

## Minicourses

**Shigeki Akiyama:** "The  $\beta$ -expansion and its extension"

**Abstract:** In this course, we review basic results on beta expansion and try to generalize it in higher dimension including rotational action. In doing so, we meet several basic problems such as ergodicity, Markoff property etc. As a result, we obtain a strong tool to produce self-similar tilings in a systematic way.

**Fabien Durand:** "Dimension groups, Bratteli diagrams, and spectral properties."

**Abstract:** These lectures are an introduction to dimension groups and topological (strong) orbit equivalence. We will start recalling basics about dimension groups of minimal Cantor systems. We will describe different ways to define such dimension groups and will show how to use them for orbit equivalence problem. We will end with spectral properties of minimal Cantor systems sharing the same dimension group.

**Mike Whittaker:** "Tilings, Smale spaces, and their  $C^*$ -algebras."

**Abstract:** In this minicourse I will introduce Smale spaces and their  $C^*$ -algebras. In their seminal work on cohomology of substitution tiling spaces, Anderson and Putnam showed that all substitution tiling spaces are Smale spaces. I will review and expand on their construction and explain the connections they made and why they are interesting. From there I will explain the philosophy of  $C^*$ -algebras and outline some recent advances that have direct consequences for Smale space  $C^*$ -algebras.

## Talks

**Pierre Arnoux:** "Tilings associated with continued fractions and S-adic systems."

**Abstract:** I will show how to generalize the classical construction of the Rauzy fractal associated to a unimodular Pisot substitution to a nonstationary framework (sequences of substitutions), and how we can use this construction for generalized continued fractions with a negative second Lyapunov exponent, such as the Brun continued fraction. This allows us to give symbolic dynamics for the Weyl chamber theorem, generalizing the classical construction for the geodesic flow on the modular surface as suspension of the natural extension of the usual continued fraction, and to give nonstationary Markov dynamics for some positive Anosov families on the torus. This is joint work with Valérie Berthé, Milton Minervino, Wolfgang Steiner and Jörg Thuswaldner.

**Karma Dajani:** "Invariant measures, matching and the frequency of 0 for signed binary expansions"

**Abstract:** We introduce a parametrized family of maps  $S_\alpha$ , the so called symmetric doubling maps, defined on  $[-1, 1]$  by  $S_\alpha(x) = 2x - d\alpha$ , where  $d \in \{-1, 0, 1\}$  and  $\alpha \in [1, 2]$ . Each map  $S_\alpha$  generates binary expansions with digits -1, 0 and 1. The transformations  $S_\alpha$  have a natural invariant measure  $\mu_\alpha$  that is absolutely continuous with respect to Lebesgue measure. We show that for a set of parameters of full measure, the invariant measure of the symmetric doubling map is piecewise smooth. We also study the frequency of the digit

0 in typical expansions, as a function of the parameter  $\alpha$ . In particular, we investigate the self similarity displayed by the function  $\alpha \rightarrow \mu_\alpha([-1/2, 1/2])$ , where  $\mu_\alpha([-1/2, 1/2])$  denotes the measure of the cylinder where digit zero occurs. This is joint work with Charlene Kalle.

**Sébastien Labbé:** “An alternative definition of Jeandel-Rao aperiodic tilings”

**Abstract:** We will present an alternate definition of Jeandel-Rao aperiodic tilings as the coding of dynamical systems defined by translations on a particular surface.

**Yasushi Nagai:** “Relations between geometric and dynamical properties of tilings and Delone sets.”

**Abstract:** The correspondence between geometric and dynamical properties of patterns such as tilings and Delone sets has been studied in the literature: for example, the repetitivity of a tiling is equivalent to the minimality of the corresponding dynamical system; the uniform patch frequency is equivalent to the unique ergodicity of the corresponding dynamical system for a tiling. In this talk we add another correspondence by showing a geometric property of certain patterns is equivalent to the property that there are “many” topological eigenvalues. Next, we introduce an abstract framework that includes tilings, Delone sets and functions as examples and describe the above-mentioned correspondences in an abstract level.

**Jamie Walton:** “Topological invariants of substitution tilings and rotations”

**Abstract:** To an aperiodic tiling one may assign cohomology groups, topological invariants which retain important combinatorial information about the tiling in question. In this talk I shall review the construction of these cohomology groups and explain some approaches to computing them for substitution tilings. I shall also explain how to visualise them using pattern-equivariant (PE) cochains. Via Poincaré duality, one may also visualise them via dual PE chains. When coupled with the action of rotation, this approach can often pick out familiar geometric features of aperiodic patterns. I shall explain recent work in determining the cohomology of the full Euclidean hull of a tiling using the rotation action on the cohomology, allowing us to determine the Čech cohomology of the Euclidean hull of Penrose tilings.