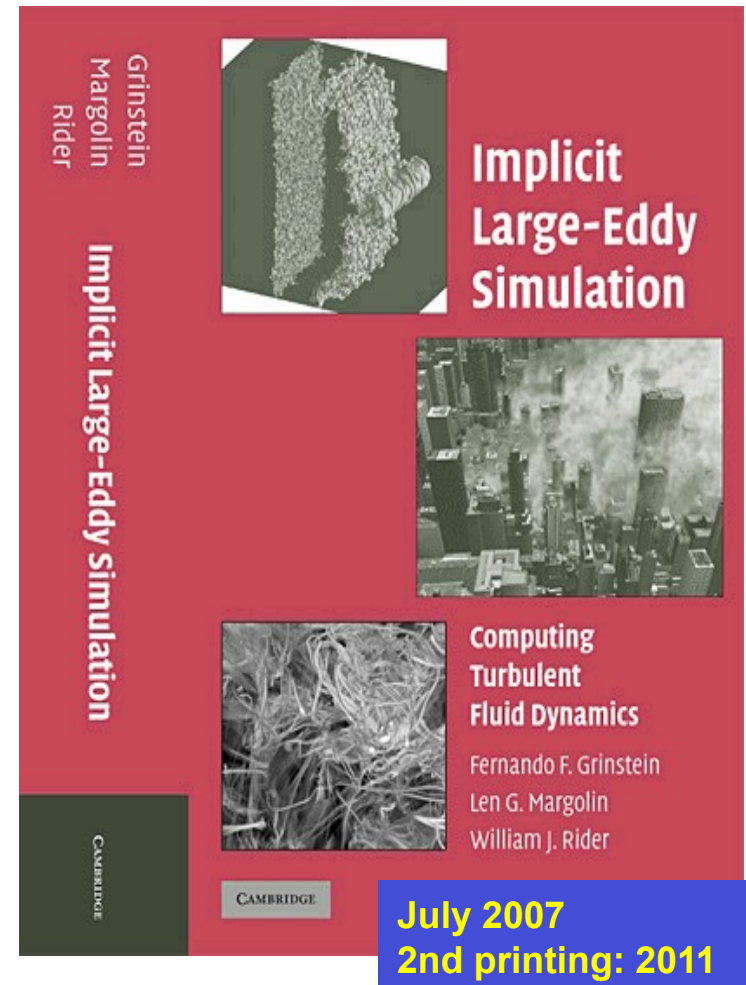
Classical LES:

explicit subgrid models
(eddy-viscosity, ..., mixed, ...)

Numerical LES (NLES):

relies on subgrid models implicitly provided by the numerics

Implicit LES (ILES, MILES):
(a very specific NLES !)



relies on non-oscillatory finite-volume numerics (FCT, PPM, Godunov, ...)
--> Boris, Youngs, ... ILES Book '07

Implicit Large Eddy Simulation

ILES, MILES --> is not free lunch !

$$\partial(\bar{\mathbf{v}}) + \nabla \cdot (\bar{\mathbf{v}} \otimes \bar{\mathbf{v}}) + \nabla \bar{p} - \nabla \cdot \bar{\mathbf{S}} =$$

$$- \cancel{\nabla \cdot \mathbf{T}} + \nabla \cdot \tau + \cancel{\mathbf{m}}$$

$$\mathbf{T} = \overline{\mathbf{v} \otimes \mathbf{v}} - \bar{\mathbf{v}} \otimes \bar{\mathbf{v}}$$

- Finite Volumes \Rightarrow discretization “error” appears in div. form “ $\nabla \cdot \tau$ ”
- **No explicit filtering:** no commutation error term “ \mathbf{m} ”,

FV discretization provides top-hat implicit filtering

$$\bar{f}_P \equiv \frac{1}{\delta V_P} \int_{\Omega_P} f dV$$

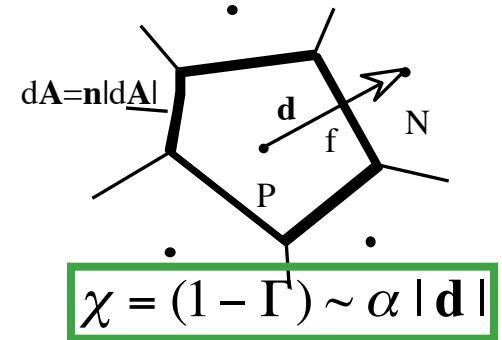
- **T=0: minimal-choice --> models convection driven physics**
(uses Non-Oscillatory FV numerics: FCT, PPM, Godunov, TVD, hybrid)

Depending on Re, Sc, Da..., additional models **and / or** numerics are needed with ILES (or any LES !) to address

near-wall flow, material mixing, combustion, ...

MEA Example: Flux-Limiting ILES vs. Classical LES
 momentum equation, 2nd. order fluxes,
 1st-order upwind / 2nd-order central

$$\mathbf{v}_f^C = \Gamma \mathbf{v}_f^{C,H} + (1 - \Gamma) \mathbf{v}_f^{C,L}, \quad \Gamma = \Gamma(\mathbf{v}, \mathbf{x}, t)$$



$$\partial_t(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla p + \nabla \cdot \left(\frac{1}{3} \mu (\nabla \cdot \mathbf{v}) \mathbf{I} - \mu (\nabla \mathbf{v} + \nabla \mathbf{v}^T) \right) + \rho \mathbf{f}$$

$$+ \nabla \cdot \left(\rho \left[\chi (\mathbf{v} \otimes \mathbf{d}) (\nabla \mathbf{v})^T + \chi (\nabla \mathbf{v}) (\mathbf{v} \otimes \mathbf{d})^T + \chi^2 (\nabla \mathbf{v}) \mathbf{d} \otimes (\nabla \mathbf{v}) \mathbf{d} \right] \right) + \dots$$

lead-order
convective
truncation

*generalized
eddy-viscosity*

*anisotropic
scale-similarity*

$$+ \nabla \cdot \left(\frac{1}{8} \rho \left[\mathbf{v} \otimes ((\nabla^2 \mathbf{v})) (\mathbf{d} \otimes \mathbf{d}) + ((\nabla^2 \mathbf{v})) (\mathbf{d} \otimes \mathbf{d}) \otimes \mathbf{v} \right] \right) + \dots$$

Clark-type term due to 2nd order FV scheme

lead-order
viscous
truncation

$$+ \nabla \cdot \left(\frac{1}{24} \mu (\nabla^3 \mathbf{v}) (\mathbf{d} \otimes \mathbf{d}) \right) + \dots$$

hyperviscosity

Grinstein & Fureby, JCP 2002, JFE 2007

**Classical LES MEA
for 2nd order central
+ explicit SGS model**

lead truncation terms

$$\partial_t(\tilde{\rho} \bar{\mathbf{v}}) + \nabla \cdot (\tilde{\rho} \bar{\mathbf{v}} \otimes \bar{\mathbf{v}}) = -\nabla \bar{p} + \nabla \cdot \left(\frac{1}{3} \mu (\nabla \cdot \bar{\mathbf{v}}) \mathbf{I} - \mu (\nabla \bar{\mathbf{v}} + \nabla \bar{\mathbf{v}}^T) \right) + \tilde{\rho} \bar{\mathbf{f}}$$

$$+ \nabla \cdot (\mathbf{T})$$

$$+ \nabla \cdot \left(\frac{1}{8} \tilde{\rho} \left[\bar{\mathbf{v}} \otimes ((\nabla^2 \bar{\mathbf{v}})) (\mathbf{d} \otimes \mathbf{d}) + ((\nabla^2 \bar{\mathbf{v}})) (\mathbf{d} \otimes \mathbf{d}) \otimes \bar{\mathbf{v}} \right] \right)$$

$$+ \nabla \cdot \left(\frac{1}{24} \mu (\nabla^3 \bar{\mathbf{v}}) (\mathbf{d} \otimes \mathbf{d}) \right) + \dots$$

$T \sim O(d^p)$, with $2/3 < p < 2$!

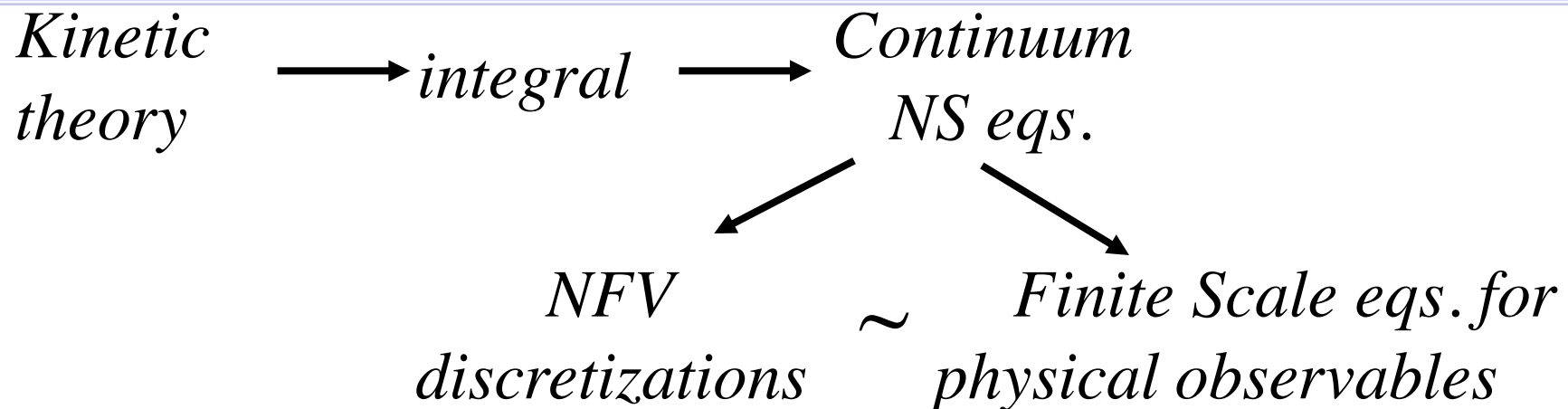
ILES Rationale: Connection with Finite-Scale Equations

Margolin & Rider, IJNMF '02; Margolin, Rider, FFG: JoT '06; ... Ristorcelli, Margolin & FFG '11

- MEA of **NFV approximations to Burgers and NS equations**
- analytically derived **finite scale (avgd. over $V=L^3$ & T) Burgers & NS eqs.**

$$\begin{aligned} \frac{\partial}{\partial t} \bar{c} + \bar{u}_j \bar{c}_{,j} &= \mathcal{D} \bar{c}_{jj} - L^2 [\bar{u}_{j,k} \bar{c}_{,k}]_{,j} - T^2 [\bar{u}_{j,t} \bar{c}_{,t}]_{,j} \\ \frac{\partial}{\partial t} \bar{u}_i + \bar{u}_j \bar{u}_{i,j} &= -\bar{p}_{,i} - L^2 [\bar{u}_{j,k} \bar{u}_{i,k}]_{,j} - T^2 [\bar{u}_{j,t} \bar{u}_{i,t}]_{,j} + \nu \bar{u}_{i,kk} \\ -\nabla^2 \bar{p} &= \bar{u}_{j,i} \bar{u}_{i,j} + L^2 \bar{u}_{j,ik} \bar{u}_{i,jk} + T^2 \bar{u}_{j,ti} \bar{u}_{i,tj} . \end{aligned}$$

“... leading order truncation “errors” introduced by **non-oscillatory finite volume (NFV)** schemes represent physical flow regularization, providing necessary modifications to the governing equations that arise when the motion of **finite scale observables** is considered”.



Coarse Grained Simulations (LES, ILES / MILES)

why (and when) do they work ?

