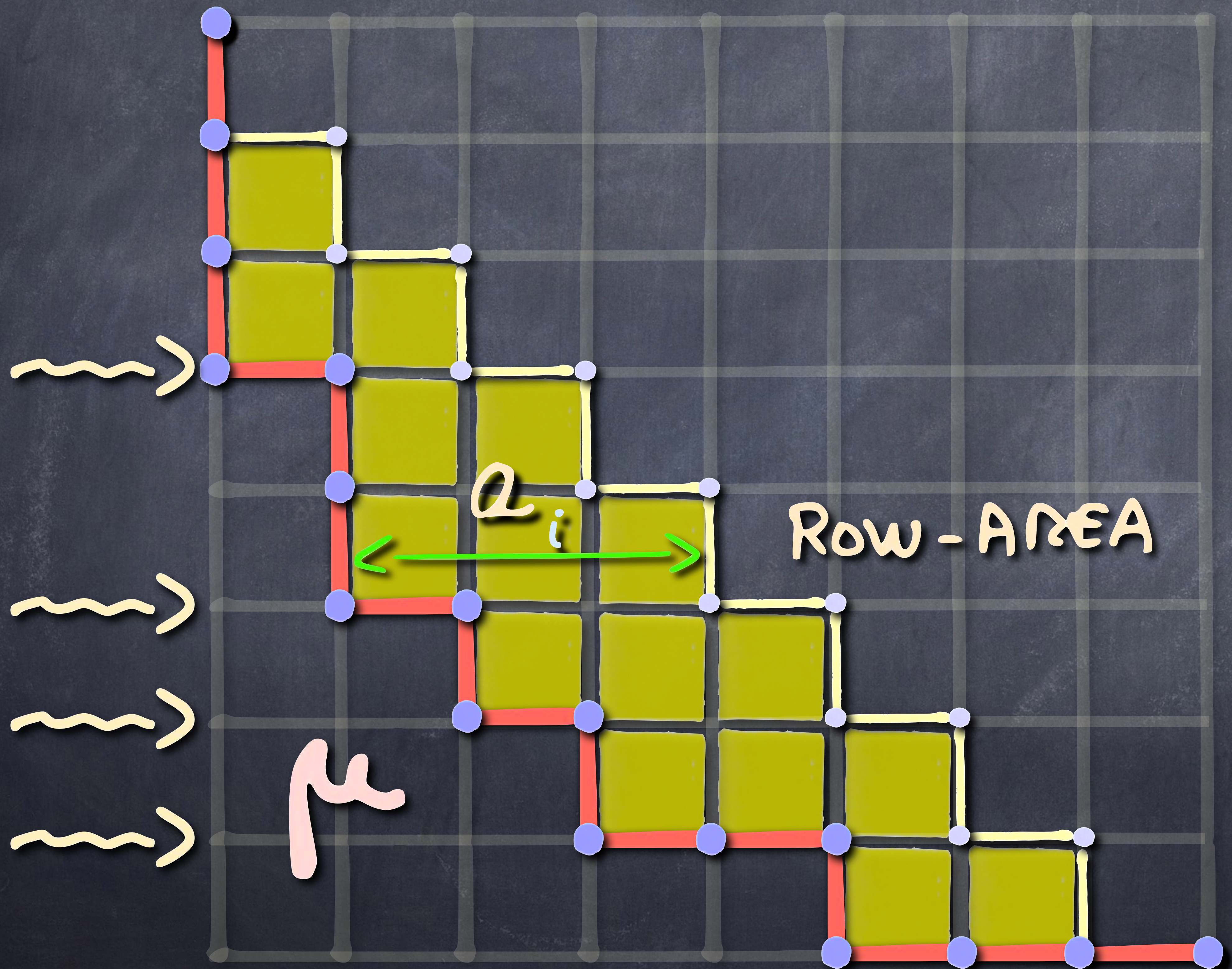
DESSERTS

FRANÇOIS BERGERON, LACIM

DESCENTS OF μ

i SUCH THAT $\mu_i > \mu_{i+1}$



$a_8 = 0$

$a_7 = 1$

$a_6 = 2$

$a_5 = 2$

$a_4 = 3$

$a_3 = 3$

$a_2 = 3$

$a_1 = 2$

FRANÇOIS BERGERON, LACIM

THE Δ-CONJECTURE


HAGLUND

$$\sum_{\mu \leq \delta_n} \Big(\sum_{J} q^{(J|a)} \Big) \mathbb{L}_\mu(t;z) = \Delta'_{e_k}(e_n)$$

$$\mathrm{Desc}(\mu) \subseteq J \subseteq \{1,2,\ldots,n-1\}$$

$$\# J = k$$

$$(J|a) = \sum_{i \in J} a_i \qquad \mathbb{L}_\mu(t;z) = \sum_{\tau \in SSYT((\mu+1^n)/\mu)} t^{\mathrm{Dinv}(\tau)} z_\tau$$

$$
\begin{aligned}
\sum_{\mu \leq \delta_4} \Big(\sum_J q^{(J|a)} \Big) \mathbb{L}_\mu(t;z) = {} & (q^3+q^4+q^5)\ \mathbb{L}_{0000}(t;z) \\
& + (q^3+q^4)\mathbb{L}_{1000}(t;z) + (q^2+q^3)\mathbb{L}_{2000}(t;z) \\
& + (q^2+q^3)\mathbb{L}_{1100}(t;z) + (q+q^2)\mathbb{L}_{3000}(t;z) \\
& + (q+q^2)\mathbb{L}_{1110}(t;z) + q^2\mathbb{L}_{2100}(t;z) \\
& + q\mathbb{L}_{3100}(t;z) + 2q\mathbb{L}_{2200}(t;z) \\
& + q\mathbb{L}_{2110}(t;z) + \mathbb{L}_{2210}(t;z) \\
& + \mathbb{L}_{3110}(t;z) + \mathbb{L}_{3200}(t;z)
\end{aligned}
$$

LLT-POLYNOMIALS

$$\mathbb{L}_{0000} = s_{1111}$$

$$\mathbb{L}_{1000} = ts_{1111} + s_{211}$$

$$\mathbb{L}_{2000} = t^2s_{1111} + ts_{211}$$

$$\mathbb{L}_{1100} = ts_{1111} + s_{211} + s_{22}$$

$$\mathbb{L}_{3000} = t^2s_{1111} + ts_{211}$$

$$\mathbb{L}_{2100} = t^3s_{1111} + \left(t^2 + t\right) s_{211} + ts_{22} + s_{31}$$

$$\mathbb{L}_{1110} = ts_{1111} + s_{211}$$

$$\mathbb{L}_{3100} = t^4s_{1111} + \left(t^3 + t^2\right) s_{211} + t^2s_{22} + ts_{31}$$

$$\mathbb{L}_{2200} = t^3s_{1111} + t^2s_{211} + ts_{22}$$

$$\mathbb{L}_{2110} = t^2s_{1111} + 2ts_{211} + s_{22} + s_{31}$$

$$\mathbb{L}_{3200} = t^5s_{1111} + \left(t^4 + t^3\right) s_{211} + t^3s_{22} + t^2s_{31}$$

