Progress & problems in the analysis of turbulence, dissipation and drag

Bounds on turbulence: what does it mean when they exist, and what does it mean when we don't know if they exist?

Charles R. Doering

Department of Mathematics, Department of Physics,
Center for the Study of Complex Systems,
University of Michigan

"Models versus physical laws/first principles, or why models work?"
Wolfgang Pauli Institute, Vienna, Austria, February 2-5, 2011

Progress & problems in the analysis of turbulence, dissipation and drag

- Energy dissipation rate bounds for body-force driven (turbulent) flows

- Bounds on (turbulent) wavenumber moments

- Energy dissipation rate bounds for boundary driven (turbulent) flows

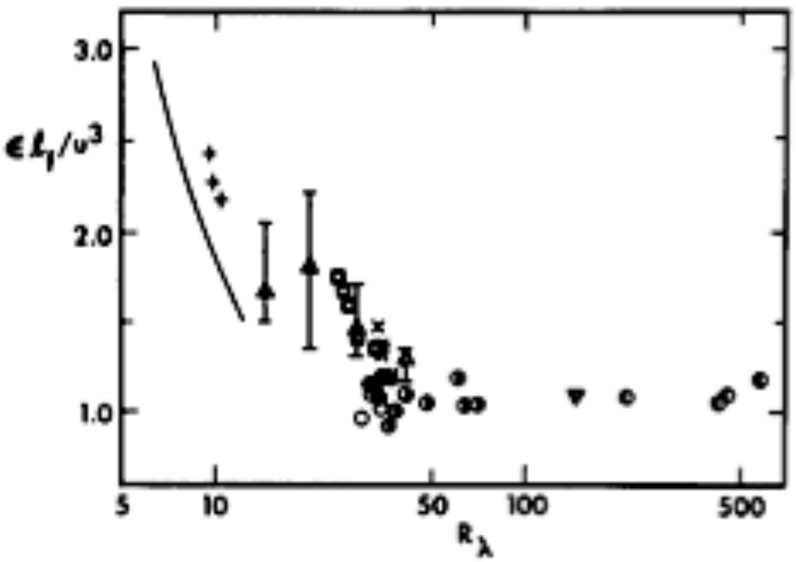
Experiments:

Direct numerical simulations:

On the scaling of the turbulence energy dissipation rate

K. R. Sreenivasan
Mason Laboratory, Yale University, New Haven, Connecticut 06520

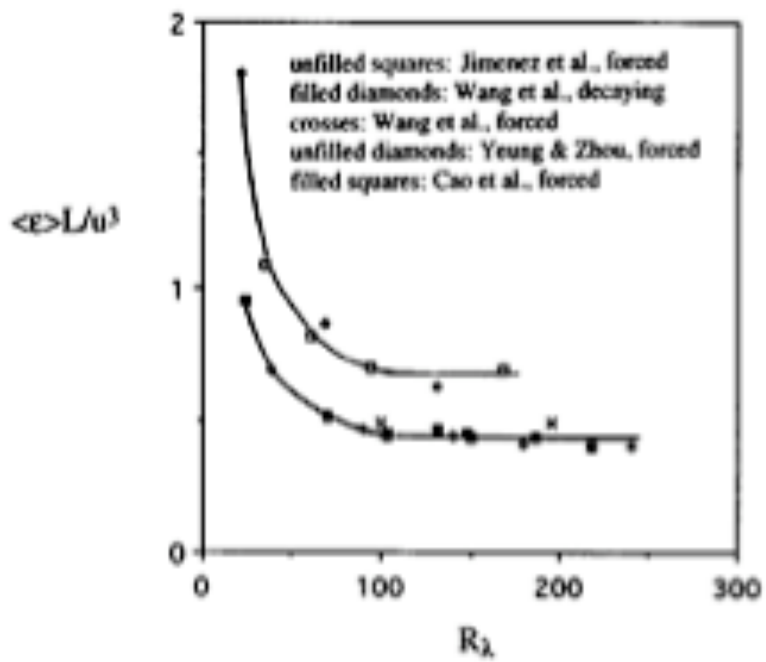
(Received 29 November 1983; accepted 23 February 1984)



An update on the energy dissipation rate in isotropic turbulence

Katepalli R. Sreenivasan
Mason Laboratory, Yale University, New Haven, Connecticut 06520-8286

(Received 29 July 1997; accepted 23 October 1997)



J. Fluid Mech. (2002), *vol.* 467, *pp.* 289–306.
DOI: 10.1017/S0022112002001386 Printed in the United Kingdom

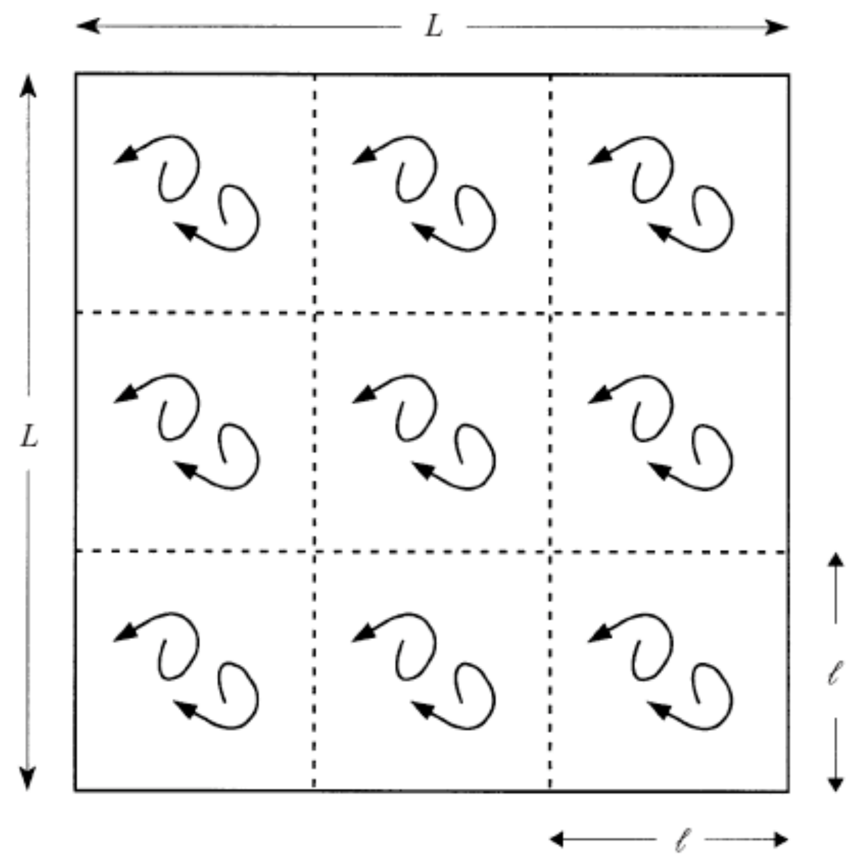
Energy dissipation in body-forced turbulence

By CHARLES R. DOERING[1] AND CIPRIAN FOIAS[2]
[1]Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1109, USA
e-mail: doering@umich.edu
[2]Department of Mathematics, Indiana University, Bloomington, IN 47405, USA

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \nabla p = \nu \Delta \boldsymbol{u} + \boldsymbol{f}(\boldsymbol{x})$$

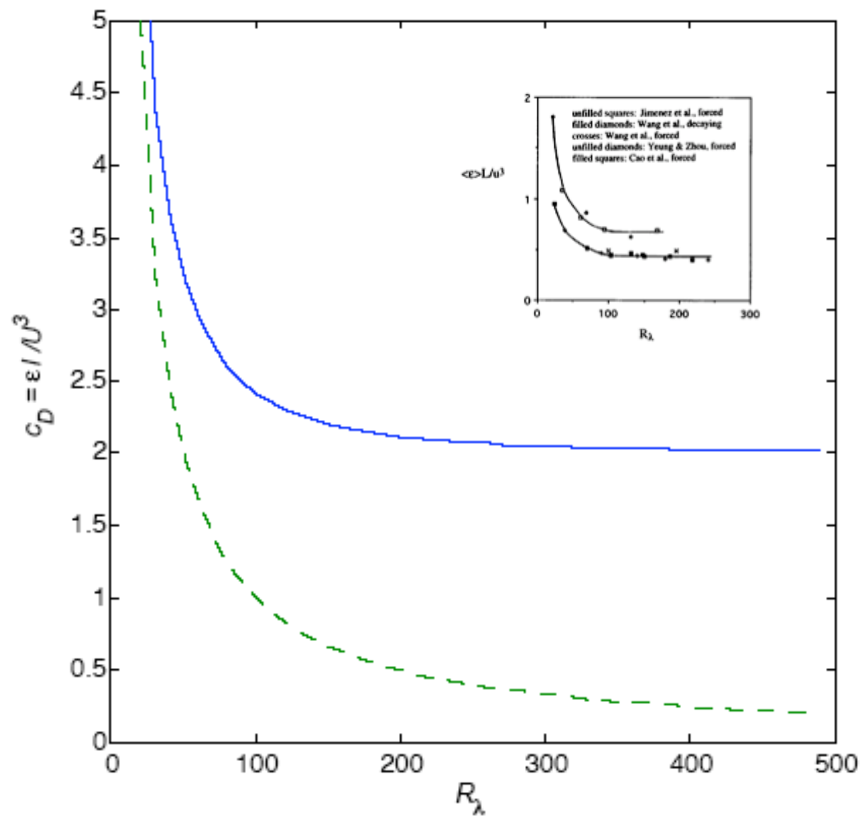
$$\nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{f}(\boldsymbol{x}) = F \Phi(\ell^{-1} \boldsymbol{x})$$