CGS: Background & Basis

- **Inhomogeneous free & wall-bounded flows**
jets, channel flow
- Transition & Decay: Taylor-Green vortex
- Material mixing: shock-driven turbulence

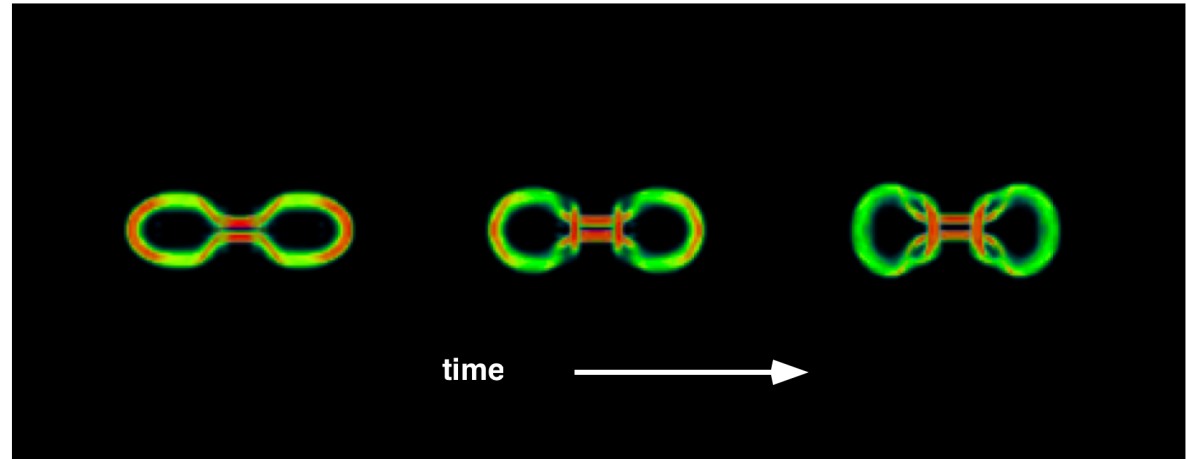
Outlook

ILES-FCT of M=0.6 Jet Vortex Dynamics

Axis-Switching, Reconnection, Bifurcation, and Transition

AR=4, simulated jet

FFG, Phys. Fluids (1995).



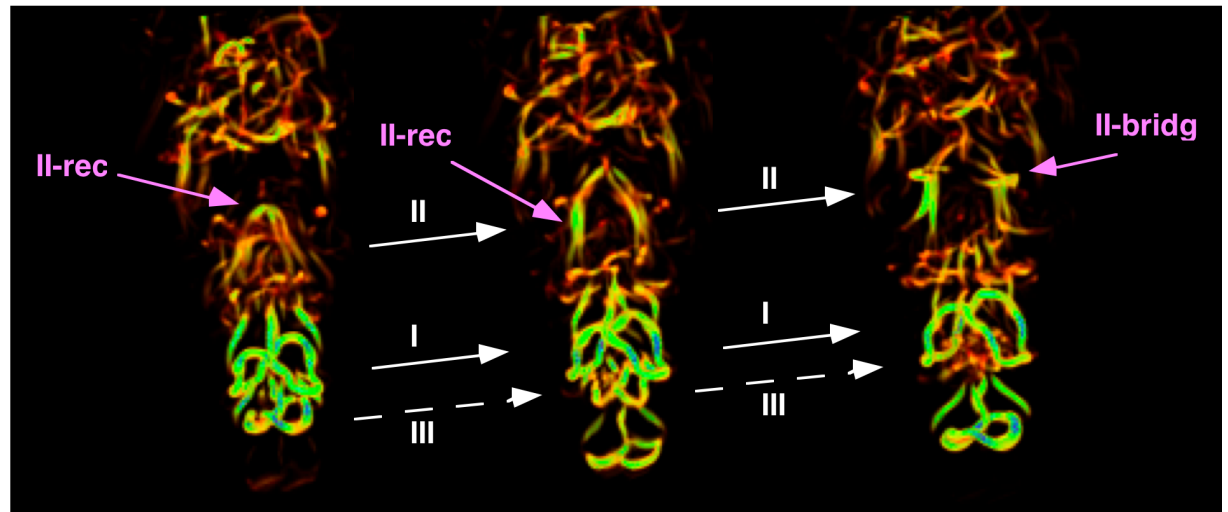
axis-switching
& bifurcation



x

AR=4, laboratory jet flow visualization
Hussain & Husain, JFM (1989).

Volume visualizations of λ_2
 $\lambda_2 =$ second-largest eigenvalue of $S^2 + \Omega^2$



AR=3, simulated jet

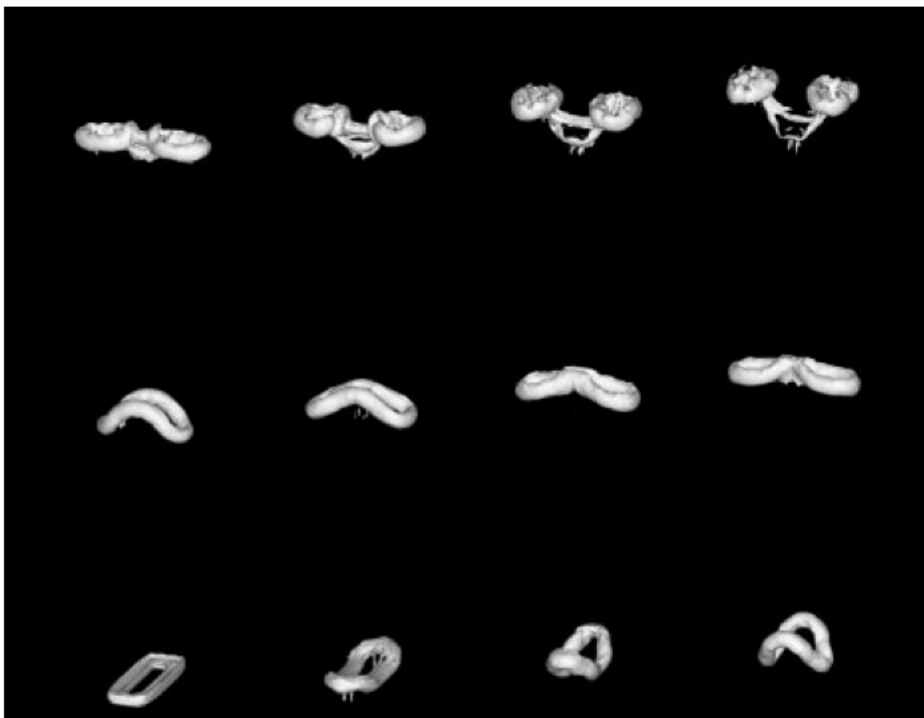
FFG, J. Fluid Mechanics (2001).

FCT-based MILES of Rectangular Jets with $2D \otimes 1D$ splitting (transverse \otimes streamwise)

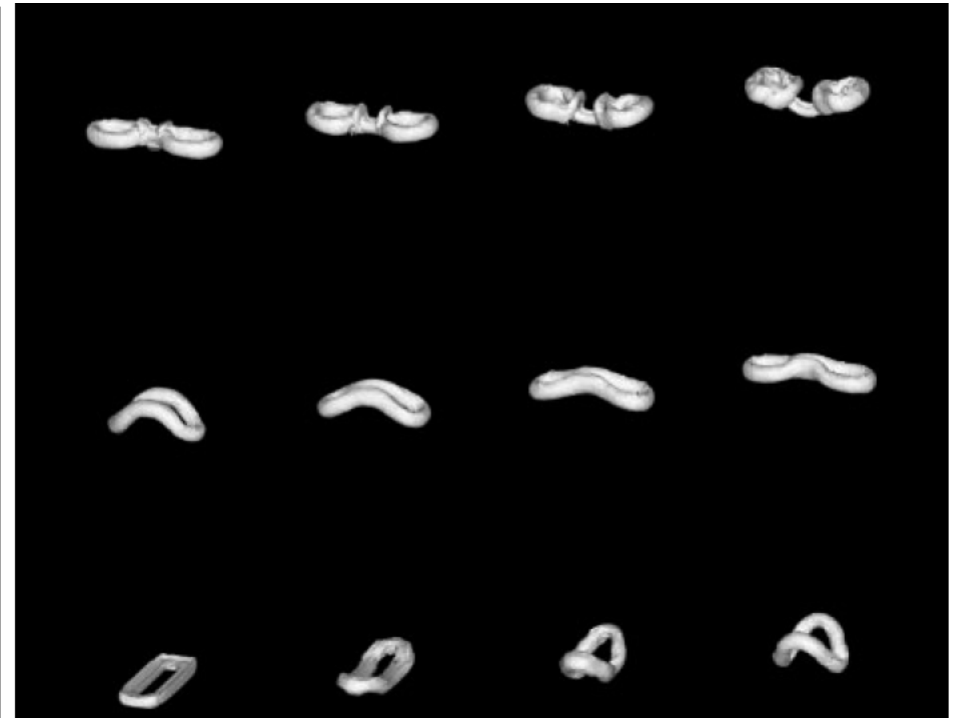
FFG, Fureby & DeVore, IJNMF 2005

positivity- but not monotonicity-preserving
(more effectively built-in backscatter ...)

with additional pre-limiting step enforcing
local monotonicity in each direction



using Zalesak's 2D FCT limiter

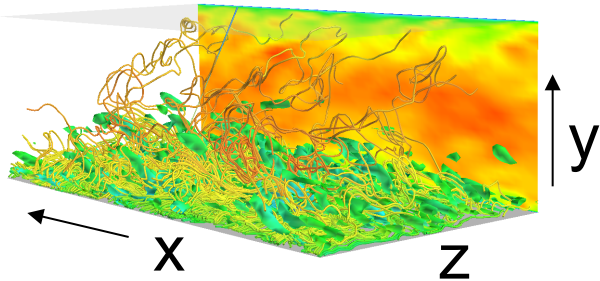


using DeVore's 2D FCT limiter

How much and what kind of backscatter is desirable ?

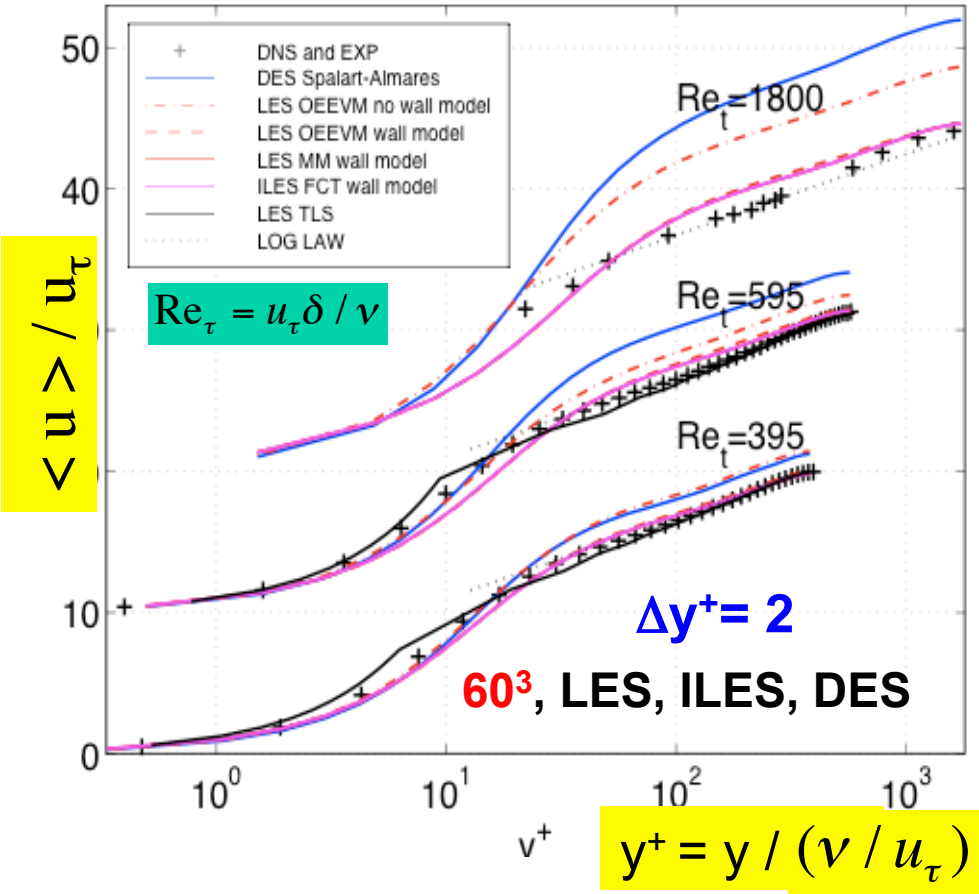
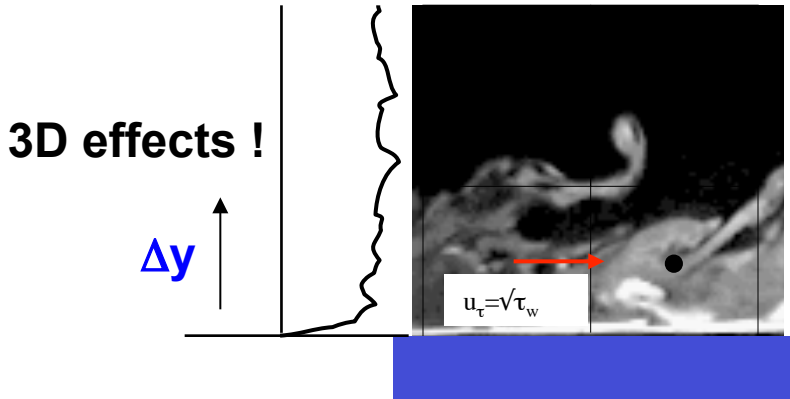
How much vortex dynamics detail should be captured ?