$$\mathbb{L}_{3110} = t^4s_{1111} + \left(t^3 + t^2\right) s_{211} + t^2s_{22} + ts_{31}$$

$$\mathbb{L}_{2210} = t^3s_{1111} + \left(t^2 + t\right) s_{211} + ts_{22} + s_{31}$$

$$\mathbb{L}_{3210} = t^6s_{1111} + \left(t^5 + t^4 + t^3\right) s_{211} + \left(t^4 + t^2\right) s_{22} + \left(t^3 + t^2 + t\right) s_{31} + s_4$$

$$\boxed{\Delta'_{e_k}(e_n)}$$

$$\begin{aligned}
\Delta'_{e_2}e_4 = {} & (1+s_1+s_2)\otimes s_{31}\\
& +(s_{11}+s_1+s_2+s_3)\otimes s_{22}\\
& +(s_{11}+s_{21}+s_1+2s_2+2s_3+s_4)\otimes s_{211}\\
& +(s_{11}+s_{21}+s_{31}+s_3+s_4+s_5)\otimes s_{1111}
\end{aligned}$$

SCHUR ⊗ SCHUR - Positivity ?

ZABROCKI

(2019)

CONJECTURED MODULE FOR

$\Delta'_{e_k}(e_m)$

$\Downarrow$

$\Delta'_{e_k}(e_m)$ SCHUR $\otimes$ SCHUR - POSITIVITY

GENERIC CHARACTER ξ_n

IRRED. FOR
GL_∞ - ACTION

$$\xi_n = \sum_{\mu\vdash n}\sum_{\lambda} c_{\lambda\mu}\,(s_\lambda\otimes s_\mu),$$

IRRED. FOR
$\mathbb{S}_n$ - ACTION

$$c_{\lambda\mu}\in\mathbb{N}$$

$$\begin{aligned}\xi_4 = 1\otimes s_4 &+ (s_1+s_2+s_3)\otimes s_{31}\\ &+ (s_{21}+s_2+s_4)\otimes s_{22}\\ &+ (s_{11}+s_{21}+s_{31}+s_3+s_4+s_5)\otimes s_{211}\\ &+ (s_{111}+s_{31}+s_{41}+s_6)\otimes s_{1111}\end{aligned}$$

FRANÇOIS BERGERON, LACIM

$$
\begin{aligned}
\xi_4 = 1 \otimes \Delta_4 &+ (\Delta_1 + \Delta_2 + \Delta_3) \otimes \Delta_{31} \\
&+ (\Delta_{21} + \Delta_2 + \Delta_4) \otimes \Delta_{22} \\
&+ (\Delta_{11} + \Delta_{21} + \Delta_{31} + \Delta_3 + \Delta_4 + \Delta_5) \otimes \Delta_{211} \\
&+ (\Delta_{111} + \underbrace{\Delta_{31} + \Delta_{41} + \Delta_6}_{q,t\text{-CATALAN}}) \otimes \Delta_{1111}
\end{aligned}
$$

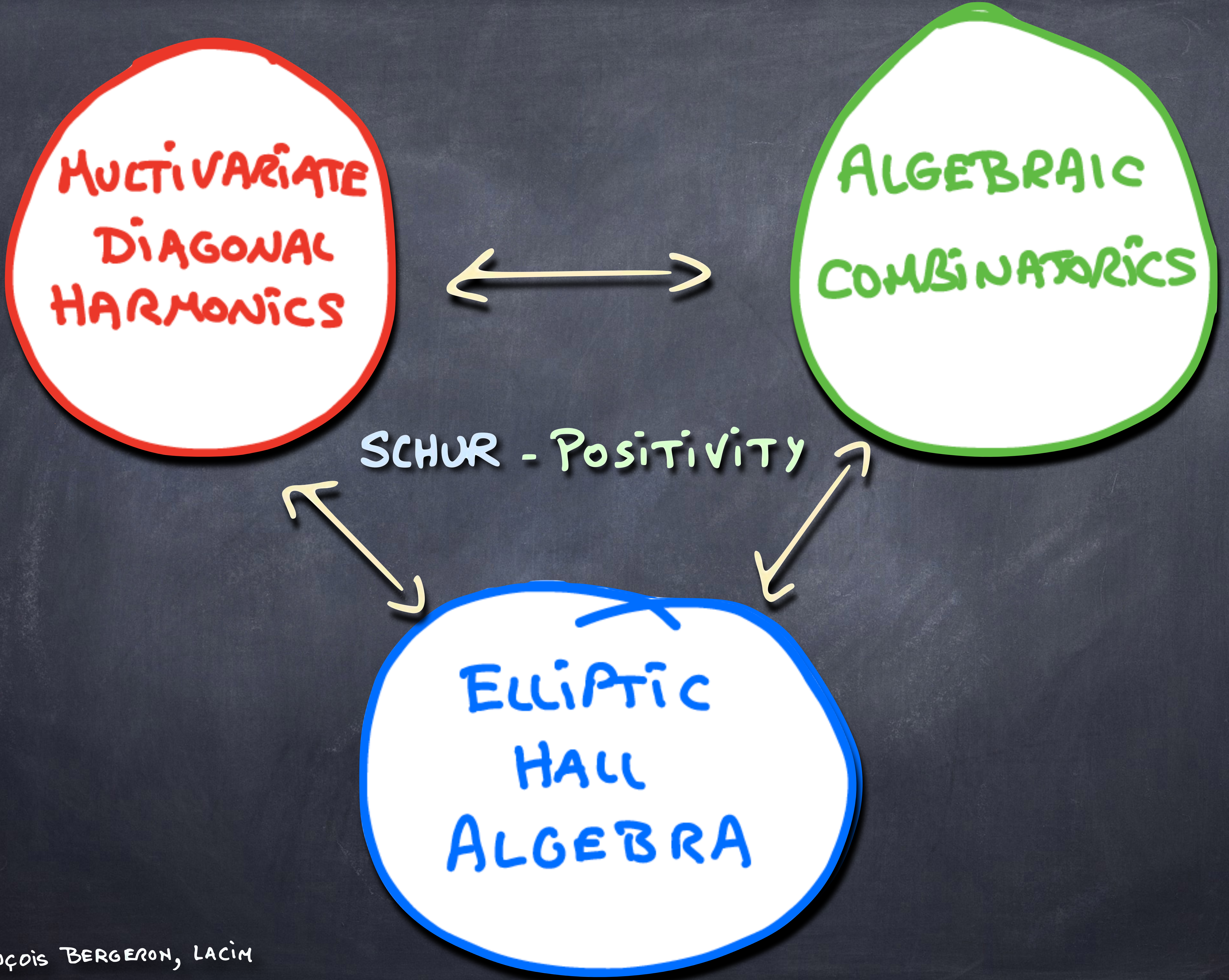
WE OBSERVE THAT

$$
\boxed{\Delta'_{e_2} e_4 = (e_1^{\perp} \otimes \mathrm{Id})\, \xi_4}
$$

$$\boxed{\Delta'_{e_2} e_4 = (e_1^{\perp} \otimes \mathrm{Id})\,\xi_4}$$

CONJECTURE (F.B. 2018)

$$\Delta'_{e_{n-1-k}}\, e_n =_{(2)} (e_k^{\perp} \otimes \mathrm{Id})\,\xi_n$$



François Bergeron, LaCIM

RECTANGULAR
CATALAN
COMBINATORICS

MONOMIAL-POSITIVITY
⟷

ELLIPTIC
HALL
ALGEBRA

THE (m, n)-SHUFFLE CONJECTURE

n


HAGLUND


HAIMAN


LOEHR


REMMEL


ULYANOV

(m, n)

F.B.