Theorem: for $\boldsymbol{\Phi}(\boldsymbol{\xi})$ in $L^2([0,1]^d)$,

$$\frac{2\pi}{\alpha R_\lambda} \leqslant \beta \leqslant \frac{b}{2}\left(1 + \sqrt{1 + \frac{4a}{b^2}\frac{1}{R_\lambda^2}}\right)$$



J. Fluid Mech. (2003), *vol.* 494, *pp.* 275–284.
DOI: 10.1017/S002211200300613X Printed in the United Kingdom

Energy dissipation in body-forced plane shear flow

By C. R. DOERING[1,2], B. ECKHARDT[3] AND J. SCHUMACHER[3]
[1]Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1109, USA
[2]Michigan Center for Theoretical Physics, Ann Arbor, MI 48109-1120, USA
[3]Fachbereich Physik, Philipps-Universität, D-35032 Marburg, Germany

$$\boldsymbol{f}(\boldsymbol{x}) = F\phi(y/\ell)\boldsymbol{e}_x$$

Journal of Turbulence
Volume 6, No. 17, 2005

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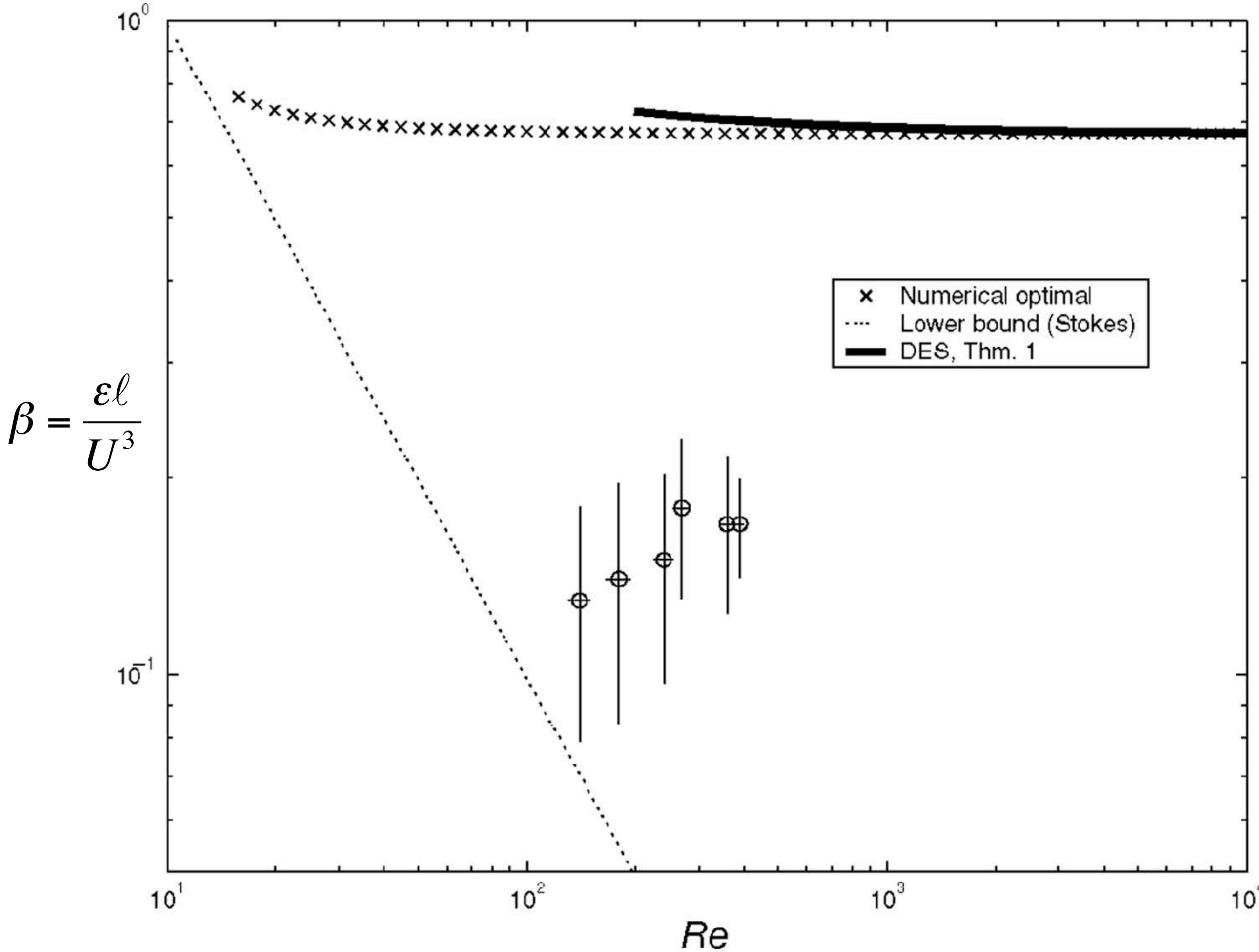
Variational bounds on the energy dissipation rate in body-forced shear flow

N. P. PETROV*†‡, L. LU† and C. R. DOERING†‡

†Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1043, USA
‡Also at the Michigan Center for Theoretical Physics, University of Michigan, Ann Arbor, MI 48109-1120, USA



$\beta = \frac{\varepsilon\ell}{U^3}$
10^0
10^{-1}
10^1
10^2
10^3
10^4
Re
Numerical optimal
Lower bound (Stokes)
DES, Thm. 1

accepted!

Variations on Kolmogorov Flow: Turbulent Energy Dissipation and Mean Flow Profiles

B. ROLLIN[1] Y. DUBIEF[1] & CHARLES R. DOERING[2]

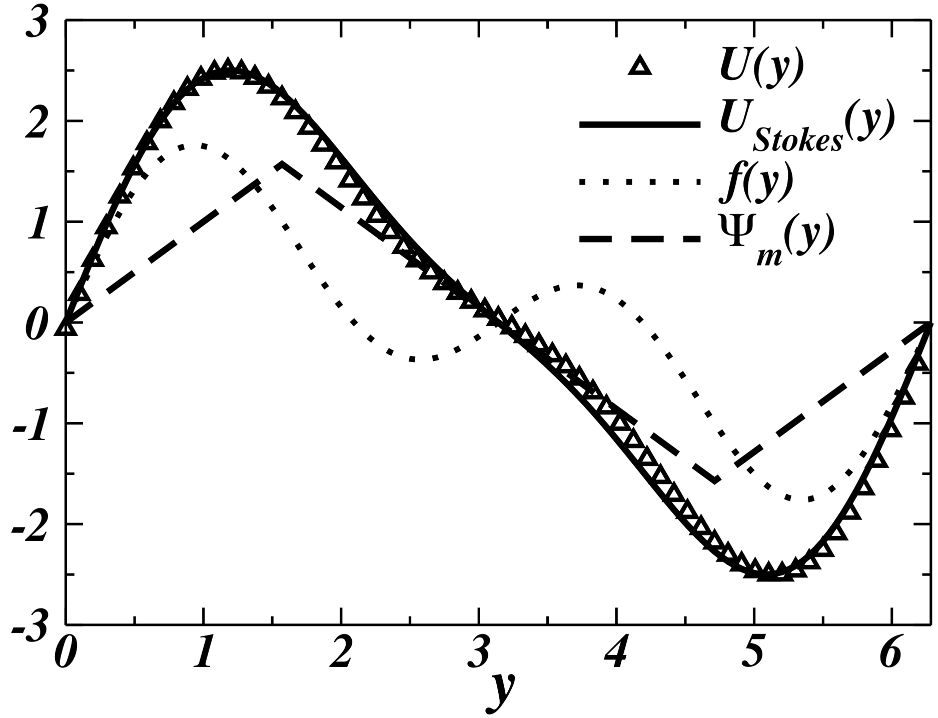
[1]School of Engineering, University of Vermont, Burlington, VT 05405, USA

[2]Department of Mathematics, Department of Physics, and Center for the Study of Complex Systems, University of Michigan, Ann Arbor, MI 48109-1107, USA