Need suitable well designed lab experiments & VVUQ metrics to decide ...



Under-Resolved Near-Wall ($Sc = \infty$) Flow
robust outer flow CGS; mature VVUQ metrics
 Developed Turbulent Channel Flow, Fureby et al. AIAA J. 2004

Prohibitive number of grid points to resolve near-wall dynamics
 for DNS: $\sim (Re_\tau)^3$, $\Delta y^+ \ll 1$;
 for LES: $\sim (Re_\tau)^2$, $\Delta y^+ < \sim 1$
 +: scaled by viscous length scale



State-of-the-art strategies

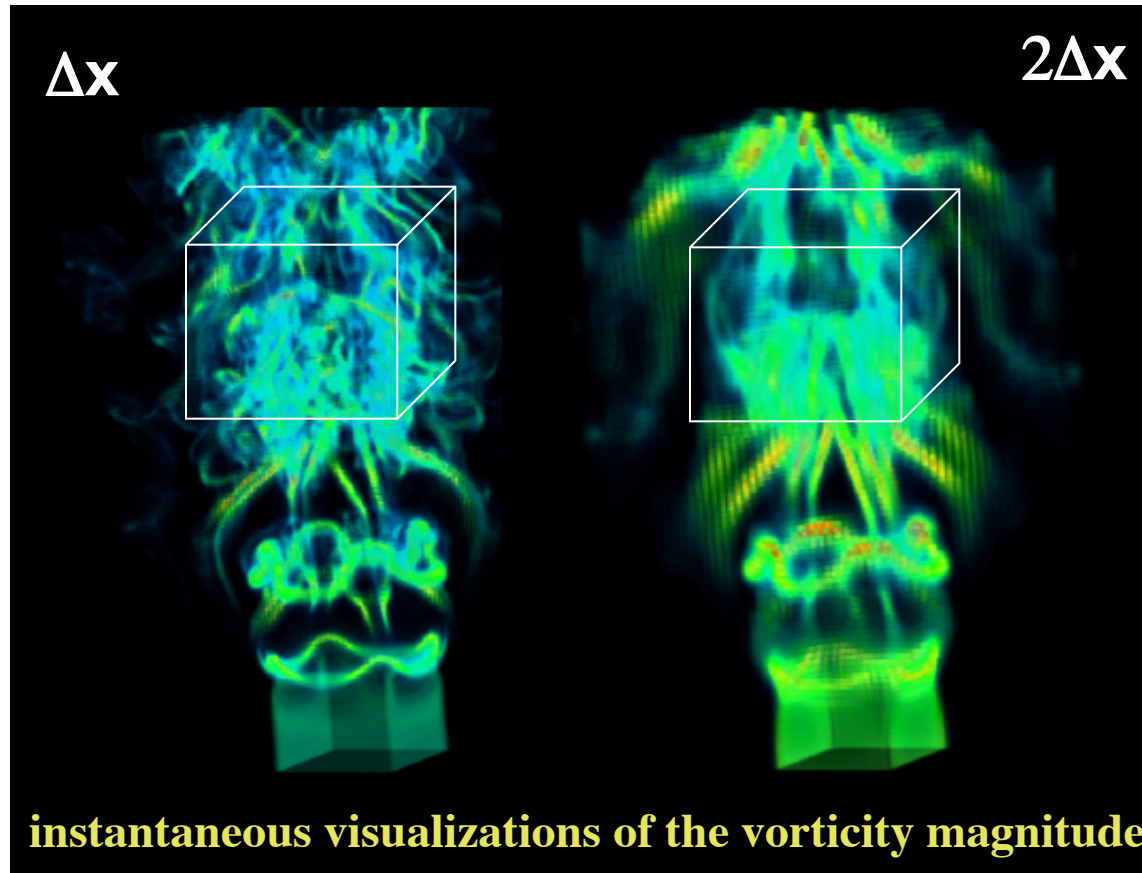
- Hybrid RANS / LES (e.g., DES)
- Wall shear stress BC models, $\tau_w = \tau_w(\mathbf{v})$

$u_\tau = \sqrt{\tau_w}$ = friction velocity
 δ = boundary layer thickness

- Re = 395, 595, DNS by Moser et al. '95
- Re = 2030-50, expts. by Wei & Willmarth '89

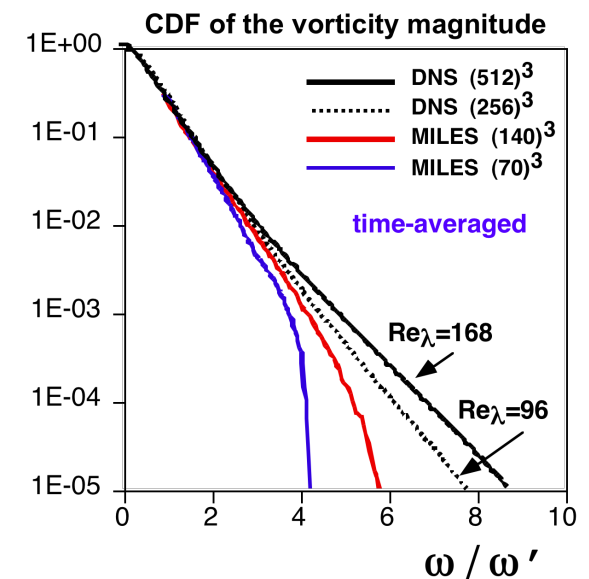
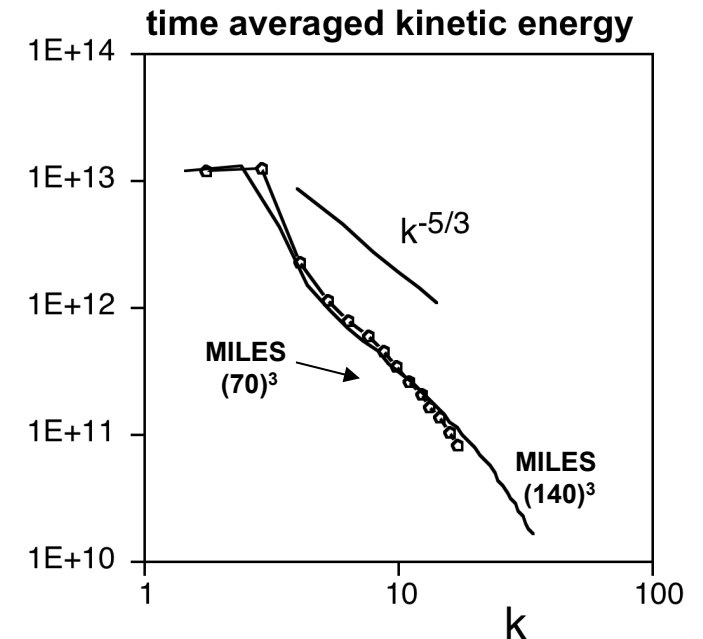
Coarse Grained Simulation “Convergence”

weakly forced jets, small-scale analysis



ILES by Grinstein & Devore PoF 1996, Grinstein JFM 2001;
DNS by Jimenez JFM 1993

Coarse Grained “observations” are affected by **(explicit or implicit)** filter-length cutoff ...
--> **must be incorporated in V&V metrics**



Coarse Grained Simulations (LES, ILES / MILES)

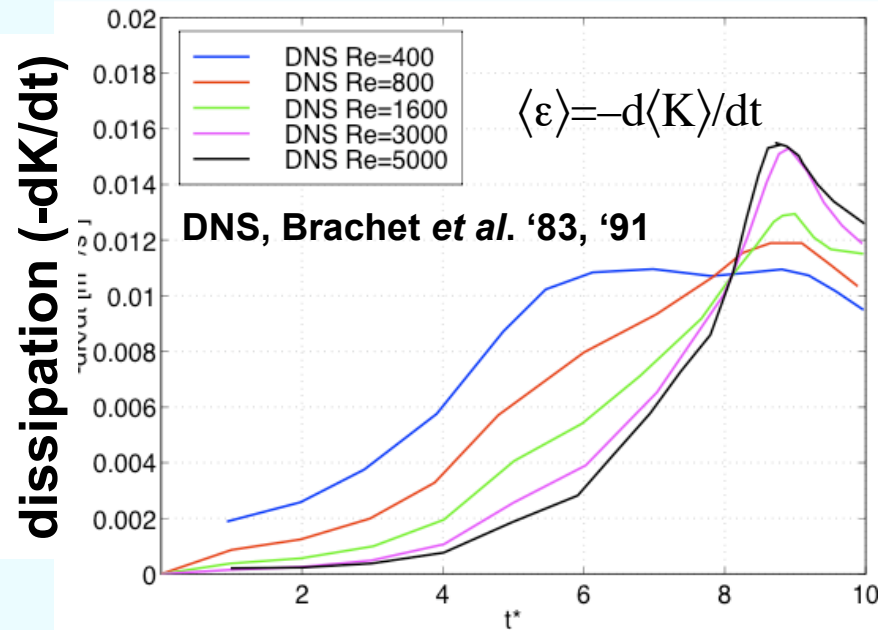
why (and when) do they work ?

- CGS: Background & Basis
- Inhomogeneous free & wall-bounded flows
 - jets, channel flow
- **Transition & Decay: Taylor-Green vortex**
- Material mixing: shock-driven turbulence

Outlook

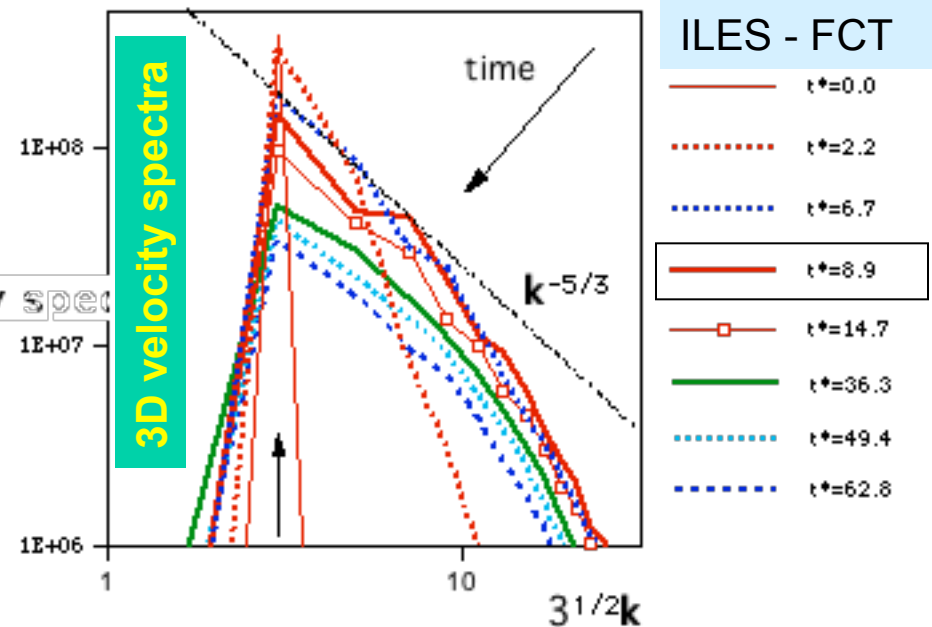
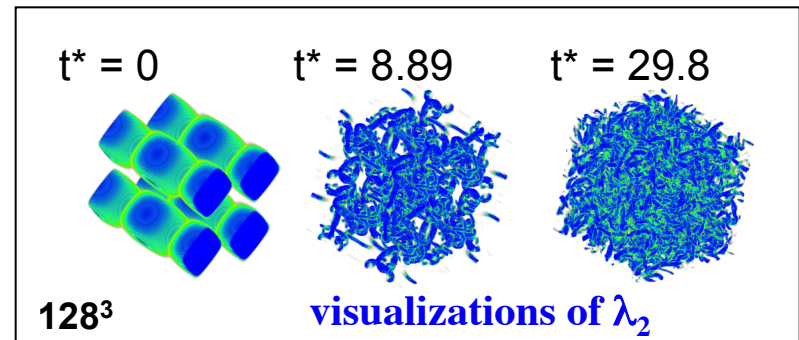
Taylor-Green Vortex