GARSIA


LEVEN


XIN

FRANÇOIS BERGERON, LACIM

RECTANGULAR
CATALAN
COMBINATORICS

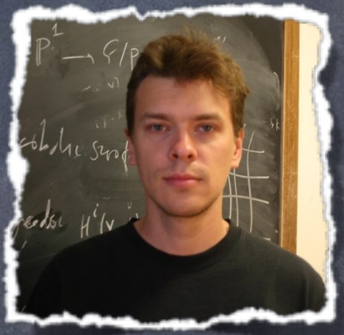
MONOMIAL-POSITIVITY

⟷

ELLIPTIC
HALL
ALGEBRA

THE (m,n)-SHUFFLE THEOREM

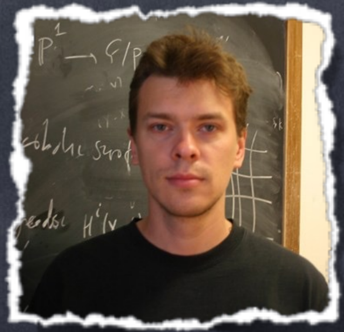
n



MELLIT

CARLSSON

(m,n)



MELLIT

FRANÇOIS BERGERON, LACIM

THE (m,n)-SHUFFLE THEOREM

ELLIPTIC HALL ALGEBRA ⟷ RECTANGULAR CATALAN COMBINATORICS

$$e_{mn}(q,t;z) = \sum_{\mu\subseteq\delta_{mn}} q^{\mathrm{AREA}(\mu)} \underbrace{\sum_{\tau\in \mathrm{SSYT}((\mu+1^m)/\mu)} t^{\mathrm{DINV}(\tau)}\, z_\tau}_{(*)}$$

IS SCHUR-POSITIVE

HERE, BOTH AREA AND DINV DEPEND ON m AND n

(*) TO BE DISCUSSED

ELLIPTIC
HALL
ALGEBRA

$e_{mn}(q,t;z)$

OBTAINED VIA AN OPERATOR REALISATION OF THE ELLIPTIC HALL ALGEBRA.


BURBAN


VASSEROT


SCHIFFMANN

MULTIVARIATE DIAGONAL HARMONICS

CONJECTURE

(F.B. 2018)

↔

ELLIPTIC HALL ALGEBRA

THERE IS A MODULE(*) $\mathcal{E}_{mn}$ SUCH THAT

FOR ALL m AND n

$$\mathcal{E}_{mn}(q,t;z) = e_{mn}(q,t;z)$$

SCHUR - POSITIVITY

(*) TO BE DESCRIBED

ELLIPTIC
HALL
ALGEBRA

$$e_{mn}(q,t;z)$$

OPEN QUESTIONS FOR OPERATORS
RELATED TO RECTANGULAR CATALAN COMBINATORICS

JOURNAL OF COMBINATORICS
VOL 8, NO 4 (2017)

FRANÇOIS BERGERON, LACIM

$$e_{m\,n}(q,t;z) := \Theta_{a,b}(e_d)(1)$$

$Q_{1,0} := D_0$ MACDONALD EIGENOPERATOR

$Q_{0,1} :=$ MULTIPLICATION BY $e_1(x)$

$$Q_{m,n} := \frac{1}{(1-t)(1-q)}\left[Q_{a,b}, Q_{c,d}\right]$$

$(m,n)=(a,b)+(c,d)$ $\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \mathrm{GCD}(m,n)$

FRANÇOIS BERGERON, LACIM

$$\boxed{e_{mn}(q,t;z) := \Theta_{a,b}(e_d)(1)}$$

$$Q_{4,3} = \frac{1}{(1-t)^6(1-q)^6}\big[[Q_1,D_0],[[Q_1,D_0],[[Q_1,D_0],D_0]]\big]$$

FACT

$$\boxed{Q_{ac,bc} \quad \text{COMMUTES WITH} \quad Q_{ad,bd}}$$

$$Q_{(a,b)\mu} := Q_{a\mu_1,b\mu_1} Q_{a\mu_2,b\mu_2} \cdots Q_{a\mu_k,b\mu_k}$$

François Bergeron, LaCIM

$$\boxed{e_{mn}(q,t;z) := \Theta_{a,b}(e_d)(1)}$$

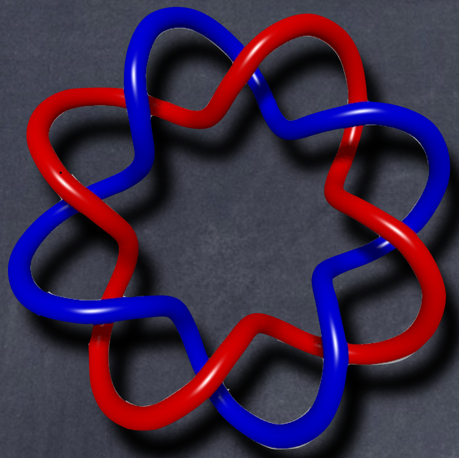
$$\Theta_{a,b}(e_d) := \sum_{\mu\vdash d} f_\mu\, Q_{(a,b)\mu}$$

$$e_d(z) = \sum_{\mu\vdash d} f_\mu\, \Pi_\mu(z)$$

$$\boxed{\begin{aligned}
\Pi_d &:= s_{11\cdots 1} - \frac{1}{qt}\, s_{21\cdots 1} + \cdots + \left(\frac{-1}{qt}\right)^{d-1} s_d\\
\Pi_\mu &:= \Pi_{\mu_1}\Pi_{\mu_2}\cdots\,\Pi_{\mu_\ell}\\
\Pi_d(x)\Big|_{qt=1} &= (-1)^{d-1}\, p_d(x)
\end{aligned}}$$