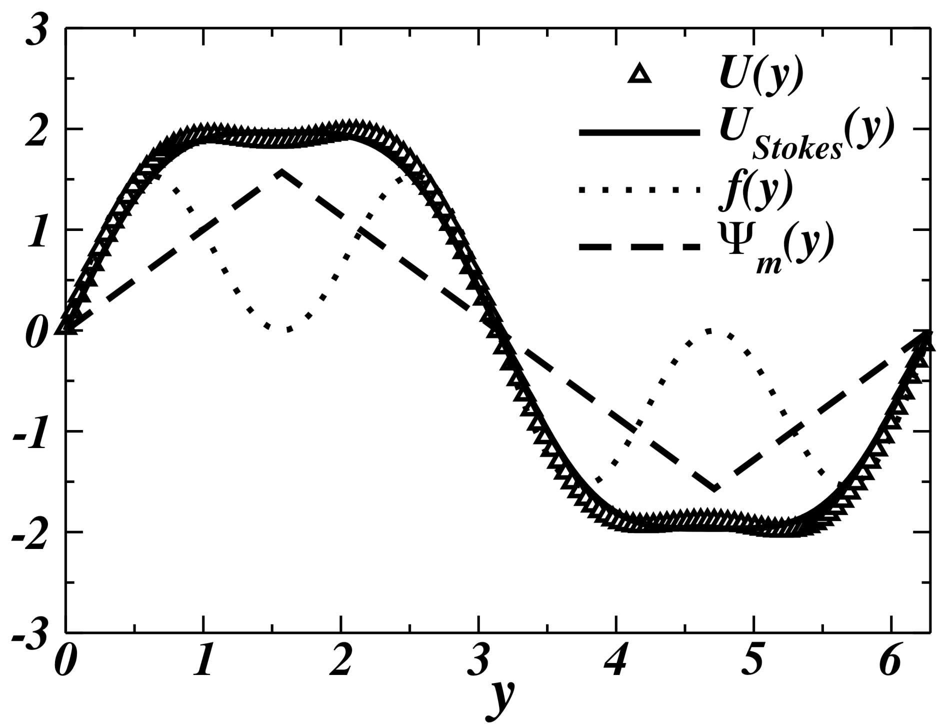
$$\boldsymbol{f}(\boldsymbol{x}) = F\phi(y/\ell)\boldsymbol{e}_x$$

$$\phi(\eta) = \sin(2\pi\eta) + A_k\sin(2\pi k\eta)$$

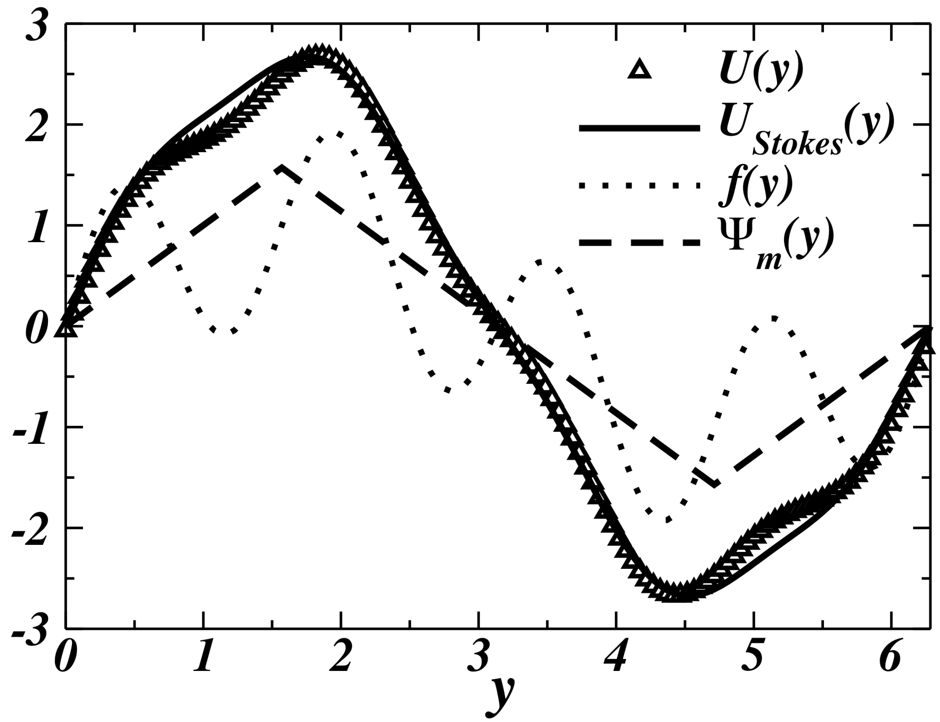
$$\phi(\eta) = \sin(2\pi\eta) + A_2\sin(4\pi\eta)$$



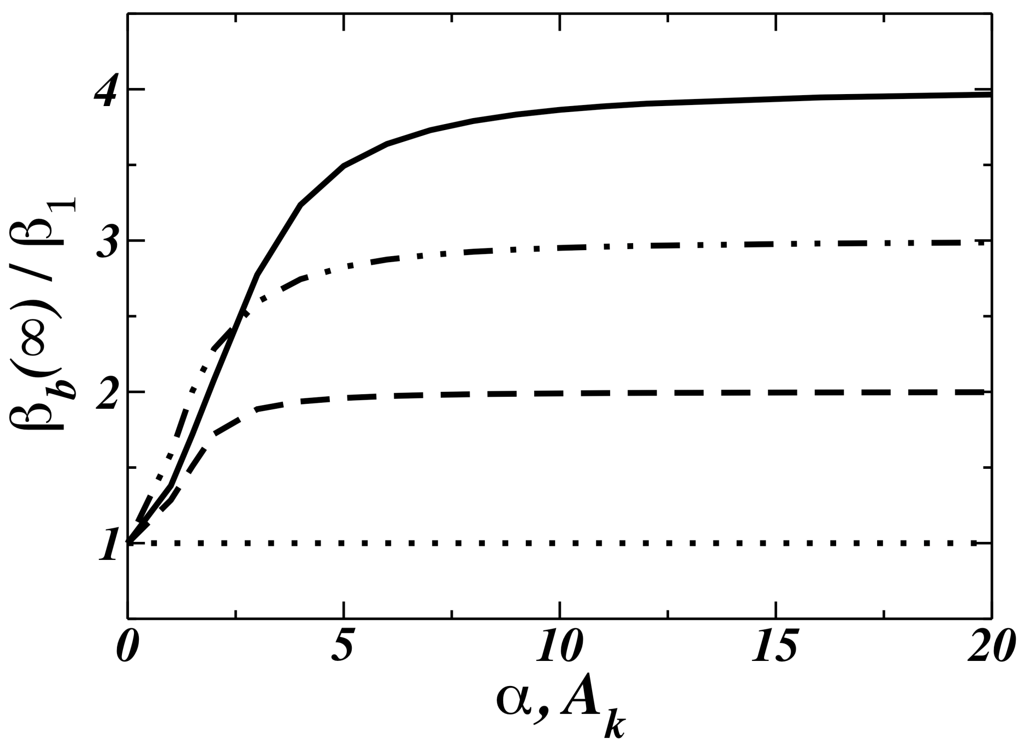
$$\phi(\eta) = \sin(2\pi\eta) + A_3\sin(6\pi\eta)$$



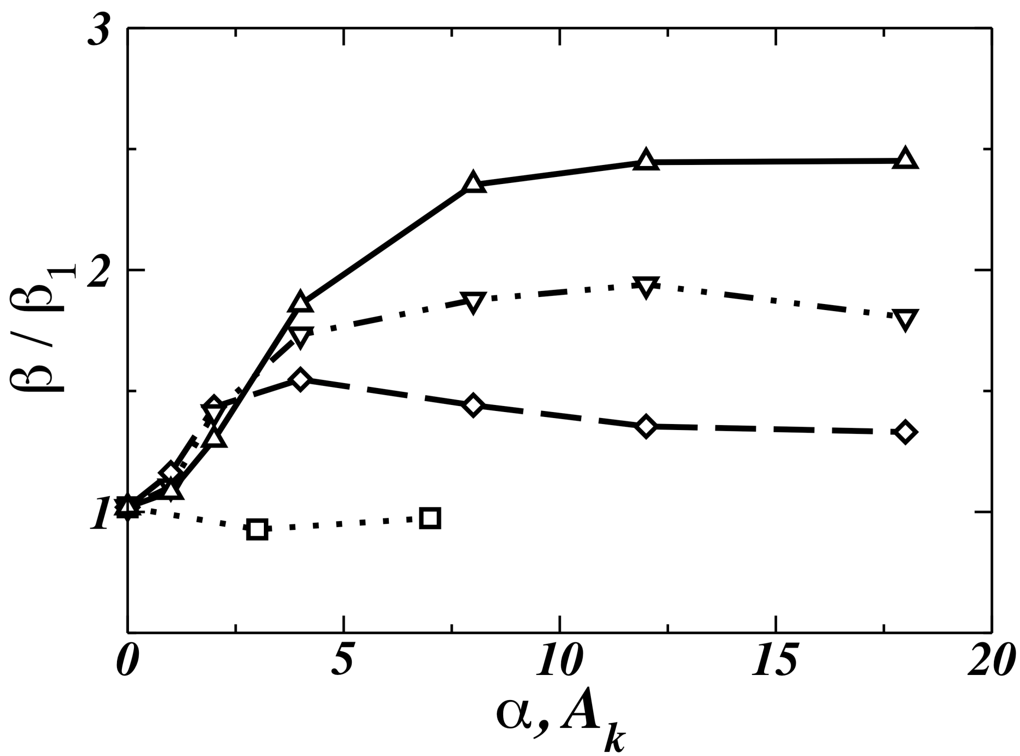
$$\phi(\eta) = \sin(2\pi\eta) + A_4\sin(8\pi\eta)$$



$$\phi(\eta) = \sin(2\pi\eta) + A_k \sin(2\pi k\eta)$$



$$\phi(\eta) = \sin(2\pi\eta) + A_k \sin(2\pi k\eta)$$



JOURNAL OF MATHEMATICAL PHYSICS **48**, 065208 (2007)