Simulations of Transition and Turbulence Decay
LES, ILES, DNS: Grinstein et al. JoT '07, **DNS,** Brachet et al. '83, '91



Robust dissipation peak at $t^* \approx 9$
 \Rightarrow onset of inviscid instability
 \Rightarrow viscosity independent limit
 \Rightarrow to be captured by CGS
 transition time, dissipation rate

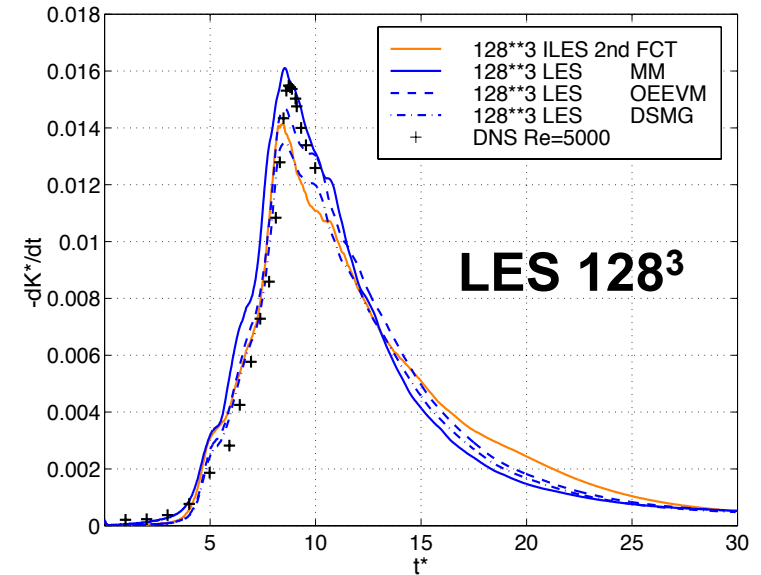
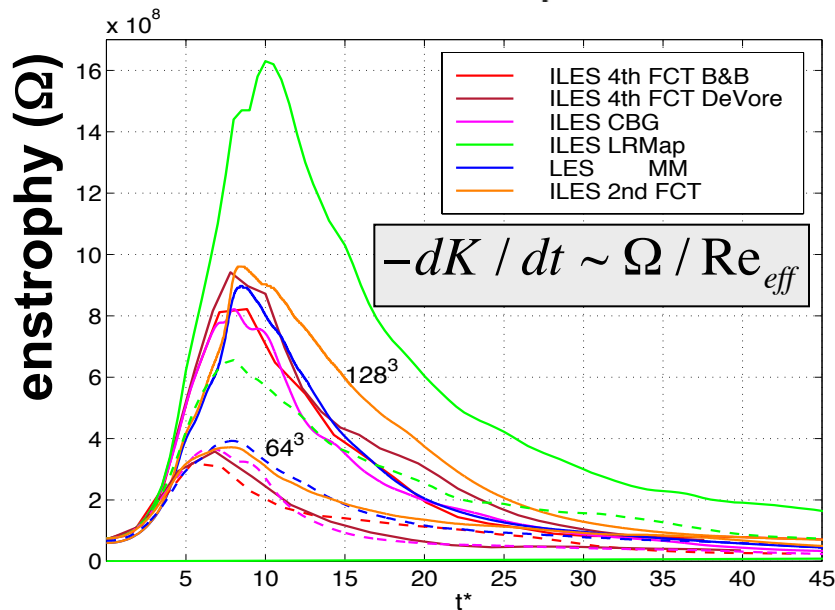
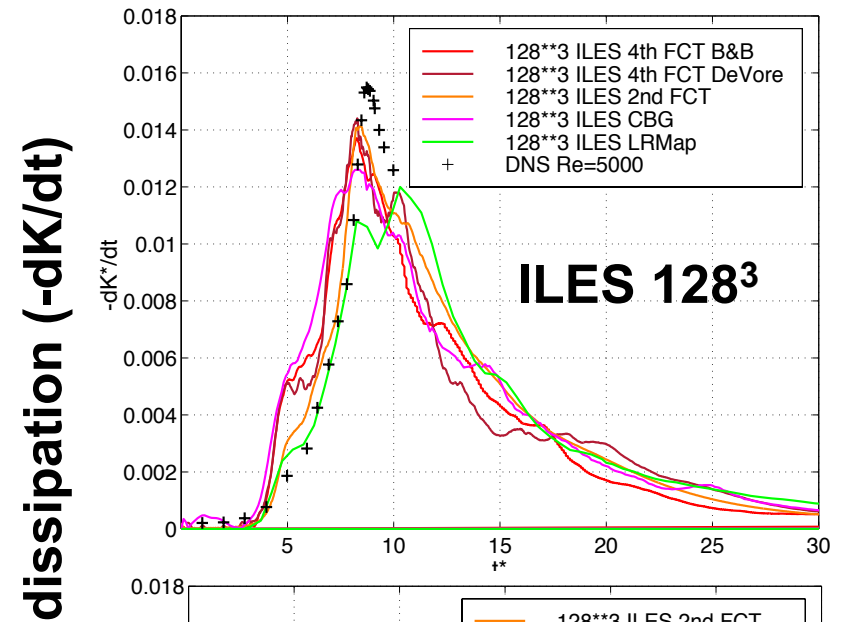
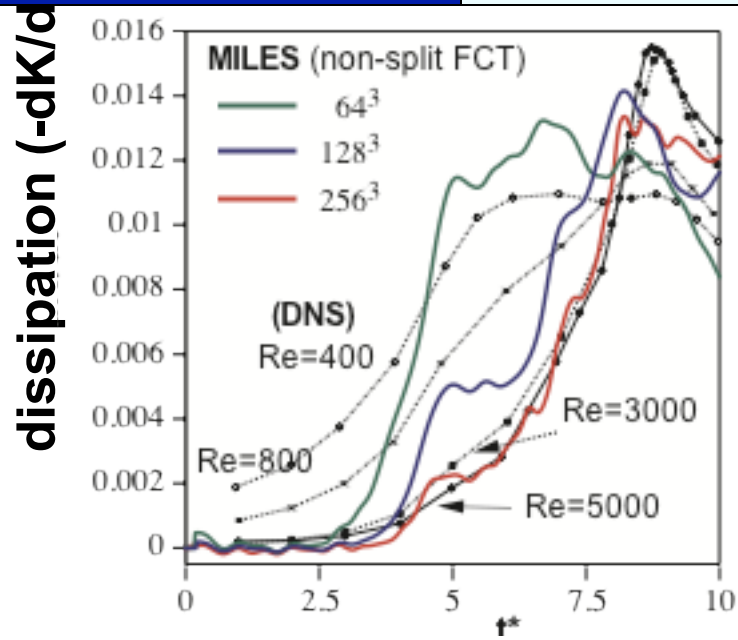
ILES effectively captures high-Re physics of transition and decay rates



Taylor-Green Vortex Integral Measures

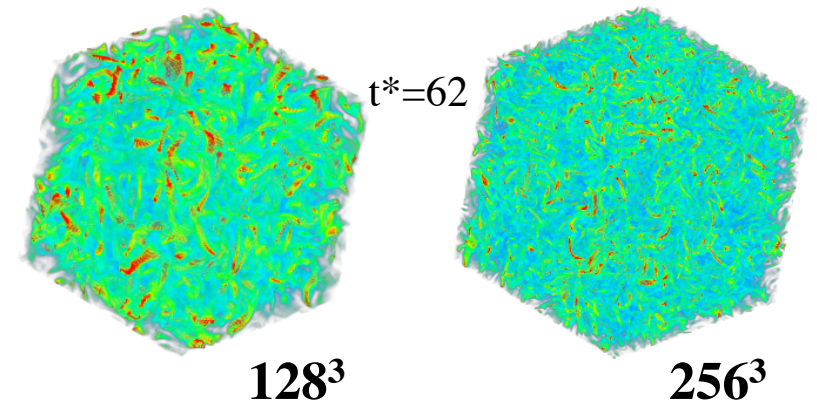
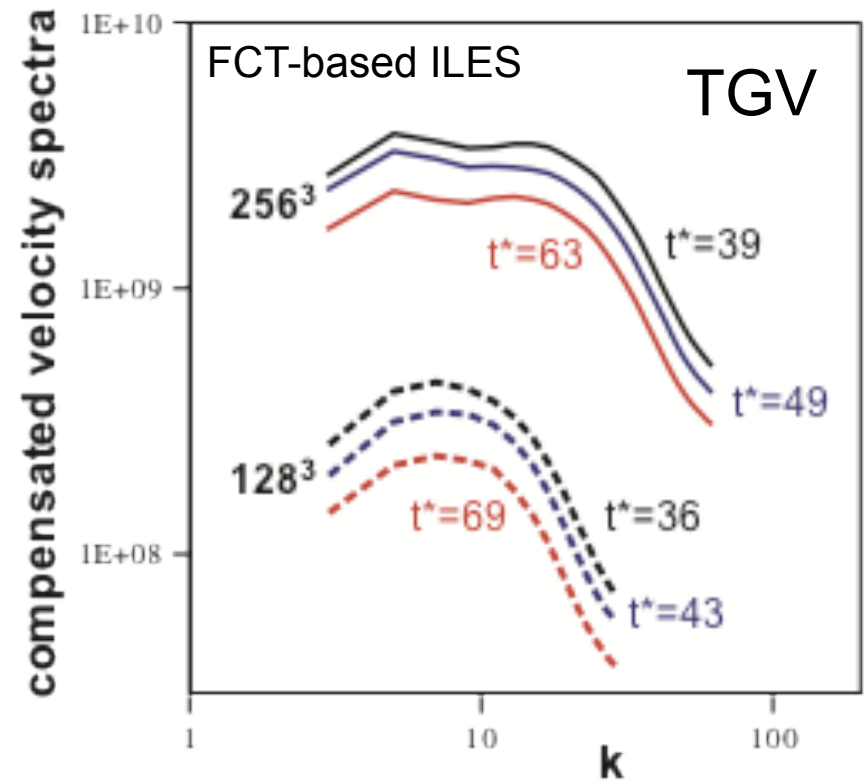
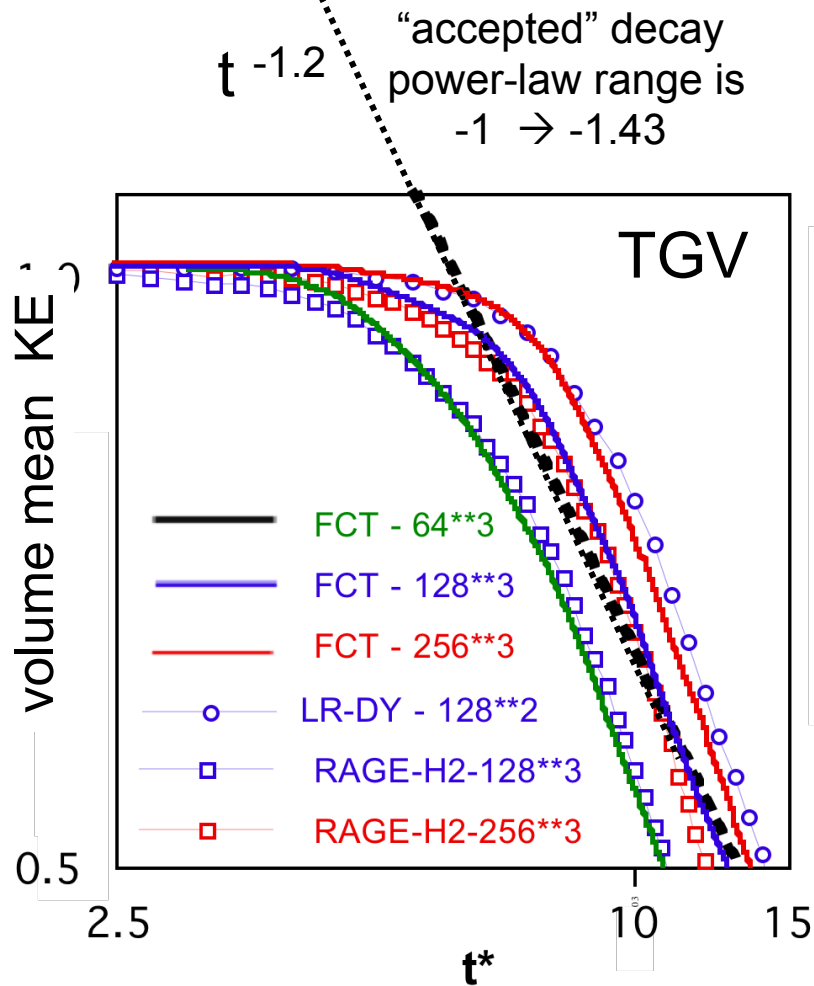
Effective Re associated with resolution

LES & ILES: Grinstein et al. JoT '07, DNS, Brachet et al. '83, '91



KE Decay Rate vs. Numerics

TGV, Grinstein et al. JoT '07



FCT (4th order), RAGE-Godunov (2nd order)
 LR: D. Youngs Lagrange-Remap, uses van
 Leer (3rd order) at remap phase

Coarse Grained Simulations (LES, ILES / MILES)

why (and when) do they work ?