LINKS TO MACDONAL POLYNOMIALS AND OPERATORS

- $\nabla e_{mn}(q,t;z) = e_{m+n,n}(q,t;z)$
- $e_{0n}(q,t;z) = e_n(z)$



THE SUPERPOLYNOMIAL OF THE (m,n)-TORUS LINK

KHOVANOV-ROZANSKY HOMOLOGY OF (m,n)-TORUS LINKS

$$(1+a)\sum_{k=0}^{n-1} \langle e_{m,n}, s_{(k+1,1^{n-k-1})}\rangle\, a^k$$

(m,n)-TORUS LINK $=$ (n,m)-TORUS LINK

GORSKY

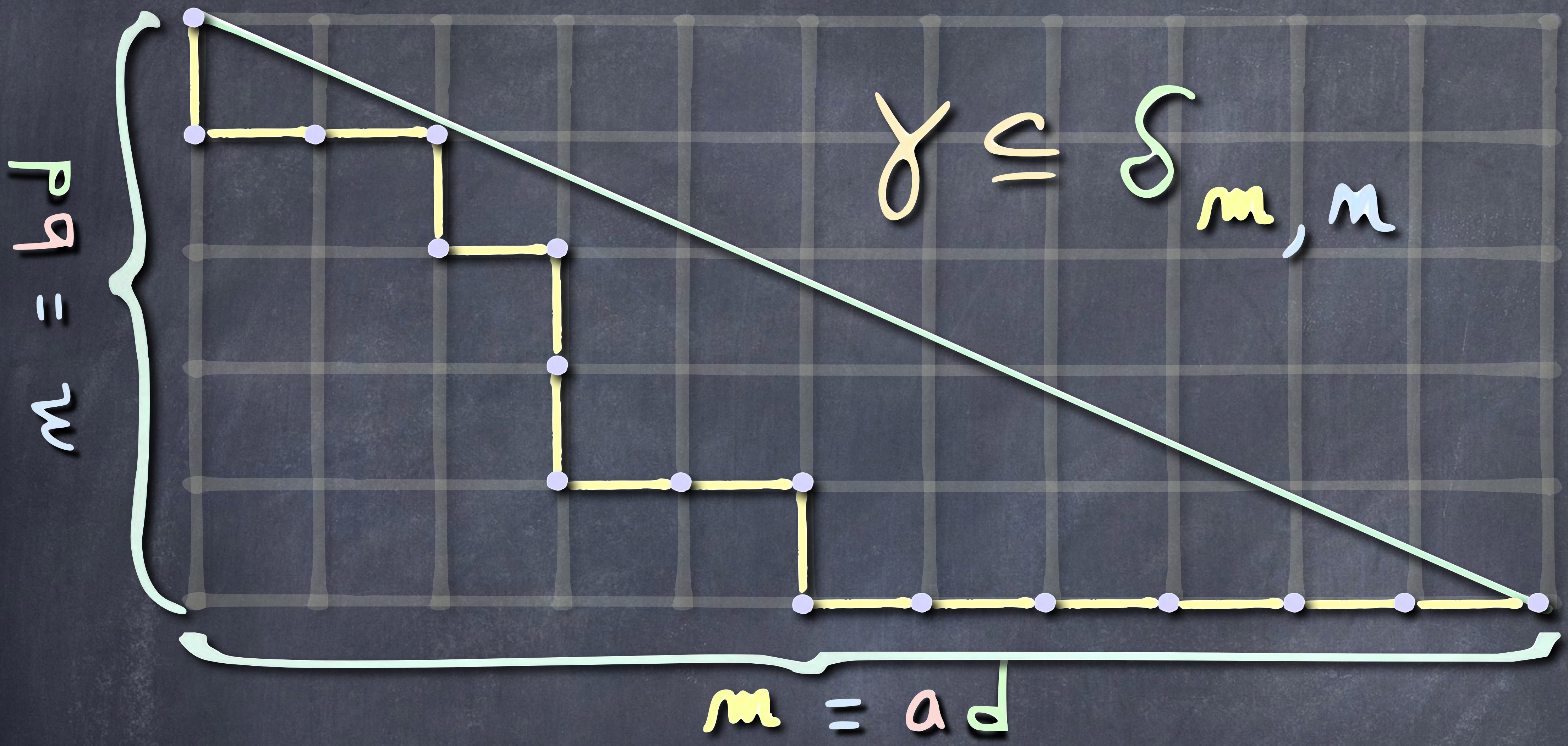
NEGUT

HOGANCAMP

COMBINATÓRIA DE CATALAN RETANGULAR

FRANÇOIS BERGERON, LACIM

$\mathrm{Cat}_{m\,n}$ NUMBER OF (m,n)-DYCK PATHS



$m = ad$

$n = bd$

$a \perp b$

COPRIME

(m,n)-STAIRCASE $\delta_{m,n} := r_1 r_2 \cdots r_n$

$$r_k := \lfloor m\,(n-k)/n \rfloor$$

FRANÇOIS BERGERON, LACIM

BIZLEY'S FORMULA

$$\boxed{\mathrm{CAT}_{(ad,bd)} = \sum_{\lambda \vdash d} \frac{1}{z_\lambda} \prod_{\substack{k \in \lambda \\ \uparrow}} \frac{1}{a+b} \binom{ak+bk}{ak}}$$

PARTS

$m = ad$

$n = bd$

$a \perp b$

COPRIME

$$z_\lambda := 1^{\kappa_1}\kappa_1!\ 2^{\kappa_2}\kappa_2!\ \cdots\ d^{\kappa_d}\kappa_d!$$

$$\kappa_i := \#\ \text{PARTS OF } \lambda \text{ OF SIZE } i$$

FRANÇOIS BERGERON, LACIM

$$e_{mn}(q,t;Z) = \sum_{\mu \subseteq \delta_{mn}} q^{\mathrm{AREA}(\mu)} \mathbb{L}_\mu(t;Z)$$

$$\mathbb{L}_\mu(t;Z) = \sum_{\tau \in \mathrm{SSYT}((\mu+1^m)/\mu)} t^{\mathrm{Dinv}(\tau)} Z_\tau$$

FRANÇOIS BERGERON, LACIM

$$\mathbb{L}_\mu(1;z) = \Delta_{(\mu+1^n)/\mu}(z)$$

$$\Delta_{(\mu+1^n)/\mu}(z) = e_{\rho(\mu)}(z)$$

$\rho(\mu)$: PARTITION WHOSE PARTS ARE THE COLUMNS OF $(\mu+1^n)/\mu$

$$m = ad$$
$$n = bd$$

$$\boxed{\begin{aligned} &e_{mn}(1,1;z) = \\ &\sum_{\mu \subseteq \delta_{m,n}} \Delta_{(\mu+1^n)/\mu}(z) = \\ &\qquad \sum_{\lambda \vdash d} \frac{1}{z_\lambda} \prod_{k \in \lambda} \frac{1}{a}\, e_{kb}[kaz] \end{aligned}}$$