Energy dissipation in fractal-forced flow

Alexey Cheskidov[a)]
Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109

Charles R. Doering[b)]
Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109 and Michigan Center for Theoretical Physics, University of Michigan, Ann Arbor, Michigan 48109

Nikola P. Petrov[c)]
Department of Mathematics, University of Oklahoma, Norman, Oklahoma 73019

(Received 29 July 2006; accepted 30 November 2006; published online 7 June 2007)

$$\mathbf{f} \in H^{-\alpha} \text{ with } \alpha \in [0,1]$$

$$\beta \lesssim \mathrm{Re}^{\alpha/(2-\alpha)}$$

Progress & problems in the analysis of turbulence, dissipation and drag

- Energy dissipation rate bounds for body-force driven (turbulent) flows

- Bounds on (turbulent) wavenumber moments

- Energy dissipation rate bounds for boundary driven (turbulent) flows

Physica D 165 (2002) 163–175

PHYSICA D

www.elsevier.com/locate/physd

Bounds on moments of the energy spectrum for weak solutions of the three-dimensional Navier–Stokes equations

Charles R. Doering [a,*], J.D. Gibbon [b,1]

[a] Department of Mathematics, University of Michigan, 525 East University Avenue, Ann Arbor, MI 48109-1109, USA

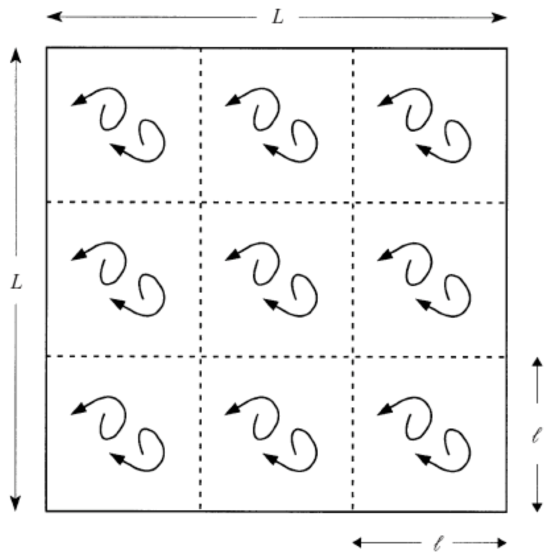
[b] Department of Mathematics, Imperial College of Science, Technology and Medicine, London SW7 2BZ, UK

Received 25 July 2001; accepted 5 March 2002

Communicated by U. Frisch

$$\langle\tilde{\kappa}_n\rangle \equiv \left\langle\left(\frac{\|\nabla^n \boldsymbol{u}\|_2}{\|\boldsymbol{u}\|_2}\right)^{1/n}\right\rangle$$

$$\langle \tilde{\kappa}_n \rangle \equiv \left\langle \left(\frac{\|\nabla^n \boldsymbol{u}\|_2}{\|\boldsymbol{u}\|_2} \right)^{1/n} \right\rangle$$



Theorem: For any $0 < \delta < \frac{1}{2}$,

$\exists\, c_n < \infty$ so that as $\nu \to 0$,

$$\ell \langle \kappa_n \rangle \leq c_n \left(\frac{L}{\ell} \right)^{3(n-1)/n} Re^{3-5/2n+\delta/n}$$

Consistent (mod δ) with $E(k) \sim k^{-q}$ up to $\ell k_c \sim R^{q_c}$ with

$q = 8/3$ and $q_c = 3$

Now *assume* that $\dfrac{\|u\|_\infty}{L^{-3/2}\|u\|_2} = O(1)$ as $\nu \to \infty$ ... then

Theorem: For any $0 < \delta < \frac{1}{2}$,

$\exists\, c_n < \infty$ so that as $\nu \to 0$,

$$\ell\langle\kappa_n\rangle \le c_n Re^{3/4-1/4n+\delta/n}$$

Consistent (mod δ) with $E(k) \sim k^{-q}$ up to $\ell k_c \sim R^{q_c}$ with

$q = 5/3$ and $q_c = 3/4$

Progress & problems in the analysis of turbulence, dissipation and drag

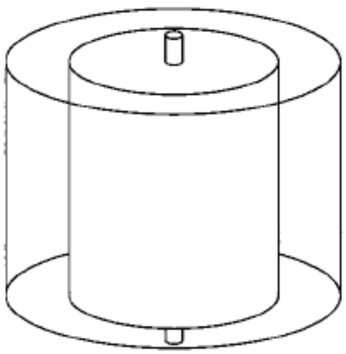
- Energy dissipation rate bounds for body-force driven (turbulent) flows

- Bounds on (turbulent) wavenumber moments

- Energy dissipation rate bounds for boundary driven (turbulent) flows

Transition to shear-driven turbulence in Couette-Taylor flow

Daniel P. Lathrop,* Jay Fineberg, and Harry L. Swinney†
Center for Nonlinear Dynamics, University of Texas, Austin, Texas 78712
and Department of Physics, University of Texas, Austin, Texas 78712
(Received 27 March 1992)



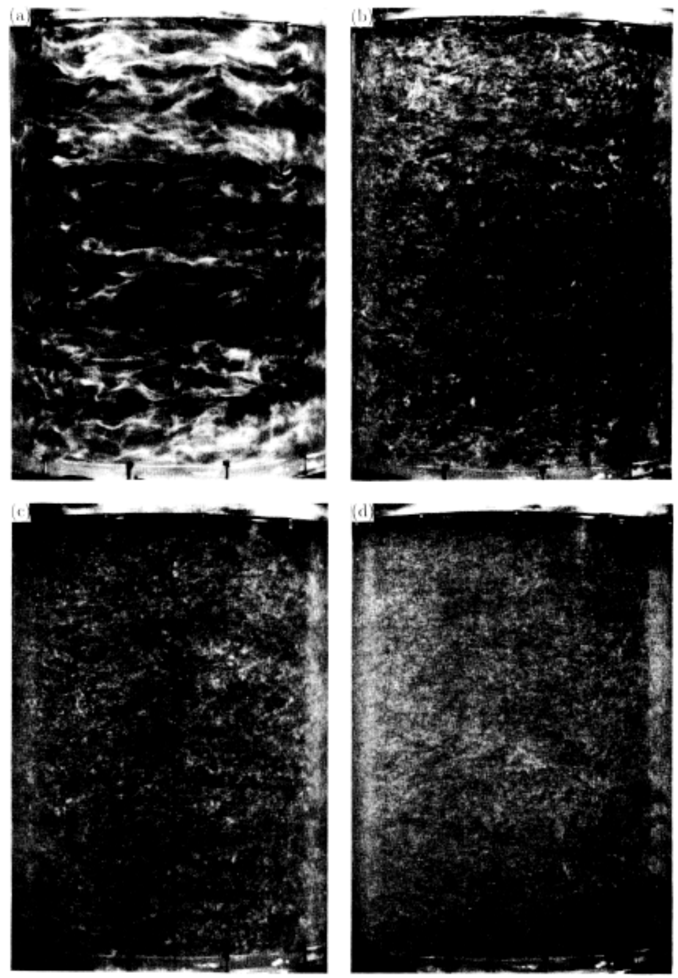


FIG. 5. Photographs of flow states at (a) $R = 6000$, (b) $R = 24\,000$, (c) $R = 48\,000$, and (d) $R = 122\,000$, obtained using Kalliroscope flow visualization. Eight vortices are visible in (a) and (b) and possibly (c), but not in (d).

Transition to shear-driven turbulence in Couette-Taylor flow

Daniel P. Lathrop,* Jay Fineberg, and Harry L. Swinney†
Center for Nonlinear Dynamics, University of Texas, Austin, Texas 78712
and Department of Physics, University of Texas, Austin, Texas 78712
(Received 27 March 1992)