CGS: Background & Basis

Inhomogeneous free & wall-bounded flows

➤ jets, channel flow

• Transition & Decay: Taylor-Green vortex

• **Material mixing**

➤ **shock-driven turbulence**

Outlook

Predictive Mix Simulation

“Numerical mix” is **unavoidable in under-resolved simulations of complex turbulent flows !**

- What **physical** mix can be emulated **numerically** ?
- When is a **subgrid mix model** needed ?

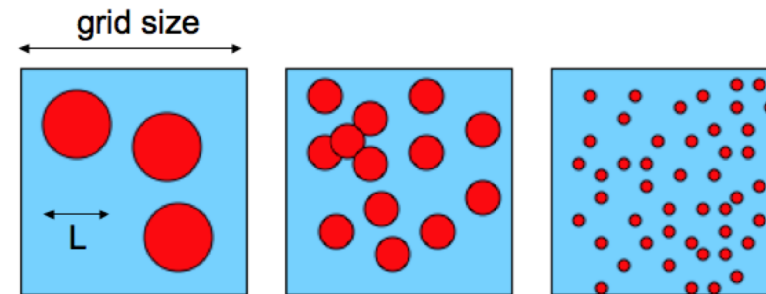
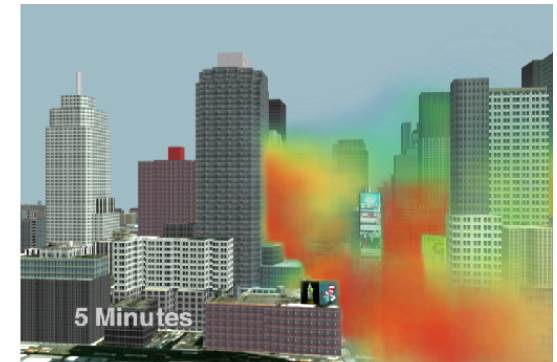
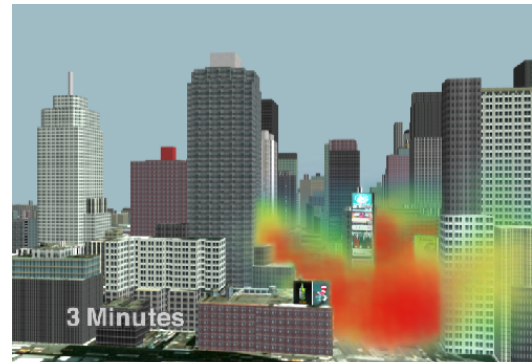


Figure 1. Three different realizations of fluid mixing that all have the same volume fractions.

Fundamental mix issues:

Can we predict *integral consequences* of small-scale scalar mix ?

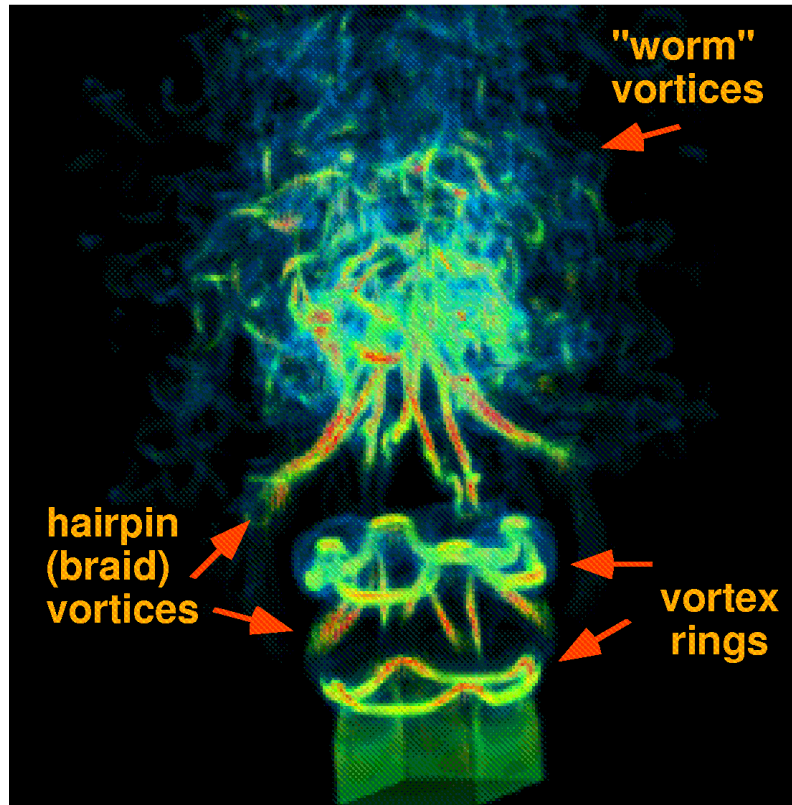
resolve *integral mix effects* due to Initial Conditions ?

How to improve under-resolved mix modeling **when it “breaks”** ?

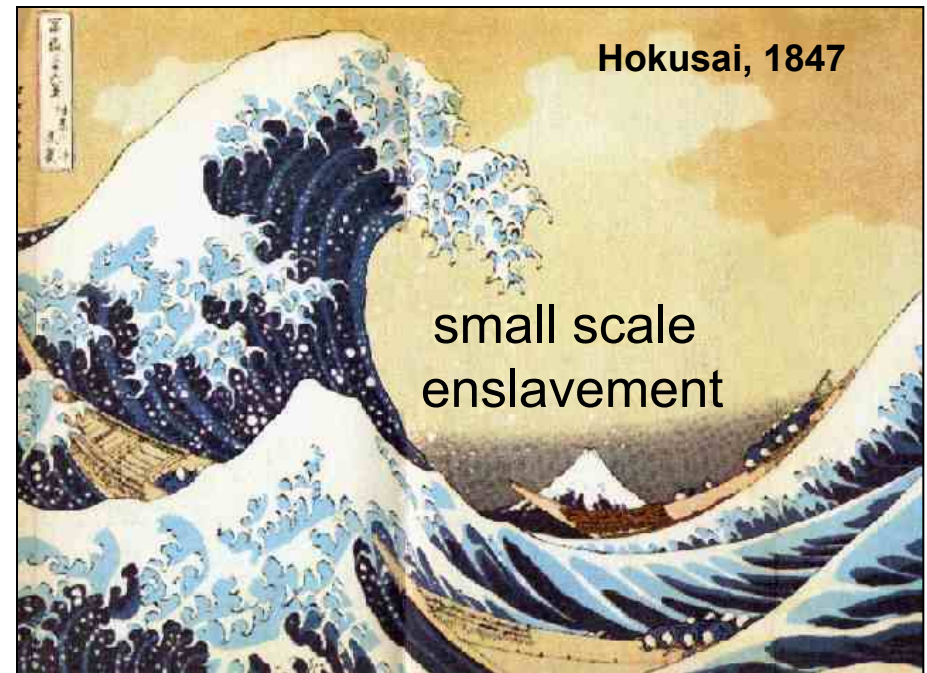
(effective *mixed implicit / explicit* subgrid modeling)

CHARACTERISTIC MIXING PROCESSES

- 1) **large scale** entrainment
- 2) **intermediate / small scale** stirring due to velocity fluctuations
- 3) **smaller scale** molecular diffusion (less important for high Re and $Sc \sim 1$)