François Bergeron, LaCIM

$$\begin{tikzcd}
 & \boxed{\displaystyle \sum_{\mu\subseteq\delta_{m,n}} \Delta_{(\mu+1^n)/\mu}(Z) = \sum_{\lambda\vdash d} \frac{1}{z_\lambda} \prod_{k\in\lambda} \frac{1}{a}\, e_{kb}[kaZ]} \arrow[dl, "{\langle -, e_n(Z)\rangle}"'] \arrow[dd, "{\langle -, p_1^n(Z)\rangle}"] \\
\boxed{\displaystyle \mathrm{CAT}_{(ad,bd)} = \sum_{\lambda\vdash d} \frac{1}{z_\lambda} \prod_{k\in\lambda} \frac{1}{a+b} \binom{ad+bd}{ad}} & \\
 & \boxed{\displaystyle \sum_{\lambda\subseteq\delta_{m,n}} f^{(\mu+1^n)/\mu} = \sum_{\lambda\vdash d} \frac{1}{z_\lambda} \binom{n}{k\lambda} \prod_{k\in\lambda} \frac{1}{a} (ka)^{kb}}
\end{tikzcd}$$

$m = ad$

$n = bd$

(m,n)-Parking Functions

François Bergeron, LACIM

LLT-POLYNOMIALS

$$\mathbb{L}_\mu(t;z) = \sum_{\tau \in SSYT((\mu+1^m)/\mu)} t^{\mathrm{Dinv}(\tau)} z_\tau$$

FRANÇOIS BERGERON, LACIM

THM

HAIMAN

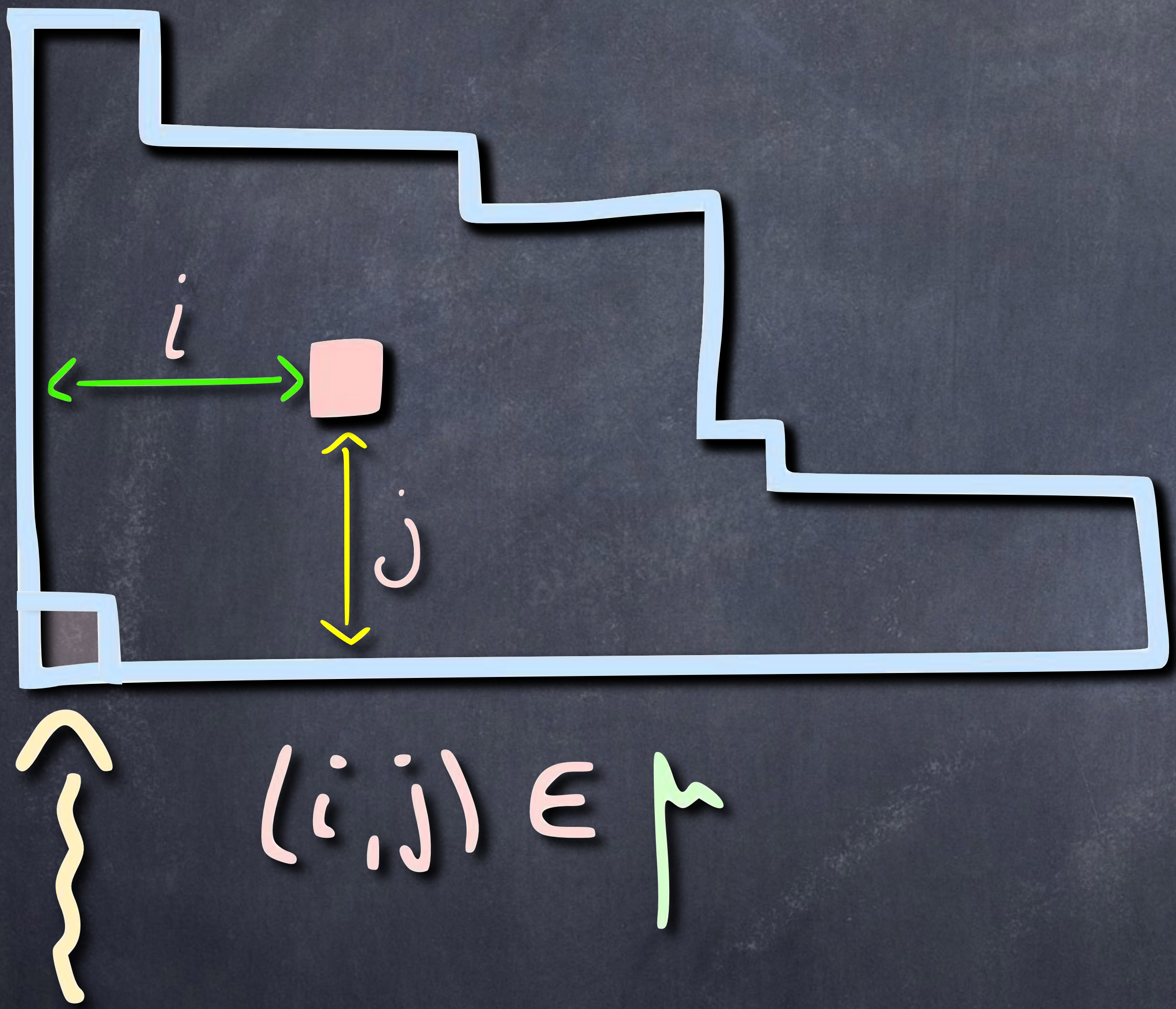
GROJNOVSKI

$\mathbb{L}_\mu(t;z)$ is SCHUR-POSITIVE

OBSERVATION

$\mathbb{L}_\mu(1+t;z)$ is e-POSITIVE

Δ_f OPERATORS WITH MACDONALD POLYNOMIALS AS JOINT EIGENFUNCTIONS



$(i,j) \in \mu$

$$\Delta_f H_\mu = \overbrace{f(\ldots, q^i t^j, \ldots)_{(i,j)\in\mu}}^{\text{EIGENVALUE}} H_\mu$$

$$\Delta'_f H_\mu = \overbrace{f(\ldots, q^i t^j, \ldots)_{\substack{(i,j)\in\mu \\ (i,j)\neq(0,0)}}}^{\text{EIGENVALUE}} H_\mu$$

FRANÇOIS BERGERON, LACIM

THE Δ-CONJECTURE


HAGLUND

$$\Delta'_{e_k}(e_n) = \sum_{\mu \subseteq \delta_n} \Big(\sum_{J} q^{(J|a)} \Big) \mathbb{L}_\mu(t;z)$$

$$\mathrm{DESC}(\mu) \subseteq J \subseteq \{1,2,\dots,n\}$$
$$\#J = k$$

$$(J|a) = \sum_{i \in J} a_i$$

$$(k = n-1) \Rightarrow (J|a) = \mathrm{AREA}(\mu)$$

$$\Longrightarrow$$ SCHUR - POSITIVITY

RECTANGULAR MULTIVARIATE MODULES OF DIAGONAL HARMONIC POLYNOMIALS

FRANÇOIS BERGERON, LACIM

THE MODULE $\mathcal{E}_{m,n}$

$\mathcal{M}_{m,n}$ SMALLEST MODULE CONTAINING

$$V_{m,n} := \det\left(x_i^a\,\theta_i^b\right)_{\substack{1\le i\le n\\ (a,b)\in \gamma_{m,n}}}$$