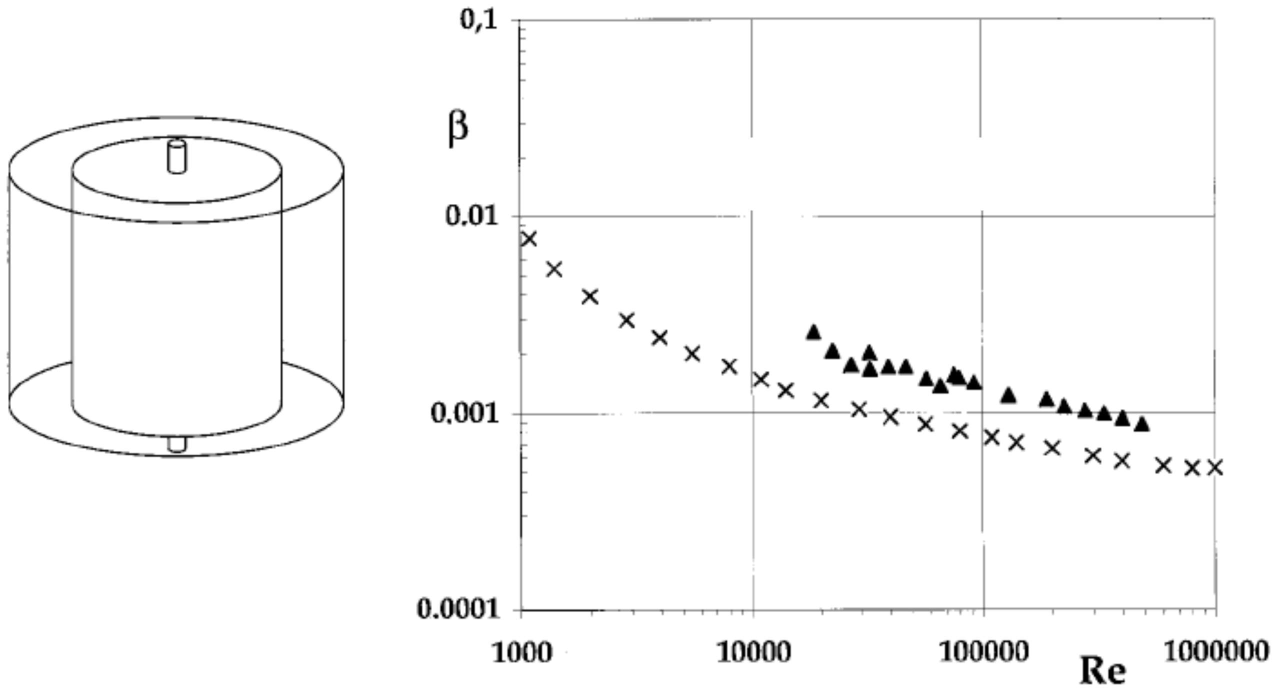


FIG. 5. Couette-Taylor experiments. Logarithmic plots of the nondimensional rates of energy dissipation β_D as a function of the Reynolds number.

J. Fluid Mech. 17 (1963) 405–432.

Heat transport by turbulent convection

By LOUIS N. HOWARD

J. Fluid Mech. 37 (1969) 457–477.

On Howard’s upper bound for heat transport by turbulent convection

By F. H. BUSSE

J. Fluid Mech. 41 (1970) 219–240.

Bounds for turbulent shear flow

By F. H. BUSSE

VOLUME 69, NUMBER 11 PHYSICAL REVIEW LETTERS 14 SEPTEMBER 1992

Energy Dissipation in Shear Driven Turbulence

Charles R. Doering[a]
Department of Physics and Institute for Nonlinear Studies, Clarkson University, Potsdam, New York, 13699-5820

Peter Constantin[b]
Department of Mathematics, University of Chicago, Chicago, Illinois 60637
(Received 21 February 1992; revised manuscript received 13 August 1992)

PHYSICAL REVIEW E VOLUME 49, NUMBER 5 MAY 1994

Variational bounds on energy dissipation in incompressible flows: Shear flow

Charles R. Doering*
Department of Physics, Clarkson University, Potsdam, New York 13699-5820

Peter Constantin†
Department of Mathematics, University of Chicago, Chicago, Illinois 60637
(Received 18 October 1993)

ELSEVIER

Physica D 101 (1997) 178–190

PHYSICA D

Improved variational principle for bounds on energy dissipation in turbulent shear flow

Rolf Nicodemus [1], Siegfried Grossmann [2], Martin Holthaus *

Fachbereich Physik der Philipps-Universität, Renthof 6, D–35032 Marburg, Germany

Received 18 July 1996; revised 2 September 1996; accepted 3 September 1996
Communicated by F.H. Busse

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Physica D 121 (1998) 175–192

PHYSICA D

Unification of variational principles for turbulent shear flows: the background method of Doering–Constantin and the mean-fluctuation formulation of Howard–Busse

R.R. Kerswell [1]

Department of Mathematics, University of Bristol, Bristol, BS8 1TW, UK

Received 15 October 1997; received in revised form 5 February 1998; accepted 27 March 1998
Communicated by F.H. Busse

Variational Bound on Energy Dissipation in Turbulent Shear Flow

Rolf Nicodemus, Siegfried Grossmann, and Martin Holthaus
Fachbereich Physik der Philipps-Universität, Renthof 6, D-35032 Marburg, Germany
(Received 25 July 1997)

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Improved upper bound on the energy dissipation rate in plane Couette flow: the full solution to Busse's problem and the Constantin–Doering–Hopf problem with one-dimensional background field

By S. C. PLASTING AND R. R. KERSWELL
Department of Mathematics, University of Bristol, Bristol, BS8 1TW, UK

Transition to shear-driven turbulence in Couette-Taylor flow

Daniel P. Lathrop,* Jay Fineberg, and Harry L. Swinney†
Center for Nonlinear Dynamics, University of Texas, Austin, Texas 78712
and Department of Physics, University of Texas, Austin, Texas 78712
(Received 27 March 1992)

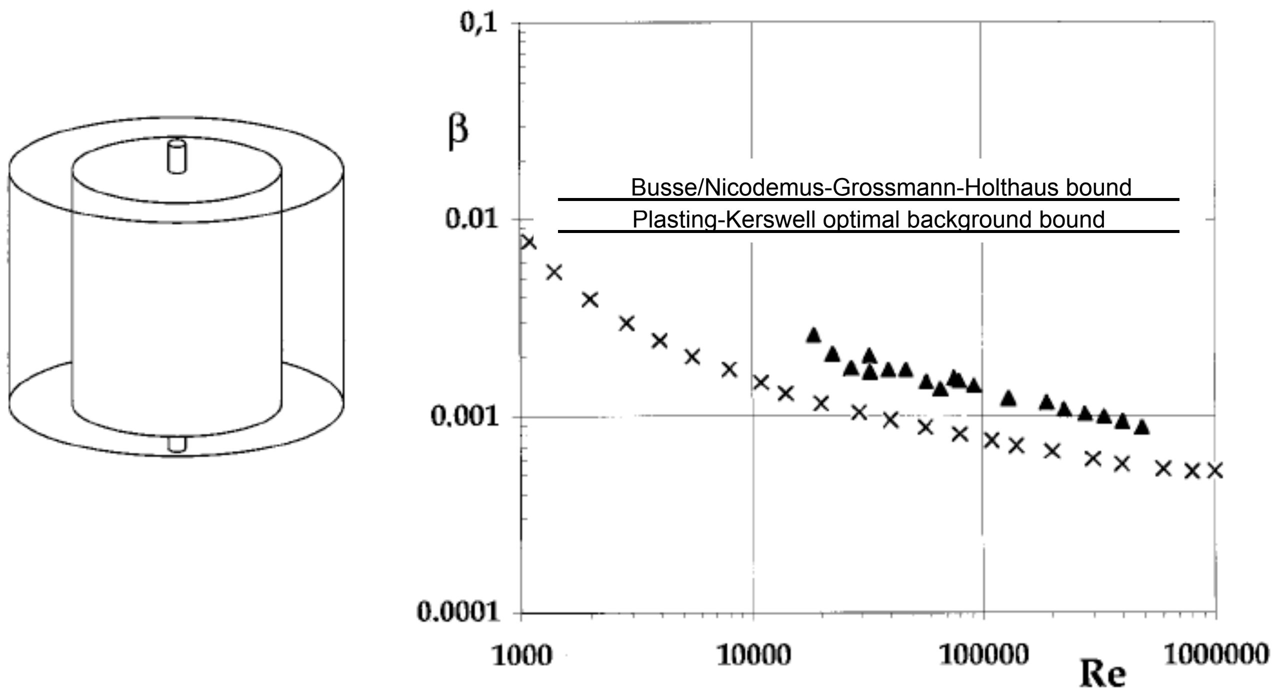
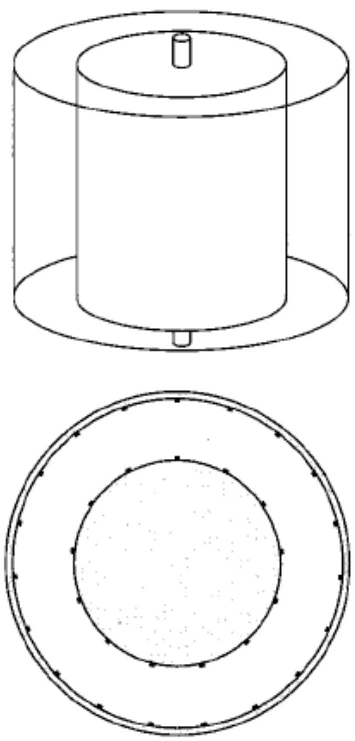


FIG. 5. Couette-Taylor experiments. Logarithmic plots of the nondimensional rates of energy dissipation β_D as a function of the Reynolds number.

