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Toward a History of Mathematics Education Reform in Soviet Schools (1960s–1980s)

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1 Introduction

The reform of mathematics education in Soviet schools during the 1960s and 1970s often is linked with the name of Andrey Kolmogorov. This view is well founded. Andrey Nikolayevich Kolmogorov was indeed the recognized leader of the reform. Not a single decision of any importance was made without his involvement. He was both the intellectual force behind the reform and its most active participant. In the rich history of Russian culture, it may be argued, there were only two great personalities who both won worldwide recognition for their achievements in their chosen fields and devoted a considerable part of their lives to the cause of Education. They were Leo Tolstoy and Andrey Kolmogorov.

For this reason, we will begin this sketch of the history of mathematics in Soviet schools with a brief discussion of the following topic: Kolmogorov and schools.

2 Kolmogorov and Schools

Kolmogorov's research in mathematics was published in widely known journals. Collectively, they form a body of work that has been thoroughly analyzed and thoroughly annotated. The fate of his vast pedagogical legacy has been altogether different. His numerous articles are scattered among disparate sources and there are many unpublished texts in the Kolmogorov archives. There is reason to hope that, in the near future, certain obvious gaps in "Kolmogorov studies" will be reduced significantly. After 20 years of studying Kolmogorov's pedagogical works on secondary school education (and he also has works on higher education), I can confirm that this part of his legacy is colossal in scope (a brief overview and an incomplete list of works appear in Abramov, 1988), and extremely rich in ideas and precise observations.

The final 24 years of Kolmogorov's life were devoted to improving mathematics education in Soviet schools. There is a certain mystery here. Why did Kolmogorov, the founder of a great school of mathematics, who by the age of 60 had attained the highest peaks in his discipline, suddenly sharply curtail his work in mathematics and devote himself wholly to education in the schools? There is an element of drama here as well: Kolmogorov's selfless devotion not only was unrecognized, but brought him serious worries. Significantly, notes of regret about his decision, and even notes of disapproval, can be heard from some of his famous students, who remain true to their teacher's memory.

My hypothesis is this:

(1) The simplest way to explain this mystery is to say that Kolmogorov was going through a certain creative crisis — that he felt a lack of fundamental new mathematical ideas. There were psychological grounds for such a crisis, too. Kolmogorov had often said that mathematicians were capable of working to the full extent of their powers until the age of 60, but it is likely, however, that everything was actually much more complicated.

Kolmogorov's genius was by no means limited to mathematics. He was interested in problems connected with everything in the world — nature, humanity, and fields that might seem far from his main interests. His deep works on history, poetics, and linguistics were not accidental. In his half-humorous "Plan for Becoming a Great Man If One Has Enough Will and Energy" (Shiryaev, 2003), which he composed for his 40th birthday (1943), he expressed the intention not only to start writing school-level textbooks at the end of his life, but also to produce a large monograph with the mysterious title "The History of the Forms of Human Thought." It is quite possible that immersing himself in problems connected with schools was an important stage in the realization of some grand design: the school, after all, is a storehouse of all of mankind's "big questions."

(2) This decision, so important for Kolmogorov's personal life, was not made on the spur of the moment. Kolmogorov was a modest person and he never made undue parade of his worldwide fame. At the same time, he knew his own worth, clearly recognized his strength, importance, and responsibility, and believed that it was his moral duty to do everything possible for science, for his homeland, and for humanity.

In this aspect, his decision appears quite logical. After becoming one of the world's most prominent mathematicians by the end of the 1930s, he began communicating his knowledge directly to his students, thus creating — like his teacher Nikolai Luzin — remarkable scientific schools. These schools were based on many ideas and projects that Kolmogorov developed as a teacher of future scientists — as a professor at Moscow State University, who spent a great deal of time working with students. The pyramid "undergraduate students — graduate students — scientists" requires a solid foundation. Therefore, Kolmogorov's involvement in working with talented schoolchildren was a logical next step, as was his subsequent work on improving the teaching of mathematics in schools. Only in this way could the Palace of Mathematics, which Kolmogorov had spent his whole life building, become a completed edifice.

(3) Finally, it should be emphasized that certain key events took place at various stages of Kolmogorov's life and had a particular influence on him. Both Kolmogorov's genius and his personality stem from his childhood, adolescence, and youth. In his articles, letters, and conversations, he often returned to the events of his early life.