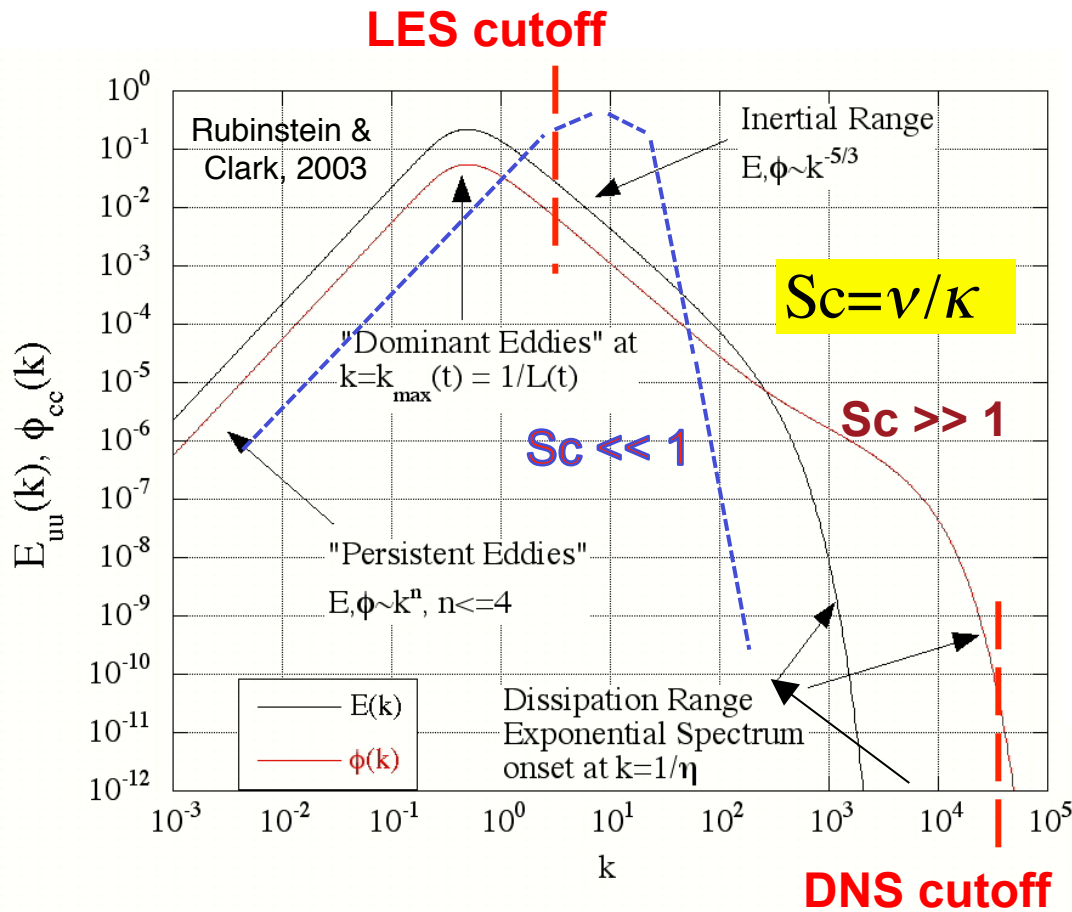


FFG, ILES, jfm '01

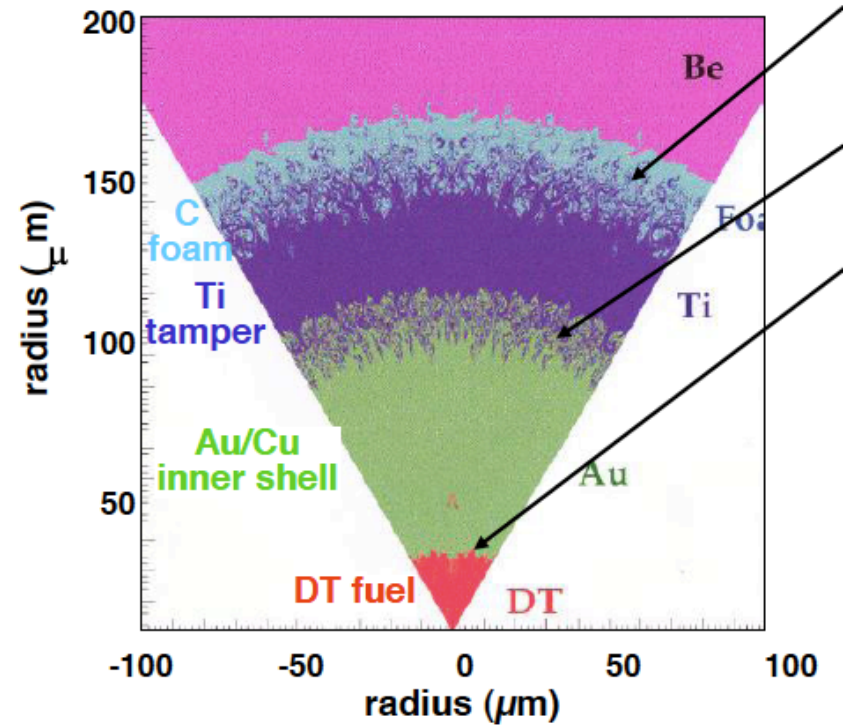


Under-Resolved Sc effects in Extremes

TKE & Scalar spectral features vs. Sc



2D HYDRA simulation of NIF-scale ignition double shell capsule



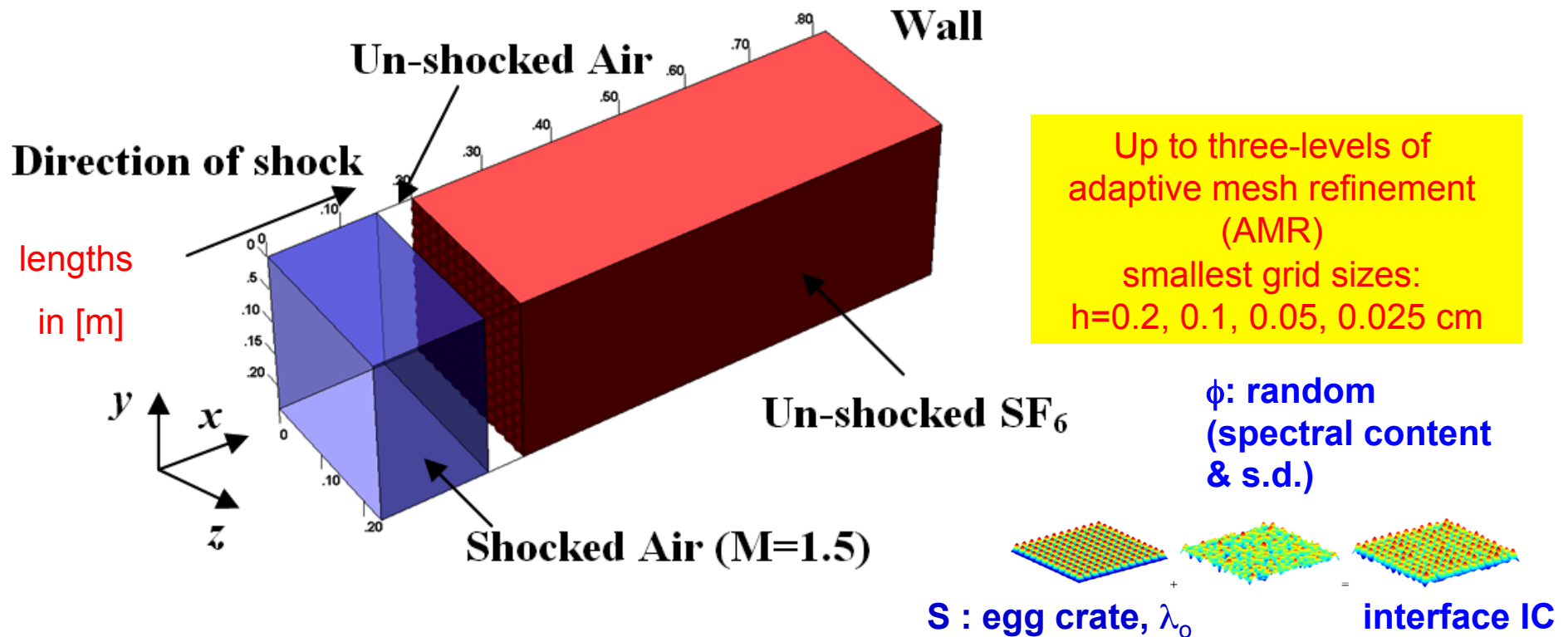
Sc $\gg 1$

Sc ~ 1

VVUQ of Material Interface Treatments (VOF, IP, ...) ?

Shock-Driven Turbulent Mixing

Shocks and Turbulence must be Captured !



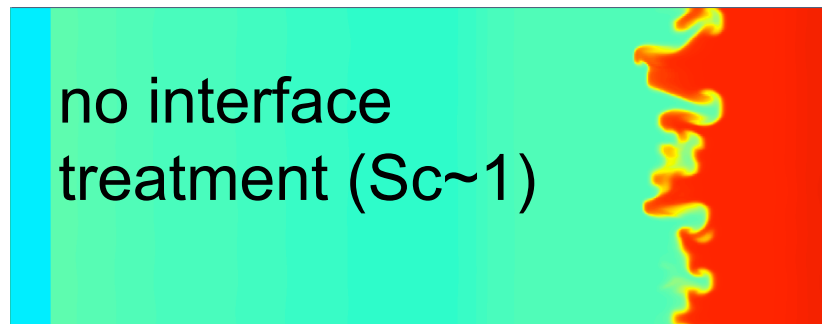
- Vetter & Sturtevant -- shocktube experiments, *Shock Waves* 1995
- Pullin et al. -- hybrid WENO / classical LES, *JFM* 2006
- Grinstein et al. -- ILES RAGE, *PoF* 2011, to appear

Challenge for any LES : simulating **under-resolved** mixing driven by **under-resolved** velocities and **under-resolved** initial conditions

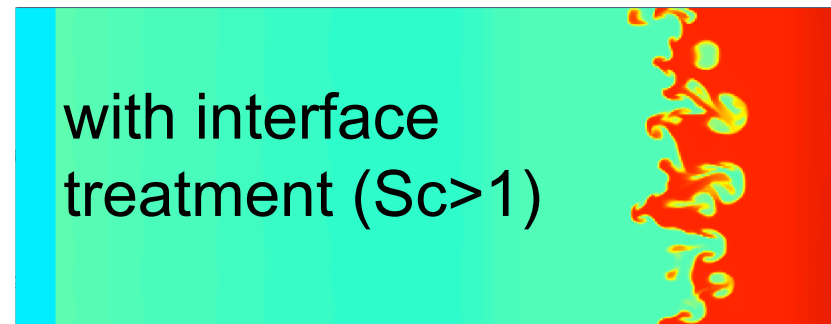
uniform grid, $h=0.2\text{cm}$, $410 \times 160 \times 160$, varying limiter & interface treatment

time = 6.33ms

minmod limiter

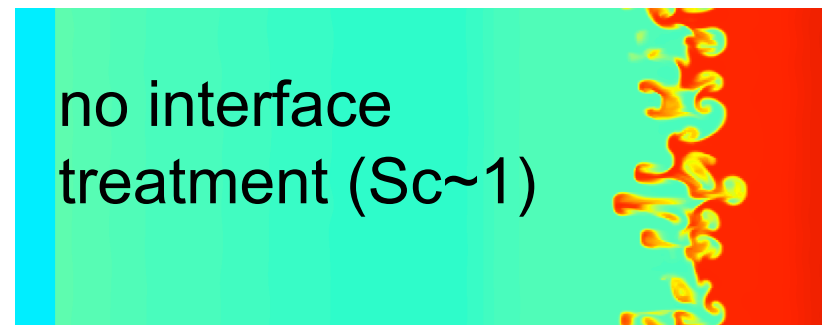


no interface
treatment ($Sc \sim 1$)

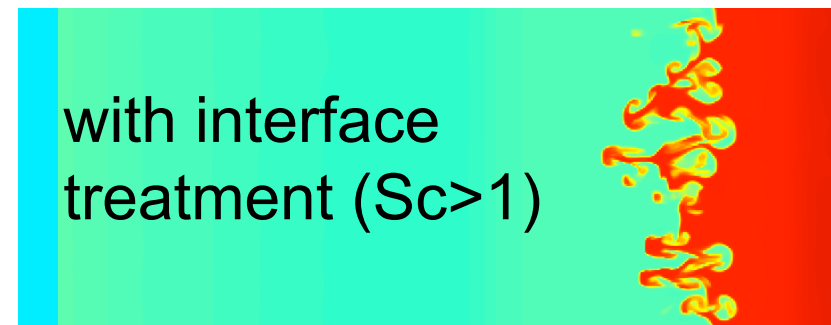


with interface
treatment ($Sc > 1$)

Van Leer limiter



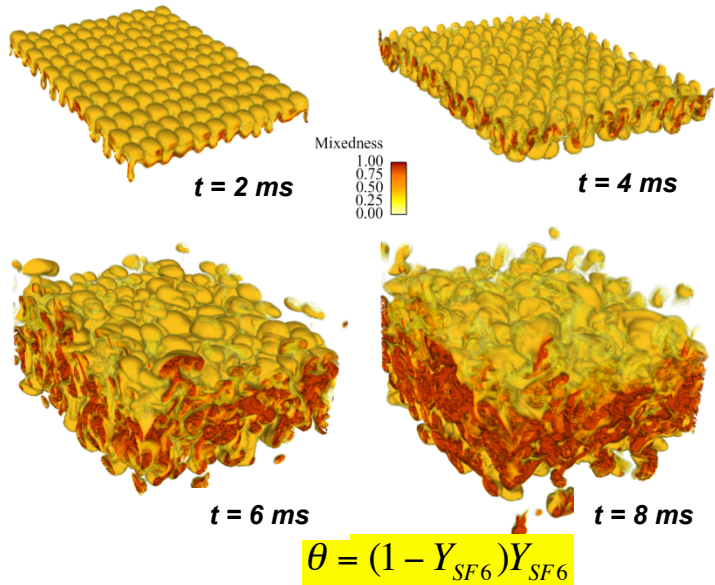
no interface
treatment ($Sc \sim 1$)