CLOSED UNDER

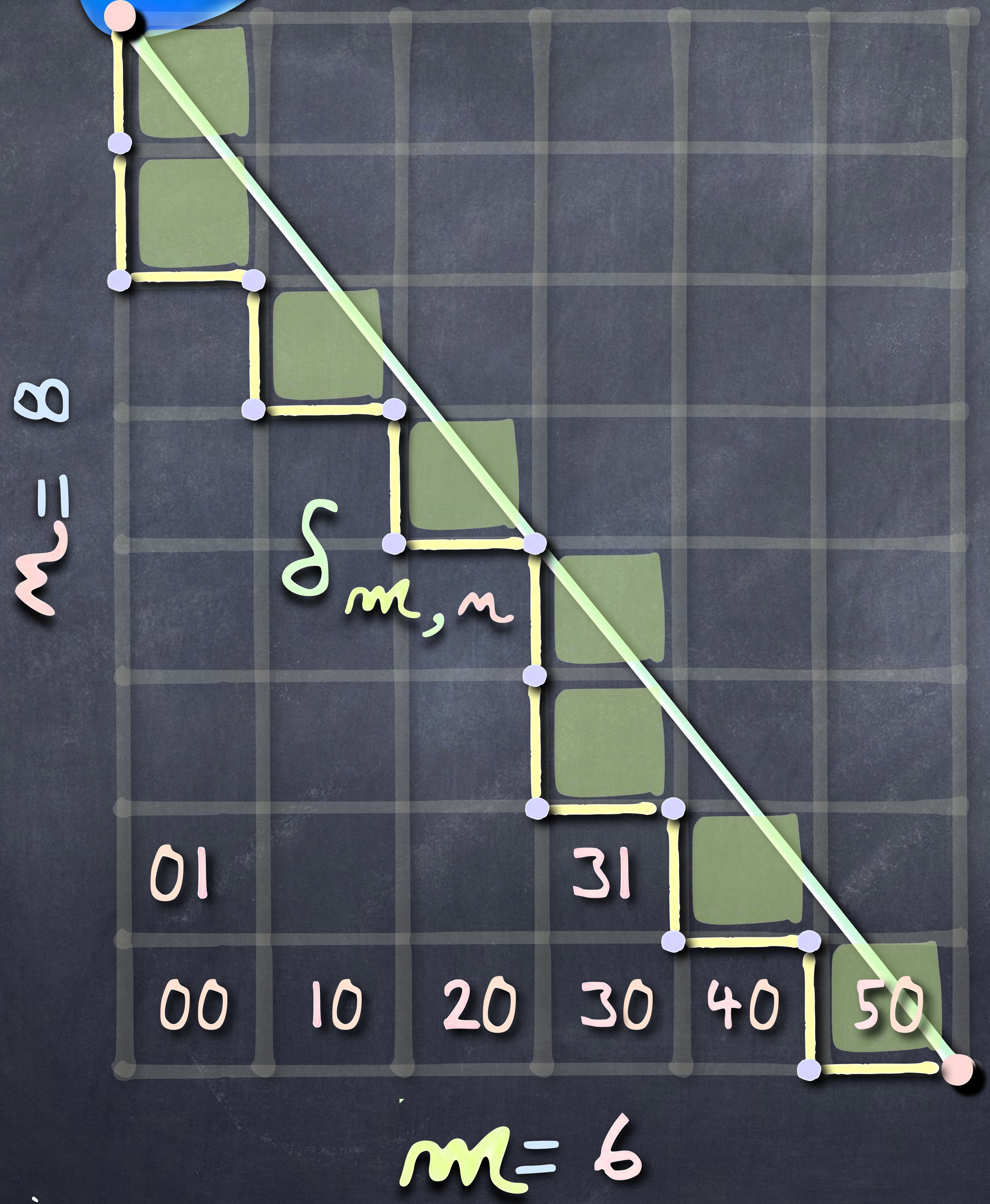
- PARTIAL DERIVATIVES
- POLARIZATION

$$\boxed{\theta_i \quad \text{INERT VARIABLES (DEGREE 0)}}$$

$\gamma_{m,n}$:= LIST OF COORDINATES ASSOCIATED TO PATH

FRANÇOIS BERGERON, LACIM

$\gamma_{m,n}$:= LIST OF COORDINATES ASSOCIATED TO PATH



FRANÇOIS BERGERON, LACIM

$V_{n-1,n}$

$$V_{n-1,n} := \det \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-2} & \theta_1 \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-2} & \theta_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n & \theta_n \end{pmatrix}$$

FRANÇOIS BERGERON, LACIM

$$\boxed{\mathcal{E}_{m,n} := \mathcal{M}_{m,n} / \Lambda^* \mathcal{M}_{m,n}}$$

$\Lambda^* \mathcal{M}_{m,n}$ SMALLEST MODULE CONTAINING

$$\sum_{i=1}^{m} \partial x_i^a \partial y_i^b \, V_{m,n} \qquad a+b \geq 1$$

$$\sum_{i=1}^{m} x_i \partial x_i^k \, V_{m,n} \qquad k \geq 2$$

FRANÇOIS BERGERON, LACIM

A TOY EXAMPLE $\mathcal{E}_{2,3}$

$$V_{2,3}(\overset{x}{y}) = \det\begin{pmatrix} 1 & \overset{x}{y}_1 & \theta_1 \\ 1 & \overset{x}{y}_2 & \theta_2 \\ 1 & \overset{x}{y}_3 & \theta_3 \end{pmatrix}$$

POLARIZATION

$$V_{2,3} = (x_2 - x_3)\theta_1 - (x_3 - x_1)\theta_2 + (x_2 - x_1)\theta_3$$

$$\partial x_1 V_{2,3} = \theta_2 - \theta_3 \qquad \partial x_2 V_{2,3} = \theta_3 - \theta_1$$

q $\quad$ t

$$\mathcal{E}_{2,3} = \mathbb{Q}\{\theta_2 - \theta_3, \theta_3 - \theta_1\} \oplus \mathbb{Q}\{V_{2,3}(x), V_{2,3}(y)\}$$

$$\mathcal{E}_{2,3}(q,t;z) = s_{21} + (q+t)\, s_{111}$$

e-POSITIVITY
AND
LOCAL Δ-CONJECTURE

FRANÇOIS BERGERON, LACIM

$$e_{mn}(q,1;z) = \sum_{\mu \subseteq \delta_{mn}} q^{\mathrm{AREA}(\mu)} \Delta_{(\mu+1^m)/\mu}(z)$$