First, there was his early childhood. Left without a mother — Maria Kolmogorova died while giving birth to him — Kolmogorov was raised in an atmosphere of love and attention in a wealthy noble family that embraced the best traditions of the Russian intelligentsia, combining a deep interest in culture with respect for work and adherence to democratic principles. Kolmogorov's diligence, inquisitiveness, and talent began to take shape at a very early age. When he was five years old, he made his first mathematical discovery, observing that the sum of consecutive odd numbers is always a perfect square (Kolmogorov, 1988, p. 7).

Second, Kolmogorov's education in E. A. Repman's private gymnasium in Moscow left a very vivid impression on him, over and above the excitement of his first encounter with science. Until the end of his life, Kolmogorov often recalled these years, his school friends, and his teachers. It was during these years, too, that he first began to dream of creating his own school.

Third, there was his time at the university. The atmosphere in Nikolai Luzin's school was the one of scientific exploration and this had an unequivocally beneficial influence on Kolmogorov. His university years witnessed his exceptionally powerful first steps in science. But a very great role was also played by the three years that Kolmogorov spent working as a teacher of mathematics and physics (as well as secretary of the school council, an elected position that he was proud to hold) at the education ministry's Potylikhinsky experimental-model school (Kolmogorov, 1988, p. 9).

In turning to the problems of school education in the 1960s, Kolmogorov was simultaneously repaying a debt to his own teachers and coming back to his old plans and dreams, which had engaged him deeply and sincerely. As he himself said, his attitude toward schools was the one of youthful enthusiasm. Kolmogorov used to express an idea that might shed light on his psychological profile: He believed that every person's development stopped at a certain age — and the lower this "psychological age" was, the more talented was the person. When asked directly: "And how old are you?" Kolmogorov replied: "I am 14." It is important to note, too, that self-conscious reflection, which was a part of his nature, influenced the decisions that he made in the course of the reform. In his youth, he had reflected deeply about his experience as a student and young teacher, and remembered them well.

By the early 1960s, Kolmogorov had already accumulated considerable experience with work on mathematics education. He had published his first article (Kolmogorov, 2007, p. 259) on the popularization of mathematics in 1929. Later on, he continued developing his ideas. Starting in the mid-1930s, on the invitation of the academician O. Yu. Shmidt, who was at that time the editor-in-chief of the *Great Soviet Encyclopedia*, Kolmogorov wrote many encyclopedia articles on mathematics (about 100 in all (Kolmogorov, 2007)). They included the classic article "Mathematics," which contained a holistic view of the development and methodology of this discipline. Many of the texts that Kolmogorov wrote for schools were connected with these encyclopedia articles. It also should be borne in mind that, in his early youth Kolmogorov had studied mathematics on his own using the Brockhaus and Efron Encyclopedia.

Starting in 1935, the year of the first Moscow mathematics olympiad for schoolchildren, Kolmogorov became actively involved in conducting olympiads, and in organizing and working in so-called "mathematics circles" (mathematics clubs for schoolchildren). At the end of the 1950s, he lent his support to the idea of so-called *Youth mathematics schools* (optional evening classes), which had originally been conceived at the Ivanovsky Pedagogical Institute, where the academician A. I. Maltsev — one of Kolmogorov's students — was working at that time (Abramov, 2008).

The present article will deal with the reform of all schools in the Soviet Union. But in order to provide a complete picture of the reform, it is necessary to describe briefly Kolmogorov's activities in facilitating the development of talented children.

In the early 1960s, Kolmogorov helped to organize the All-Russian and then the All-Union mathematics olympiads; he served as chairman of the Jury at the olympiads, and repeatedly traveled to the cities where the final rounds of the olympiads were held. In 1970s, together with I. K. Kikoin, Kolmogorov founded the magazine *Kvant*, remaining until the end of his life the head of the magazine's mathematics department and its deputy editor-in-chief.