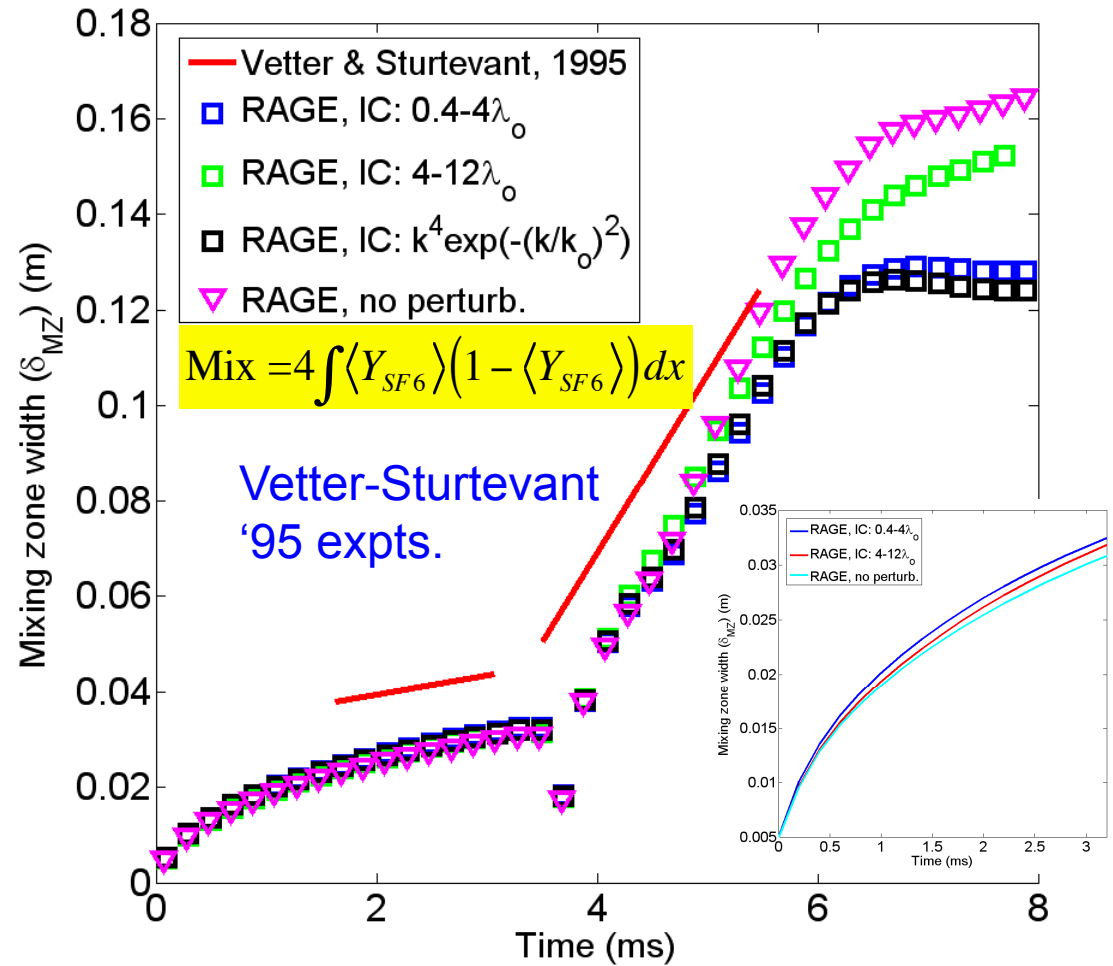
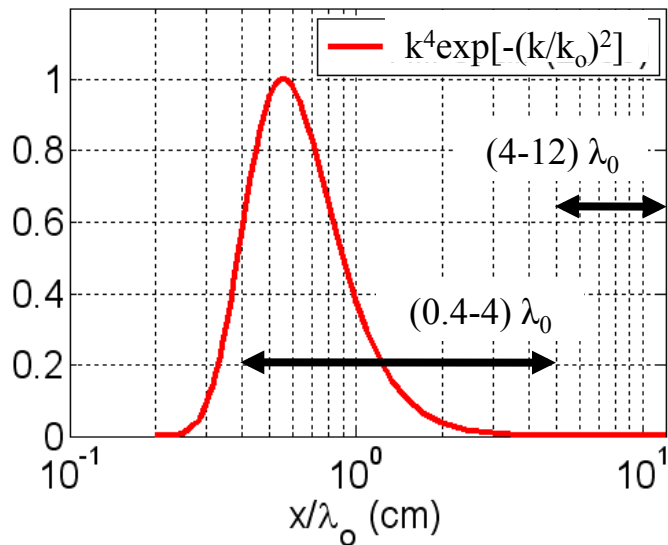
with interface
treatment ($Sc > 1$)

(x,y) plane

what are the “right” (limiter, interface) choices for the problem of interest ? --> VVUQ metrics ?



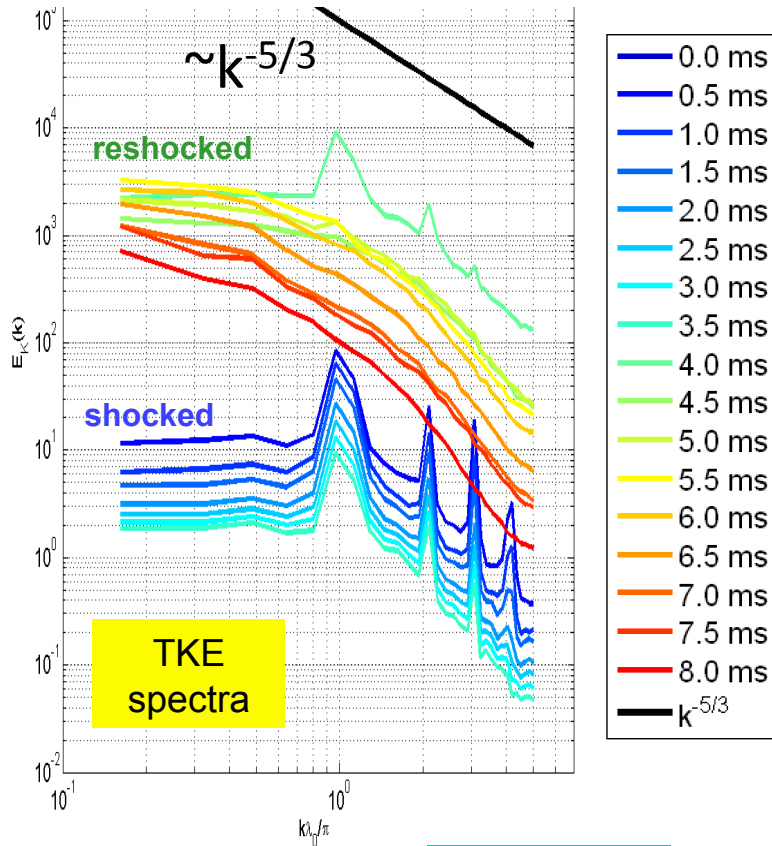
Initial interface perturbation



- consistent **growth rates**
- late-time sensitive to IC specifics
faster growth associated with “long” ICs
(as known for RT)

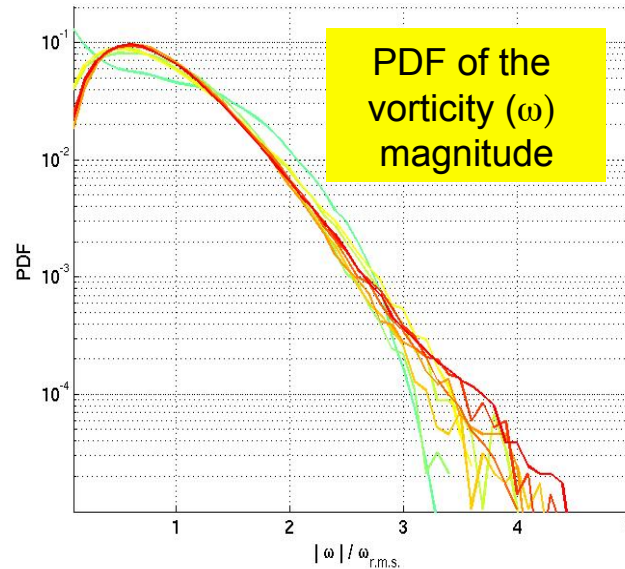
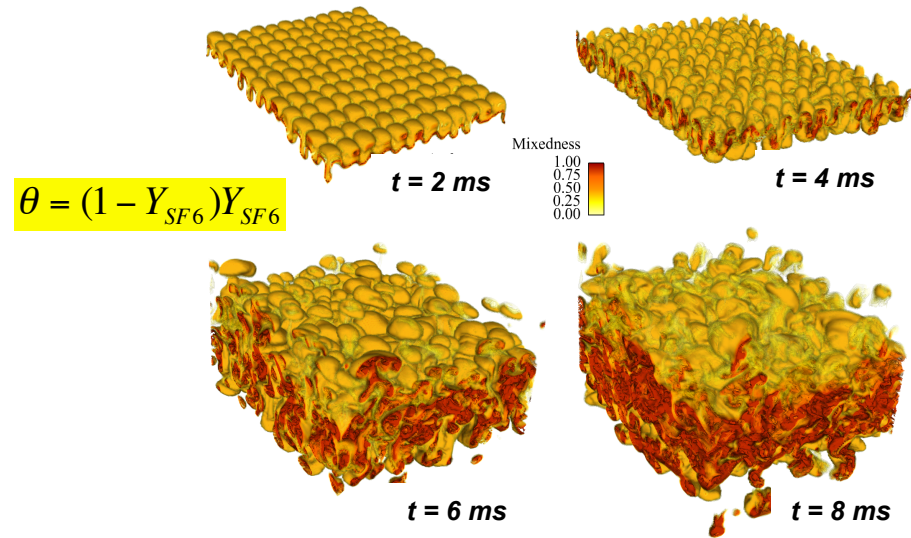
Planar RM -- RAGE Simulations

short-perturbed ICs : transition to turbulence



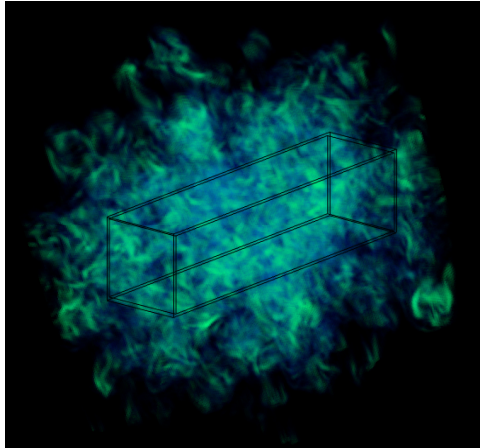
0.1 cm grid

late-time self-similarity suggested

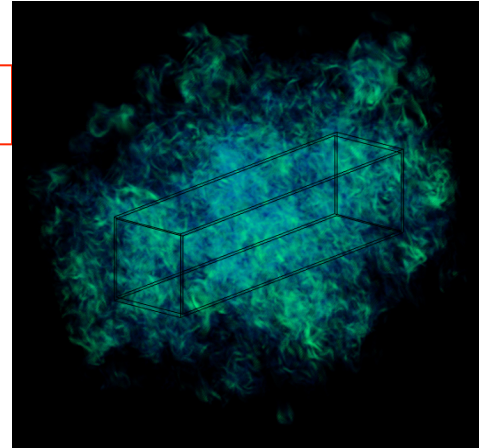
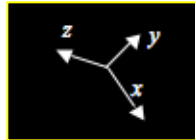


Planar RM – ILES RAGE

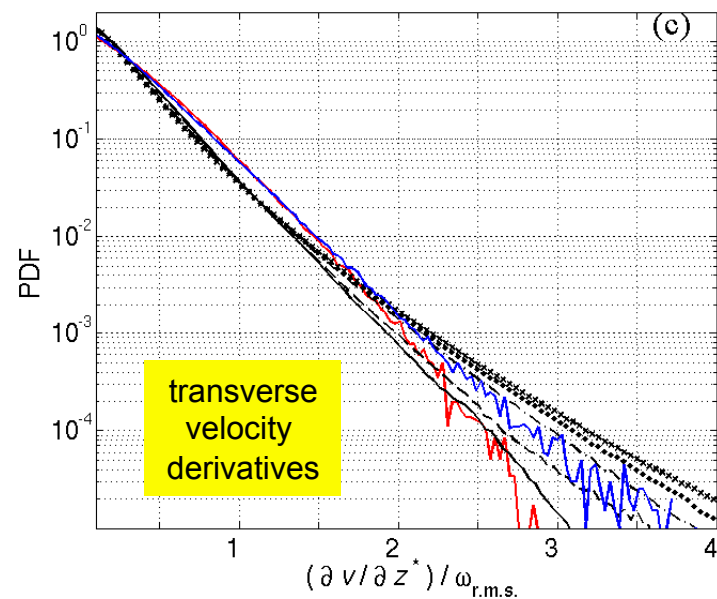
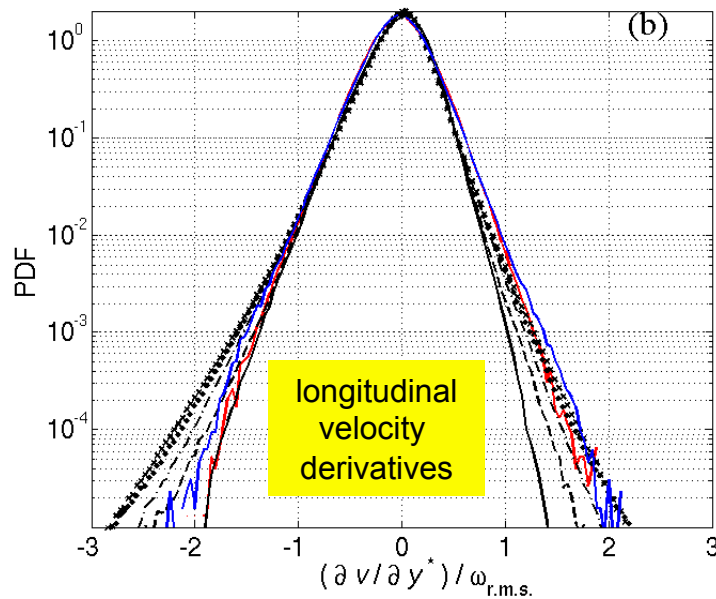
short IC, late time velocity-derivative PDF analysis



t = 8ms



- late-time self-similarity
- **non-gaussian** PDF tails
(consistent with DNS)
- higher resolution
→ higher effective Re