OBSERVATION

$e_{mn}(q,1+t;z)$ is e-Positive

e-Positivity Phenomenon

$$\mathscr{F}_n := \sum_{k\geq 0} h_k^\perp \, \xi_n$$

$$= \sum_{\mu} \sum_{\lambda} d_{\lambda\mu} \, \Delta_\lambda \otimes e_\mu$$

$$d_{\lambda\mu} \in \mathbb{N}$$

$$
\begin{aligned}
\mathscr{F}_{13} &= 1 \otimes e_3,\\
\mathscr{F}_{23} &= s_1 \otimes s_3 + 1 \otimes e_{21},\\
\mathscr{F}_{33} &= (s_{11} + s_3) \otimes e_3 + (2s_1 + s_2) \otimes e_{21} + 1 \otimes e_{111},\\
\mathscr{F}_{53} &= (s_{21} + s_4) \otimes e_3 + (s_1 + s_{11} + 2s_2 + s_3) \otimes e_{21} + (1 + s_1) \otimes e_{111},\\
\mathscr{F}_{63} &= (s_{22} + s_{41} + s_6) \otimes e_3 + (2s_2 + 2s_{21} + s_3 + s_{31} + 2s_4 + s_5) \otimes e_{21}\\
&\qquad +(1 + 2s_1 + s_{11} + s_2 + s_3) \otimes e_{111},\\
\mathscr{F}_{83} &= (s_{32} + s_{51} + s_7) \otimes e_3\\
&\qquad +(s_2 + s_{21} + s_{22} + 2s_3 + 2s_{31} + s_4 + s_{41} + 2s_5 + s_6) \otimes e_{21}\\
&\qquad +(1 + 2s_1 + s_{11} + 2s_2 + s_{21} + s_3 + s_4) \otimes e_{111},
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{F}_{14} &= 1 \otimes e_4,\\
\mathscr{F}_{24} &= s_2 \otimes e_4 + s_1 \otimes e_{31} + 1 \otimes e_{22},\\
\mathscr{F}_{34} &= (s_{11} + s_3) \otimes e_4 + (s_1 + s_2) \otimes e_{31} + s_1 \otimes e_{22} + 1 \otimes e_{211},\\
\mathscr{F}_{44} &= (s_{111} + s_{31} + s_{41} + s_6) \otimes e_4 + (2s_{11} + s_{21} + 2s_3 + s_{31} + s_4 + s_5) \otimes e_{31}\\
&\qquad +(s_{11} + s_2 + s_{21} + s_4) \otimes e_{22} + (3s_1 + 2s_2 + s_3) \otimes e_{211} + 1 \otimes e_{1111},\\
\mathscr{F}_{64} &= (s_{311} + s_{42} + s_{51} + s_{61} + s_8) \otimes e_4\\
&\qquad +(s_{21} + s_{211} + 2s_{31} + s_{32} + s_4 + 2s_{41} + 2s_5 + s_{51} + s_6 + s_7) \otimes e_{31}\\
&\qquad +(s_{11} + s_{111} + s_2 + s_{21} + s_{22} + s_{31} + s_4 + s_{41} + s_6) \otimes e_{22}\\
&\qquad +(2s_1 + 2s_{11} + 2s_2 + 2s_{21} + 4s_3 + s_{31} + 2s_4 + s_5) \otimes e_{211}\\
&\qquad +(1 + s_1 + s_2) \otimes e_{1111},
\end{aligned}
$$

THERE SEEMS TO EXISTS A FAMILY OF MONOMIAL-POSITIVE FUNCTIONS $\sigma_m(\mu)$ SUCH THAT

$$\mathcal{F}_m = \sum_{\mu} \sigma_m(\mu) \otimes \Delta_{(\mu+1^m)/\mu}$$

F.B.