But his main concern was with the Physics–Mathematics School that opened under the aegis of Moscow State University in 1963 and now bears Kolmogorov's name. Over the course of 15 years of active work, Kolmogorov taught many vivid courses and delivered numerous lectures at this school, encompassing the whole spectrum of the various fields of mathematics that are accessible to schoolchildren. In 1978, serious illness (Parkinson's disease) severely impaired his speech and limited his mobility. But prior to this, Kolmogorov very frequently visited summer schools to select students for the Physics–Mathematics School, went on camping trips with students, organized literary and musical events. From 1963 until the end of his life, he remained the head of the school's board of advisors. The history of the creation of the Physics–Mathematics School has been described in a recently published book (Abramov, 2008).

It is often forgotten that, by the time that Kolmogorov became the leader of the reform movement, in addition to his experience as a school teacher (during the 1920s), he already had considerable experience as the author of a school textbook. In the years before WWII (1937–1941), Kolmogorov co-authored an algebra textbook with P. S. Alexandrov. Its first part, for grades 6 and 7 (Alexandrov and Kolmogorov, 1940), came out in 1940. Right before the war, the journal *Matematika v shkole* ("Mathematics in the School") published sample chapters from the second part (Alexandrov and Kolmogorov, 1941a,b). An outline of the book's overall plan has survived, which indicates that A. Ya. Khinchin would have also been involved in the project. The war got in the way, however, and when the war ended, work on the textbook was not resumed.

Kolmogorov's earliest sketches for a school curriculum in mathematics date from the years before the war to the late 1940s. It appears that Kolmogorov and Alexandrov played a substantial role in the heated discussions about the teaching of mathematics at the end of the 1930s. The fate of A. P. Kiselev's textbooks, which were later acknowledged to be classics, was far from clear at the end of the 1930s: they were adopted merely on a temporary basis. Kolmogorov once told me that before the war, A. P. Kiselev had visited him and Alexandrov at their "dacha" (vacation cottage) in Komarovka in order to discuss the fate of his textbook and to ask for their support.

Kolmogorov's first programmatic article on school-related issues appeared in 1958 in the newspaper *Trud* (10 December 1958), as part of a discussion of the "Khrushchev education reform" project. Apparently, Kolmogorov made a fundamental decision to begin working with schoolchildren seriously at the end of 1962, shortly before his

60th birthday. In the 31 December edition of the newspaper *Izvestiya*, responding to a question about his plans for the new year, Kolmogorov wrote:

> Let me try, however, to formulate my long-time dreams:
>
> 1. To formulate the general logical foundations of mathematics in a way that would allow them to be taught to fourteen- and fifteen-year-old youngsters.
> 2. To eliminate the distinction between the "rigorous" methods of pure mathematicians and the "non-rigorous" methods of pure reason, employed by applied mathematicians, physicists, engineers.
>
> These two problems are closely linked...

The first part of this plan was largely carried out during the course of the reform.

3 The Pre-History of the Reform

In order to understand the factors that determined the course of the reform of school mathematics education, the initial state of mathematics education in the Soviet Union must be described briefly. On the whole, the situation that had taken shape by the beginning of the 1960s must be characterized as one that was favorable to transformations.

First, by that time, a pronounced atmosphere of respect for education and science had developed in the Soviet Union. In various levels of society, people clearly began to develop an appreciation of the significance of education. Elementary and secondary schools were supplemented by a large-scale network of evening schools, which allowed adults to continue their education and which enjoyed widespread popularity. The government was actively engaged in building and equipping schools, and preparing teachers; the number of publications aimed at students, teachers, and institutions of higher learning continued to grow rapidly. The growing respect for and popularity of science and technology were in large part fueled by the unquestionable achievements of the time: the launching of the first satellite into outer space, the first manned space flight, and so on. It is no secret that the authorities' attention to mathematics and science education was motivated first and foremost by the need to train personnel for the defense industry.

Second, at that time a certain balance was maintained between the number of high-school graduates who were prepared to continue their education in colleges and the number of qualified teachers. During the 1950s, not more than 20% of the students who entered first grade went on to obtain a full secondary school education (10 years). On the whole, teachers who were given positions in high schools were well qualified. A certain number of teachers from pre-Revolutionary gymnasia and "real-schools" had also survived, and they were able to transmit their knowledge to teachers of younger generations. Curiously, strong teachers (and hence, strong students as well) were distributed rather evenly across the country. This was an unexpected consequence of the Gulag system: highly-qualified people were sent to various remote places (one such teacher, for example, was Alexander Solzhenitsyn).