— Short IC: coarse — Short IC: fine — $Re_\lambda = 36$ --- $Re_\lambda = 60$ --- $Re_\lambda = 96$ • $Re_\lambda = 142$ • $Re_\lambda = 168$

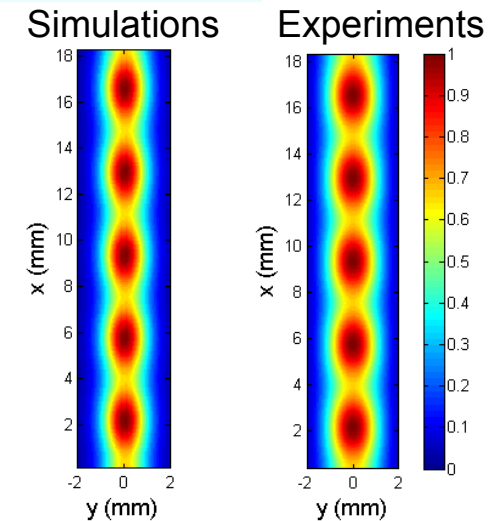
ILES

DNS (isotropic turbulence)
Jiménez et al. JFM '93

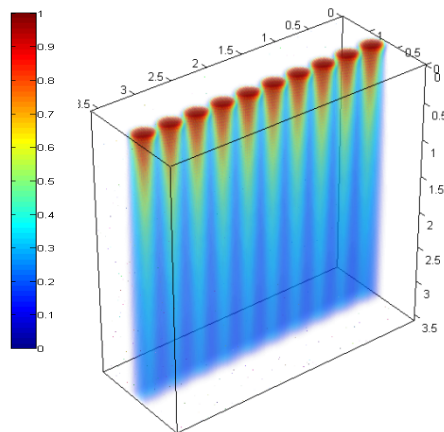
ILES of Shocked Gas-Curtain

FFG et al., AIAA ASM (2010), PoF in preparation

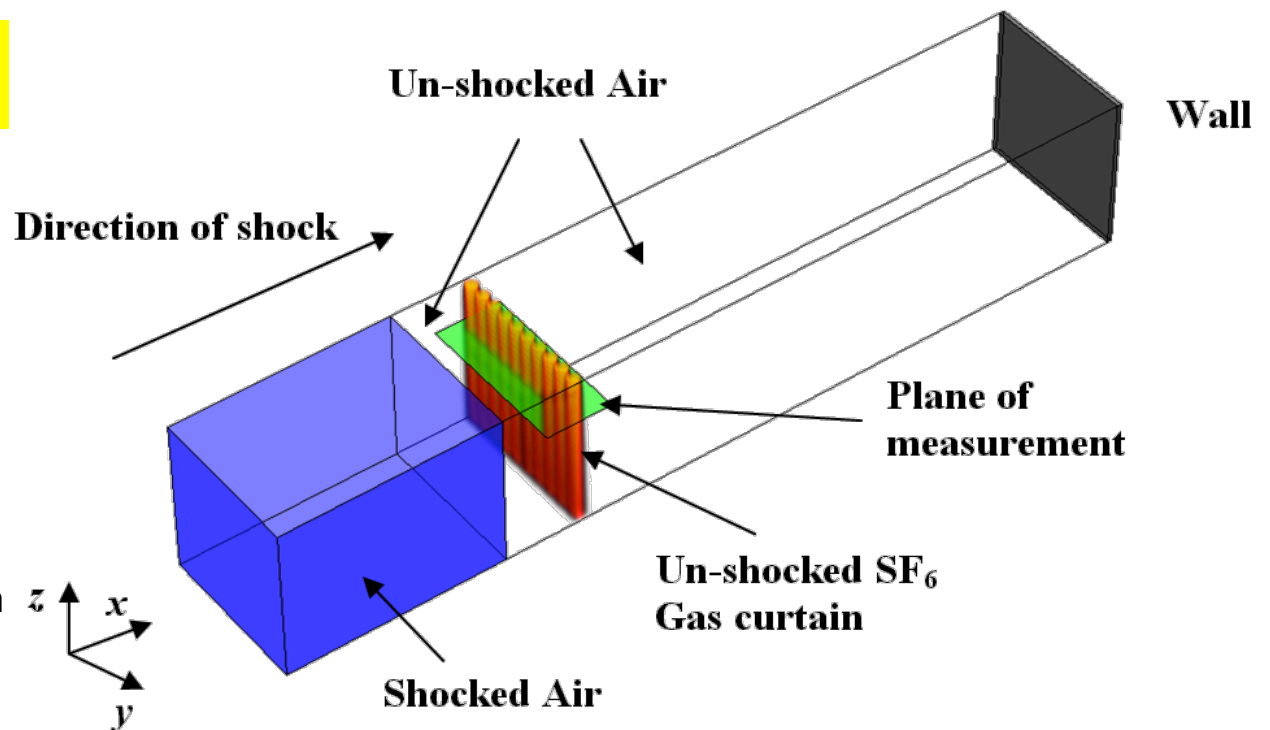
- **3D gas curtain** (ICs for RM simulations) is insufficiently characterized in laboratory expts.
- **Initial 3D gas curtain simulated** using separate incompressible NS-Boussinesq code and available info from LANL P-23 expts.



NS-Boussinesq simulated initial 3D Gas curtain

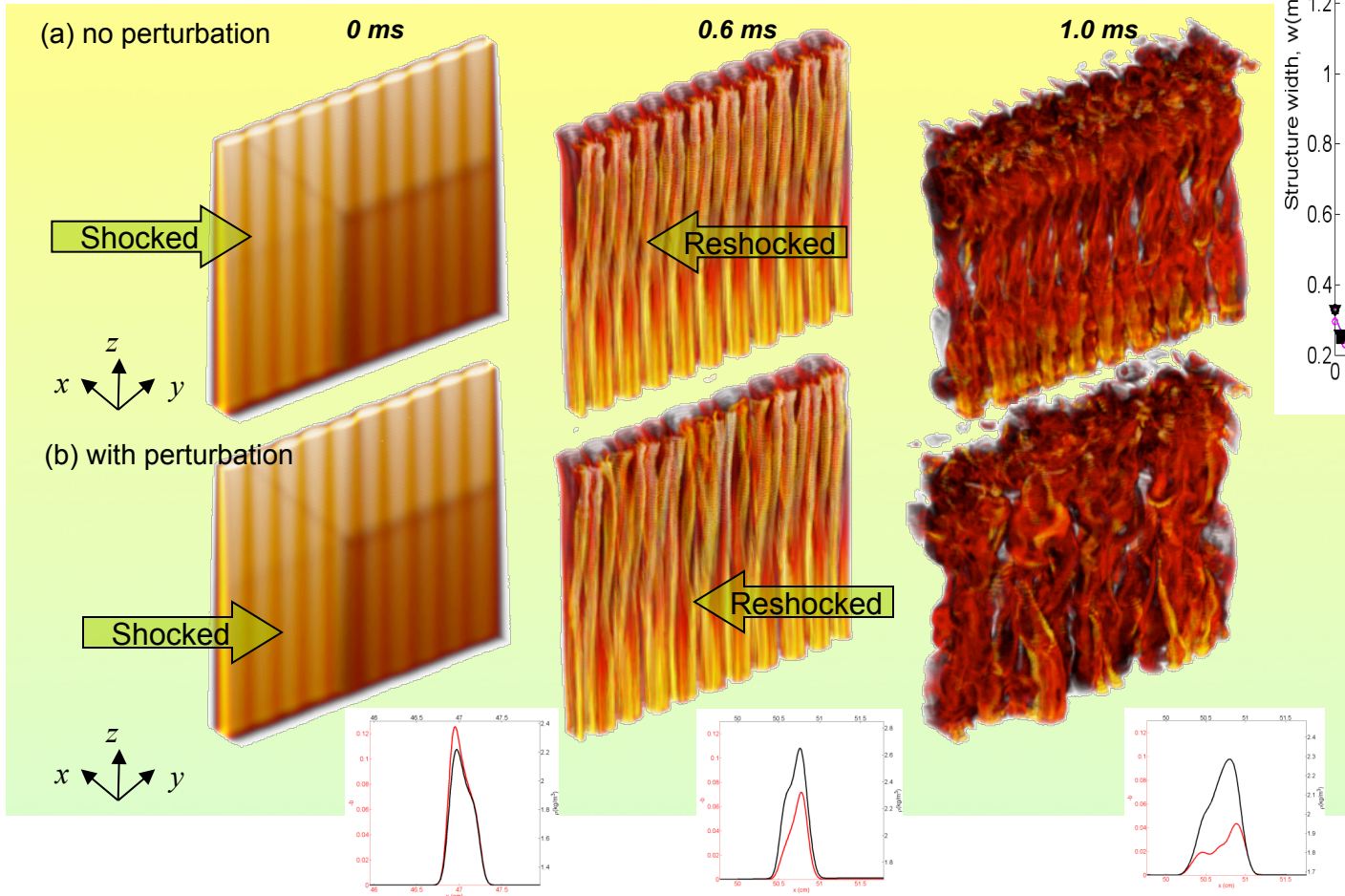
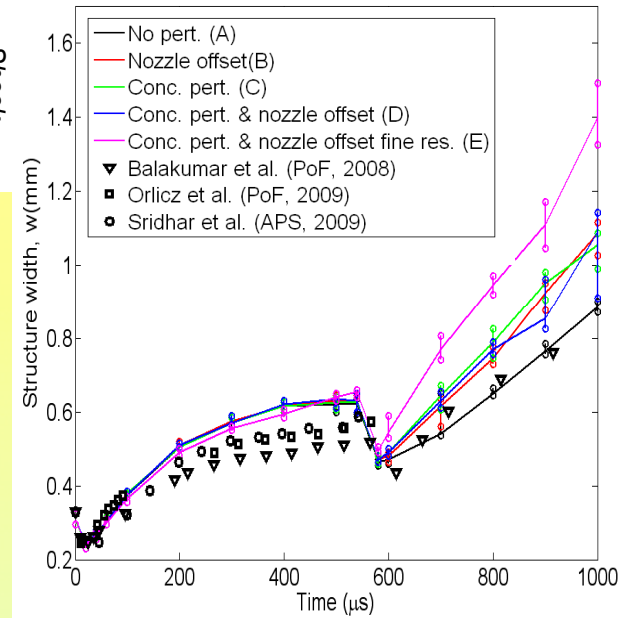
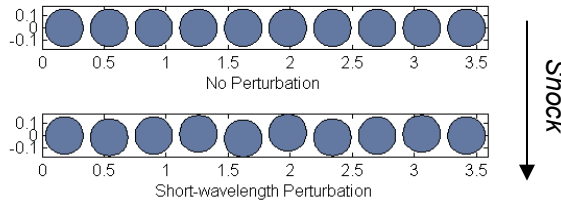


SF₆ Volume Fraction distributions



ILES RAGE of Shocked Gas-Curtain IC Effects on Mixing

Nozzle centers are randomly offset in the shock direction in the perturbed IC case



Mean mass density ρ ,
mass density
self-correlation b
(\rightarrow for BHR turbulent
mixing modeling)

$$b = -\rho' \times \left(\frac{1}{\rho} \right)'$$

Significantly more complex ICs in perturbed arrangement lead to enhanced vortex interactions and mixing

Transition & Material Mixing in **high-Re inhomogeneous, under-resolved** Extreme Turbulent Flows --> **CGS**

• Modified Equation Analysis

to assess / reverse-engineer subgrid features

- finite scale vs. continuum
- mixed explicit / implicit models

• Material Interface dynamics VVUQ

- difficult: mathematics is sketchy
- extreme sensitivity to ICs
- equations of state ... expts. ...

• UQ for “predictive” simulations

characterize & model intrusiveness of laboratory & computational expts.

2D HYDRA simulation of NIF-scale ignition double shell capsule