Energy injection in closed turbulent flows: Stirring through boundary layers versus inertial stirring

O. Cadot,* Y. Couder, A. Daerr, S. Douady, and A. Tsinober†
Laboratoire Physique Statistique, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris, France



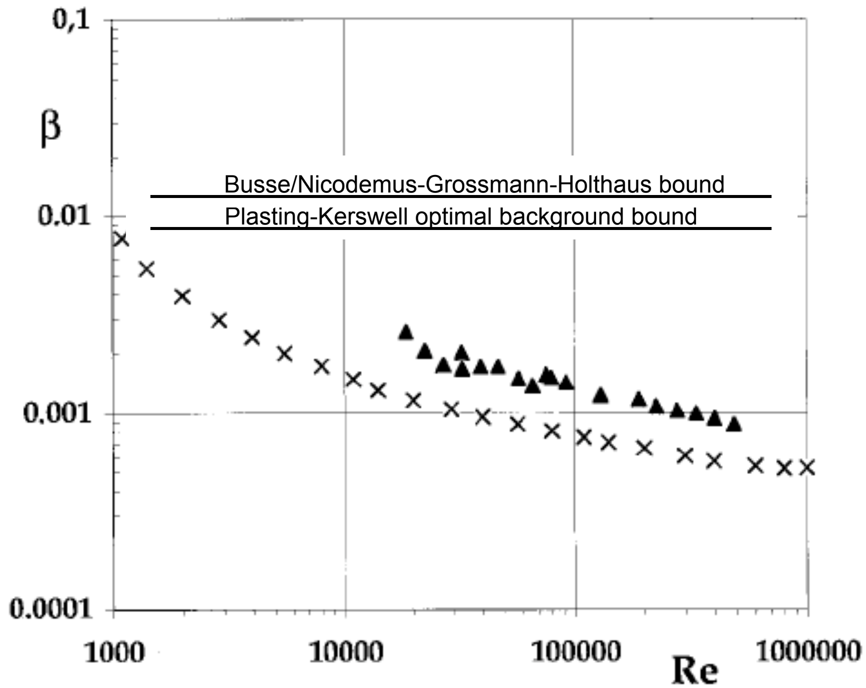


FIG. 5. Couette-Taylor experiments. Logarithmic plots of the nondimensional rates of energy dissipation β_D as a function of the Reynolds number. The black triangles (▲) are the results obtained with smooth cylinders, and the open ones (△) correspond to those obtained with the ribbed ones. The crosses (×) show for comparison the rates of energy injection β_D deduced from the data obtained with smooth cylinders by Lathrop, Finenberg, and Swinney [8].

PHYSICAL REVIEW E **68**, 036307 (2003)

Smooth and rough boundaries in turbulent Taylor-Couette flow

Thomas H. van den Berg,[1,4] Charles R. Doering,[2,3] Detlef Lohse,[1] and Daniel P. Lathrop[4]
[1]*Department of Applied Physics and J. M. Burgers Centre for Fluid Dynamics, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands*
[2]*Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109-1109, USA*
[3]*Michigan Center for Theoretical Physics, Ann Arbor, Michigan 48109-1120, USA*
[4]*Department of Physics, IREAP and IPST, University of Maryland, College Park, Maryland 20742, USA*

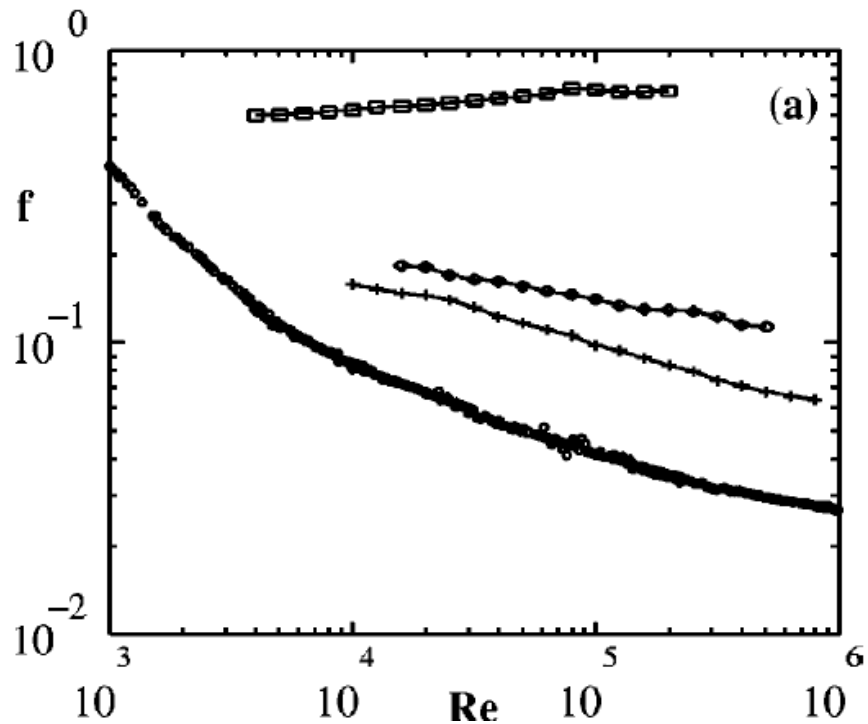


FIG. 3. (a) Skin friction coefficient f vs Reynolds number Re for the four cases (○) ss, (+) sr, (◇) rs, and (□) rr, bottom to top.

Energy dissipation in a shear layer with suction*

Charles R. Doering[a)]
Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109-1109

Edward A. Spiegel
Department of Astronomy, Columbia University, New York, New York 10027

Rodney A. Worthing
Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109-1109

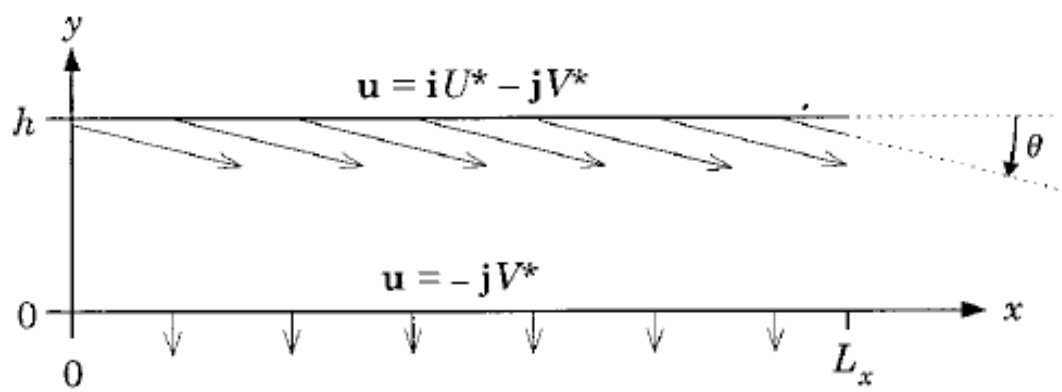


FIG. 1. Sketch of the boundaries and boundary conditions for the flows under consideration.

A simple exact steady solution of the problem is the laminar flow $\mathbf{u}_\ell = U_\ell(y)\mathbf{i} - V^*\mathbf{j}$ with

$$U_\ell(y) = U^* \frac{1-e^{-V^* y/\nu}}{1-e^{-\mathrm{Re}\tan\theta}}.$$

$$\delta_\ell = \frac{\nu}{V^*} = \frac{h}{\mathrm{Re}\tan\theta}$$

$$\varepsilon = \frac{\nu}{hA}\langle ||\nabla \mathbf{u}||^2 \rangle$$

$$\varepsilon_\ell = \frac{\nu}{h}\int_0^h \left(\frac{\partial u_x}{\partial y}\right)^2 dy$$

$$= \frac{U^{*2}V^*}{2h} \frac{1}{\tanh((1/2)\mathrm{Re}\tan\theta)}$$

$$= \frac{U^{*3}}{h} \frac{\tan\theta}{2\tanh((1/2)\mathrm{Re}\tan\theta)}.$$

$$\lim_{\nu\to 0} \varepsilon_\ell = \frac{\tan\theta}{2}\frac{U^{*3}}{h} \quad (\theta \neq 0).$$

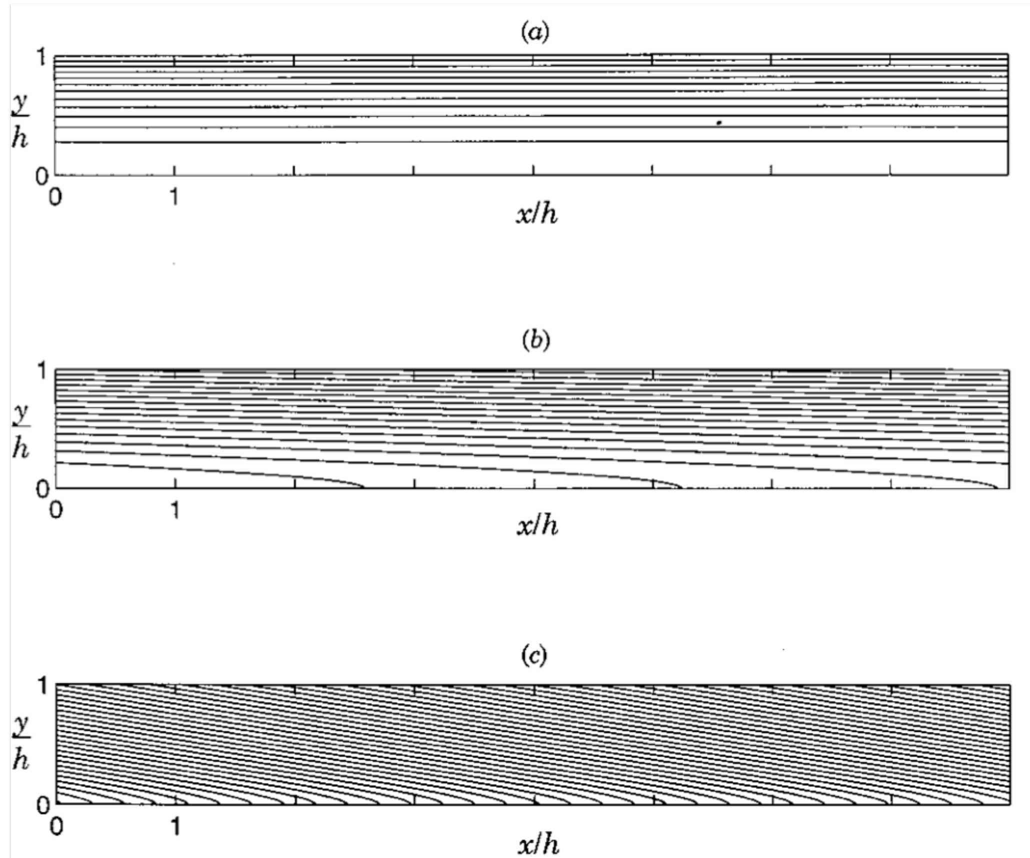


FIG. 2. Streamlines for the steady laminar flow at several parameter values. (a) Plane Couette flow, Re = 100 and $\theta = 0$. (b) Re = 99.99 and $\theta = 0.9°$, with laminar boundary layer thickness $\delta_\ell \approx 0.64h$. (c) Re = 98.77 and $\theta = 9°$, with laminar boundary layer thickness $\delta_\ell \approx 0.064h$.