Teacher preparation in pedagogical institutes and continuing education institutes for teachers were aimed specifically at future teachers. The teacher preparation program included a large course in elementary geometry and workshops in solving problems, which — as is well known — forms the substance of mathematics education. Courses in methodology were developed and supplemented with textbooks. In general, it must be said that methodology was worked out quite robustly. Since the mid-1930s, there had been no revolutions in the schools — the same textbooks remained continually in use.

In principle, the decision in the 1930s to create a system of universal education on the model of pre-Revolutionary gymnasia and "real-schools" was a risky move since universal and elite (gymnasium) education are two fundamentally different things. But all of the aforementioned factors made it possible by and large to maintain high standards and to achieve quite decent results.

Thus, computational skills were normally learned by solving problems involving elaborate and numerous computations (above all, problems involving "multilayered fractions"). Problem books in arithmetic contained many intricate problems that developed students' mental agility. Rybkin's famous geometry problem book (Rybkin, 1960) consisted of rather difficult problems, while the course in solid geometry (Kiselev, 1960) actively developed both students' spatial imagination and their logical reasoning ability, necessary for substantiating many theorems and solving problems. Lastly, the development of algorithmic skills and knowledge was facilitated by straightedge-and-compass problems, problems involving the transformation of algebraic and trigonometric expressions, problems involving putting expressions into a form convenient for logarithmic representation, and so on. Not all schoolchildren attained the required levels, but colleges' needs for strong applicants were completely satisfied. Problems on college entrance exams were based on school curricula.

The first attempts at reform began in the late 1950s. The "Khrushchev reform" introduced mandatory eight-year education (replacing mandatory seven-year education). For a brief period (1962–1967), high-school education was expanded to include an 11th grade (instead of ending with 10th grade).

Substantively, the changes were not so great. New textbooks in geometry by I. N. Nikitin (1956) and algebra by A. N. Barsukov (1956) for grades 6–8 were introduced in the late 1950s; and the subject "trigonometry" appeared in the curriculum (Novoselov, 1956). These textbooks were criticized actively, but in their basic conception they differed little from the earlier textbooks.

An attempt to change curricula and textbooks was undertaken in 1962, when an open competition for new mathematics textbooks was announced. The chairman of the panel of judges was B. V. Gnedenko; the chairmen of the committees on arithmetic, algebra, and geometry were Professor V. I. Levin from the Moscow State Pedagogical Institute, the famous algebraist A. G. Kurosh from Moscow State University, and Professor N. F. Chetverukhin.

Eighty six authors' groups participated in the competition. Most of them produced patently weak work. The only textbook that won the top prize and was recommended for large-scale publication was E. S. Kochetkova's and E. S. Kochetkov's textbook in algebra for upper grades (1965). This textbook also introduced elements of calculus.

Several other authors' groups also won recognition in the competition and subsequently played notable roles in the reform. The second prize was awarded to an authors' group comprised of three authors (A. F. Semyonovich, F. F. Nagibin, and R. S. Cherkasov) for a geometry textbook for grades 6–8. Honorable mentions were awarded to V. M. Klopsky and M. I. Yagodovsky for a geometry textbook for grades 9 and 10, as well as to K. S. Barybin. B. Ye. Veits and I. T. Demidov were likewise awarded an honorable mention for a textbook on algebra and beginning calculus for upper grades. The results of the competition were published in the journal *Matematika v shkole* (nos. 1 and 3, 1964).

In 1964, V. G. Boltyansky and I. M. Yaglom's ninth-grade geometry textbook was published in a large edition (Boltyansky and Yaglom, 1964). It introduced students to new topics: "Geometric Transformations" and "Vectors." This fundamentally new textbook was clearly still rough. It drew much criticism from both scientists and teachers, and survived in schools for only two years. It became evident that updating the school course in mathematics was a difficult problem that would require a systematic approach.

The reform was preceded by a broad and substantive discussion, mainly among university teachers. Articles were published in the journals *Matematika v shkole* and *Matematicheskoye prosveschenie* ("Mathematics Education," series no. 2). N. Ya. Vilenkin, A. A. Lyapunov, V. G. Boltyansky, and others actively participated in the discussion.

Although opinions about details differed, mathematicians and college teachers agreed that the course in mathematics had become timed. Substantive suggestions for updating the mathematics curriculum boiled down to the following: it was necessary to introduce elements of calculus and analytic geometry, vector algebra, and geometric transformations into the high-school mathematics program. Methodological articles and pedagogical texts demonstrating different approaches to presenting these new topics began to appear.