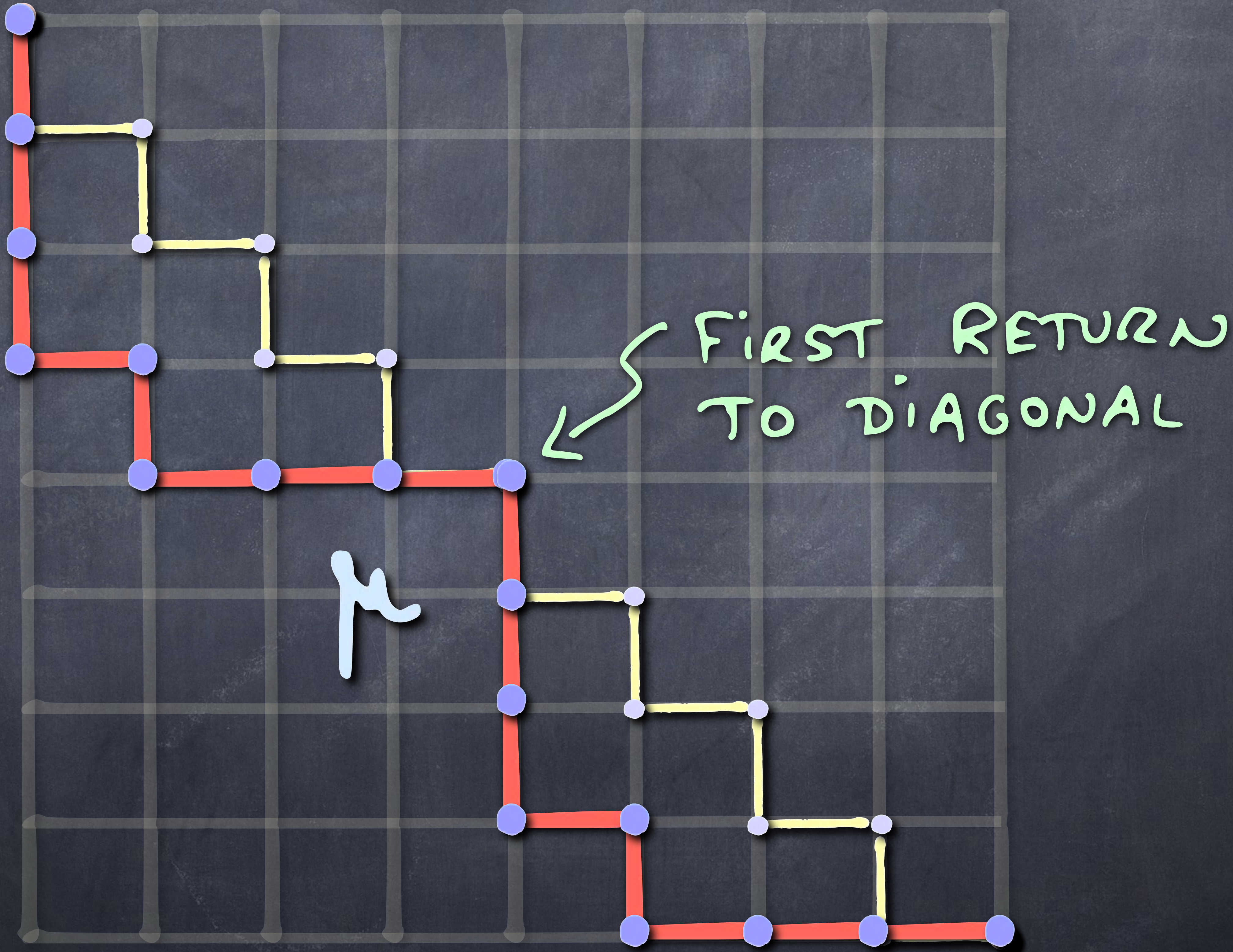
NANTEL

CEBALLOS

PILAUD

PROPERTIES

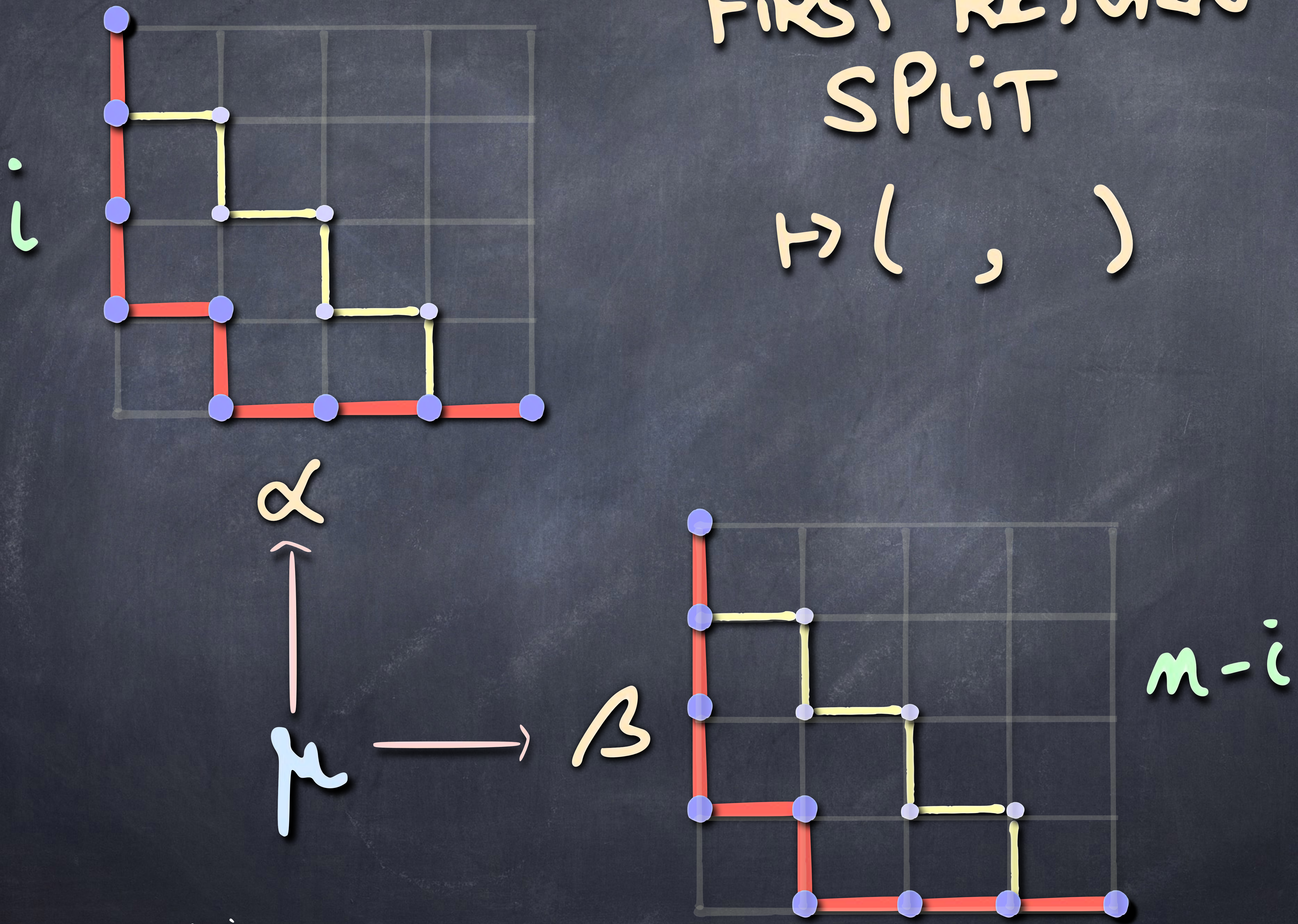
- $\ell(\sigma_n(\mu)) = n - \ell\left(\Delta_{(\mu+1^n)/\mu}\right)$
- $\sigma_n(0) = \langle \xi_{mn}, \Delta_{11-1} \rangle$
- $\sigma_n(\mu) = \sigma_n(\mu')$
- $e_k^\perp \sigma_n(0) = \sum_{\mathrm{DESC}(\mu)=[k]} \sigma_n(\mu)$
- $\sigma_n(0)[Q+1] = \sum_{\mu \leq \delta_n} \sigma_n(\mu)$



François Bergeron, LACIM

FIRST RETURN SPLIT

$\mapsto (\quad , \quad)$



François Bergeron, LACIM

MULTIPLICATION PROPERTY

$$\sigma_n(\mu) = \sigma_i(\alpha)\,\sigma_{n-i}(\beta)$$

FIRST RETURN
SPLIT

FRANÇOIS BERGERON, LACIM

LOCAL Δ-CONJECTURE

$$e_k^\perp \sigma_m(\mu) =_1 \sum_J \Delta_{(J|a)}$$

$\#J = m-1-k$

$\mathrm{DESC}(\mu) \subseteq J$

in PARTICULAR

$$e_0^\perp \sigma_m(\mu) =_1 \Delta_a$$

$a = \mathrm{AREA}(\mu)$

FRANÇOIS BERGERON, LACIM

$$\sigma_n(0) = s_{111} + s_{31} + s_{41} + s_6,$$

$$\sigma_n(2) = s_{21} + s_4,$$

$$\sigma_n(3) = s_{11} + s_3,$$

$$\sigma_n(21) = s_3,$$

$$\sigma_n(31) = s_2,$$

$$\sigma_n(32) = s_1,$$

$$\sigma_n(311) = s_1,$$

$$\sigma_n(1) = s_{31} + s_5,$$

$$\sigma_n(11) = s_{21} + s_4,$$

$$\sigma_n(111) = s_{11} + s_3,$$

$$\sigma_n(22) = s_{11} + s_2,$$

$$\sigma_n(211) = s_2,$$

$$\sigma_n(221) = s_1,$$

$$\sigma_n(321) = 1.$$

$$\mathcal{F}_n = \sum_{\mu} \sigma_n(\mu) \otimes \Delta_{(\mu+1^n)/\mu}$$

$$\xi_n = \sum_{\mu} \sigma_n(\mu)[Q-1] \otimes \Delta_{(\mu+1^n)/\mu}$$

$$Q = q_1 + q_2 + \cdots$$

FRANÇOIS BERGERON, LACIM

- e-POSITIVITY PHENOMENON
- THE Δ-CONJECTURE
- KHOVANOV-ROZANSKY HOMOLOGY OF (m,n)-TORUS LINKS
- THEORETICAL PHYSICS
  SUPERSYMMETRIC
  GAUGE THEORY
  AGT RELATIONS (BOSONS ↔ FERMIONS)

Fim