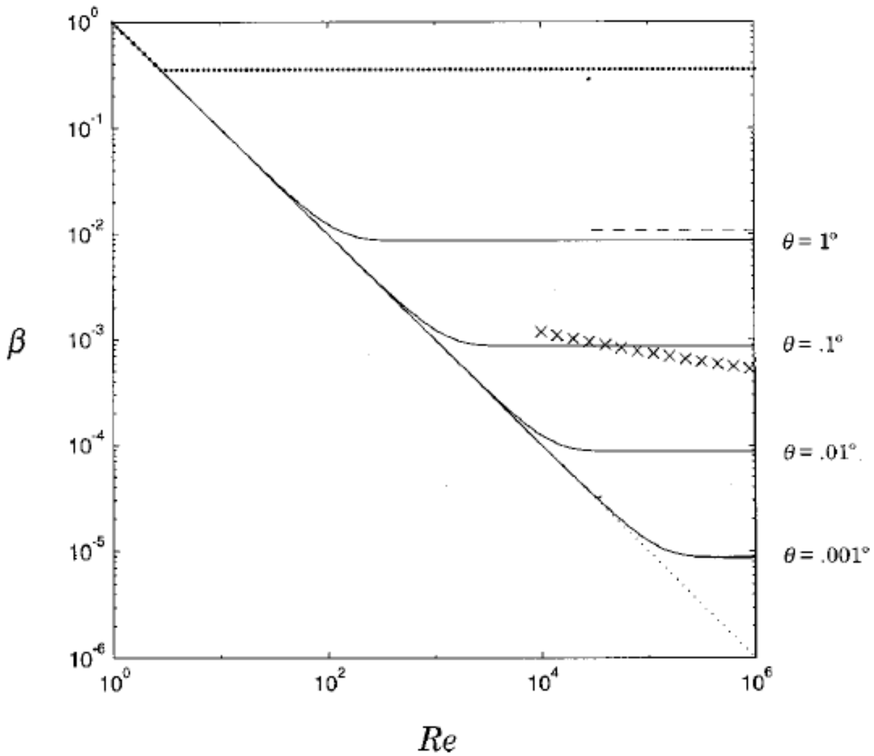


FIG. 6. The dissipation factor $\beta=\varepsilon\times h/U^{*3}$ vs Reynolds number Re. The discrete data $(\cdots)$ at the top are the rigorous upper bound $\beta_B(\mathrm{Re},\theta)$ in Eq. (5.2) for injection angles $\theta=1°,0.1°,0.01°$, and $0.001°$ (there is very little sensitivity of the bound to changes in θ at small angles). The dashed line segment (- - -) is the best known high Re bound for turbulent Couette flow, $\beta_B(\mathrm{Re},0)\approx 0.01087$ (from Ref. 26). The crosses $(\times)$ show the fit in Eq. (5.4) to experimental data. The solid lines (—) are, from top to bottom, the dissipation factor in Eq. (5.6) for injection angles $\theta=1°,0.1°,0.01°$, and $0.001°$. The lower envelope to the curves is the dissipation factor for plane Couette flow, the only rigorous lower bound available for the dissipation factor for arbitrary injection angles.

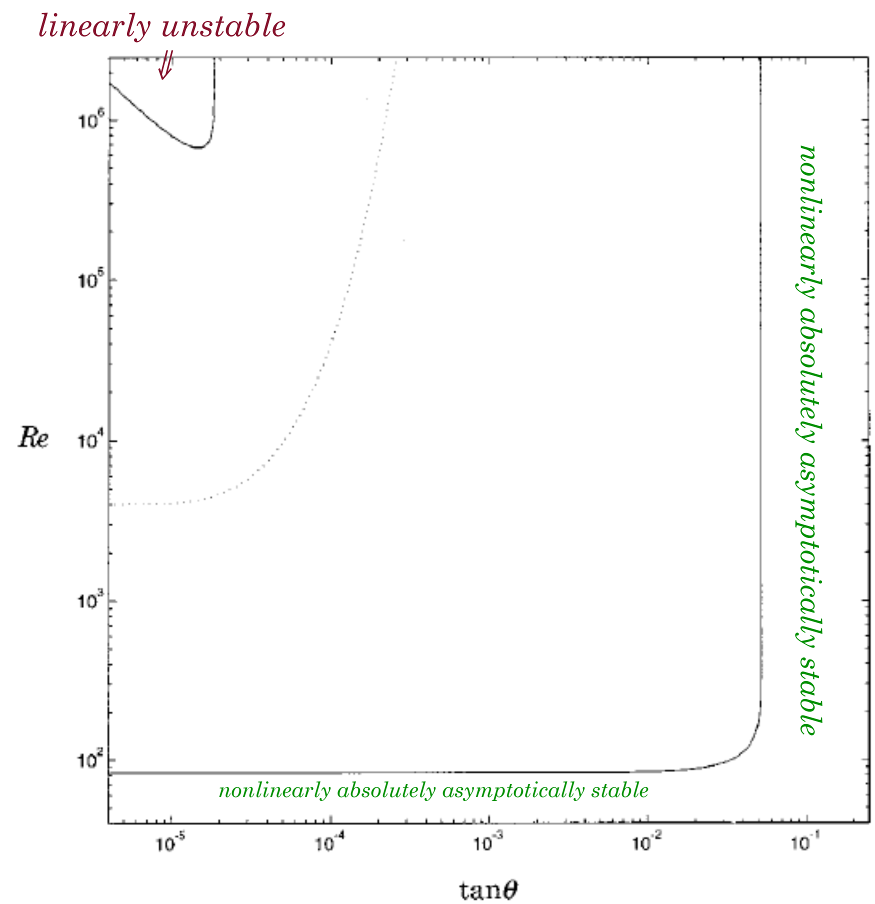


FIG. 5. Summary of the stability portrait in the Re$-\theta$ plane. The steady laminar flow is *absolutely stable* according to the energy method for Re <82 or $\theta>3°$ (with $\tan\theta\approx 0.05$). The steady laminar flow is *linearly unstable* in the indicated region in the upper left hand corner where $\theta<0.001°$ (with $\tan\theta\approx 0.0002$) and Re$\geqslant$700 000. The dotted line is a sketch of the conjectured nonlinear stability boundary for the steady laminar flow.

PHYSICS OF FLUIDS **22**, 034105 (2010)

Destabilizing Taylor–Couette flow with suction

Basile Gallet,[1] Charles R. Doering,[2] and Edward A. Spiegel[3]
[1]*Laboratoire de Physique Statistique, Ecole Normale Supérieure, 75231 Paris, France*
[2]*Department of Mathematics, Department of Physics, and Center for the Study of Complex Systems, University of Michigan, Ann Arbor, Michigan 48109-1043, USA*
[3]*Department of Astronomy, Columbia University, New York, New York 10027, USA*

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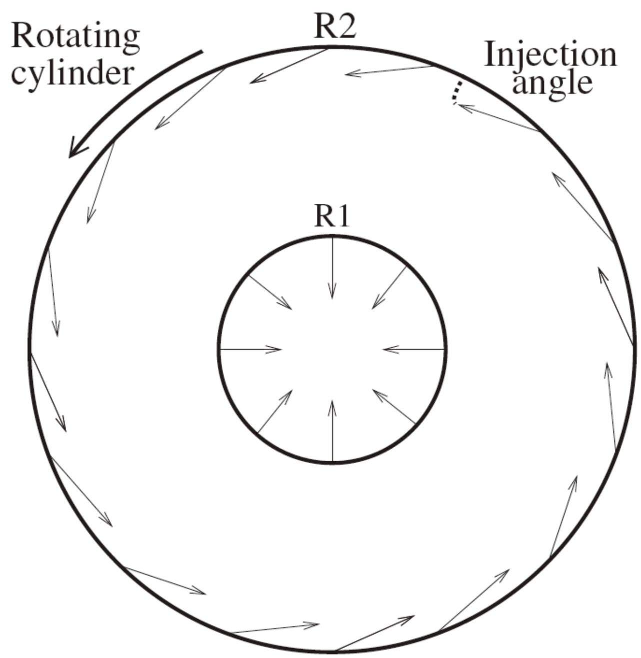
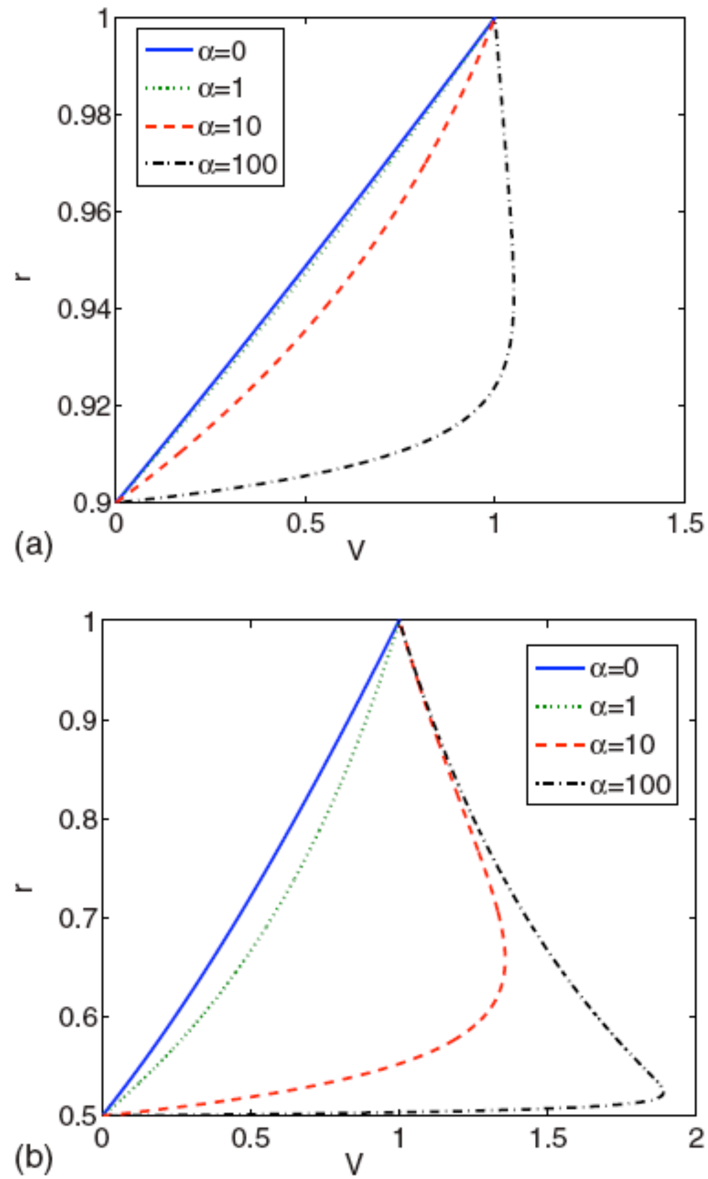


FIG. 1. The outer cylinder rotates at angular velocity Ω. Fluid is injected at the outer boundary with an entry angle $\Theta = \arctan[\varphi/(R_2^2\Omega)]$ and removed uniformly on the surface of the inner cylinder.



$$\alpha = \frac{\mathrm{Re} \tan \Theta}{1 - \eta}$$

FIG. 2. (Color online) Azimuthal velocity profiles for different values of the radial Reynolds number (top: η=0.9, bottom: η=0.5).

The geometrical factor is $\eta = R_1/R_2$. When $\eta \rightarrow 1$ we approach the narrow-gap limit where $(R_2 - R_1) \ll R_1$, and expect to find results similar to those from the slab geometry—namely, plane Couette flow with suction.

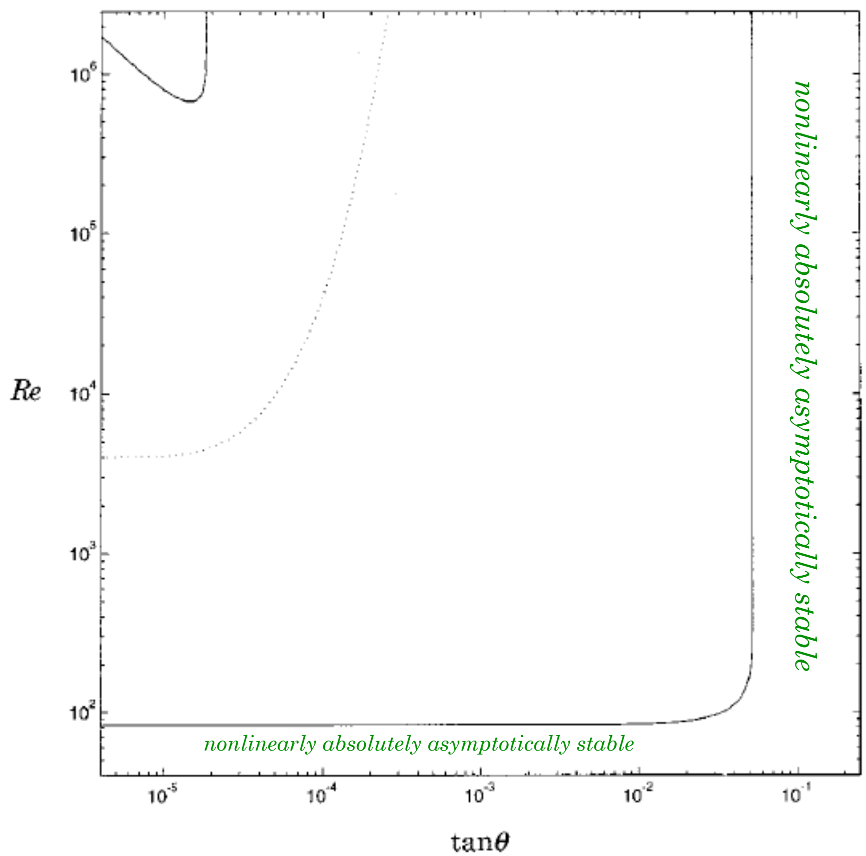


FIG. 5. Summary of the stability portrait in the Re$-\theta$ plane. The steady laminar flow is *absolutely stable* according to the energy method for Re <82 or $\theta > 3°$ (with $\tan\theta \approx 0.05$). The steady laminar flow is *linearly unstable* in the indicated region in the upper left hand corner where $\theta < 0.001°$ (with $\tan\theta \approx 0.0002$) and Re$\geqslant 700\,000$. The dotted line is a sketch of the conjectured nonlinear stability boundary for the steady laminar flow.

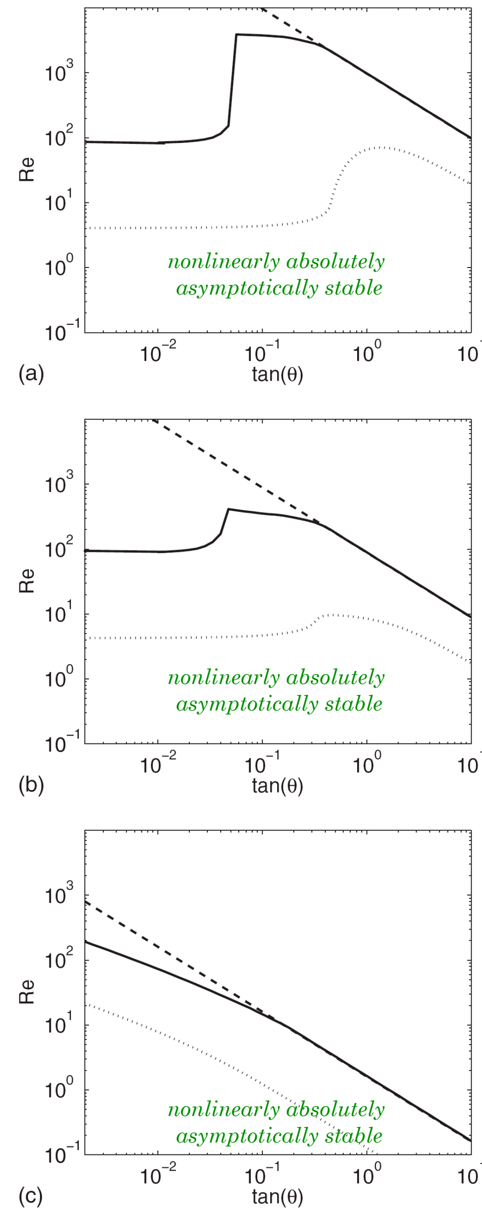


FIG. 9. Marginal energy stability boundary for different values of η (top: $\eta = 0.99$, middle: $\eta = 0.9$, bottom: $\eta = 0.02$). The solid line is the numerically computed energy stability boundary, the dashed line is the upper bound Re_1 on the location of that curve, and the dotted line is the lower bound Re_2.

Some questions & challenges:

- Does steady shear-suction *minimize* dissipation?
- $\$2^6$ prize problem = **$64** question!
- Does $\varepsilon = \mathcal{O}(1)$ as Re $\rightarrow \infty$ with flux at boundaries?
- $\$2^7$ prize problem = **$128** question!
- How do we bound the turbulent drag on a body?
- 

- Other turbulent transport & mixing problems …

Thanks for your attention!