4 The Curriculum of 1968

The ideology and principal aims of the reform of school mathematics education were largely determined during the preparation of the new mathematics curriculum, which was approved in 1968. The history of the creation of this curriculum deserves special attention.

It should be noted that the reform affected not only the course in mathematics, but the entire contents of school-level education. In December 1966, the Central Committee of the CPSU and the Council of Ministers passed a resolution that determined school policies for many years to come. At the time, it was customary to prepare for political decisions well ahead of time. By the beginning of 1965, a Central Committee for Developing the Content of School Education was established under the aegis of the USSR Academy of Sciences and the USSR Academy of Pedagogical Sciences, chaired by Academy of Pedagogical Sciences vice president A. I. Markushevich. The choice of chairman could hardly have been better. A. I. Markushevich had a great deal of experience in organizational work (since the end of the 1950s, he had been deputy minister of education), and most importantly, he was a highly cultured person, a well-known mathematician — a specialist in complex analysis and a Moscow State University professor — and a wonderful author and popularizer. Markushevich was highly respected both in academic circles and in the educational system. With respect to the reform, it was also significant that Markushevich and Kolmogorov were linked by long-standing relations of mutual respect.

Within the Central Committee, subject committees were formed. Like the mathematics committee, which was chaired by Kolmogorov, the other subject committees were chaired by well-known scientists–academicians: I. K. Kikoin (physics), M. V. Nechkina (history), D. D. Blagoy (literature), and so on. Such participation by major scholars facilitated the aims of the reform: freeing the courses from archaic, second-rate materials, and making them more rigorously scientific (this was motivated, of course, by a wish to accelerate scientific–technological progress and to surpass the USSR's principal Cold War adversary, the United States).

The first document of the reform — "The Scope of Knowledge in Mathematics for the Eight-Year School" (*Matematika v shkole,* 1965, no. 2) — was prepared by members of the Committee on Mathematics Education at the mathematics division of the USSR Academy of Sciences (I. M. Gelfand, A. N. Kolmogorov, A. I. Markushevich,

A. D. Myshkis, D. K. Faddeev, and I. M. Yaglom), i.e., without the participation of methodologists or teachers.

By contrast with ordinary programs, this text did not contain a detailed presentation of topics arranged by grades and subjects in some determined sequence. Rather, in an extremely concise fashion, it described the key ideas that students were required to absorb by the end of their eight years of schooling. The decision to present the program in such a brief form made it possible for people with widely differing views to agree on a common position. Arguments about the contents of school-level education can go on indefinitely. In order to avoid this, it is necessary to agree on key principles, which was the aim of "The Scope of Knowledge." It was expected that a broad discussion would follow and that a detailed program would then be formulated.

Authors' texts have survived that show that the main work on preparing the section on "Arithmetic and Algebra" was done by Kolmogorov; the section on "Geometry" was written by I. M. Yaglom. Drafts for a "Scope of Knowledge in Mathematics for Grades 9–10" have survived in Kolmogorov's archives; these were supposed to be published during the same year. But this plan was changed due to the more active role assumed by A. I. Markushevich's committee.

A "General Explanatory Brief on the Draft of the Curriculum and Programs for Secondary Schools" was published in 1965, followed by curricula in all subjects, including mathematics. But work continued for a long time to come. A pamphlet with the text of the mathematics curriculum was published in 1966 in an edition of 4000 copies (Mathematics Curricula, 1966), which were distributed in all the major cities of the Soviet Union. The pamphlet was discussed very widely, with a great number of people voicing their opinions, which were mainly positive. After some not very substantial revisions, the draft was published for a large-scale audience in *Matematika v shkole* (1967, no. 1), and only at the beginning of 1968 and after another discussion did a final document appear with the endorsement of the Ministry of Education (*Matematika v shkole*, 1968, no. 2).

Thus, work on the curriculum took about three years, which was accompanied by broad discussions, and largely reflected the consensus of the professional community.

The group of individuals who developed the curriculum included scientists, methodologists, and teachers. The mathematicians were represented by V. G. Boltyansky, Kolmogorov, A. I. Markushevich, and I. M. Yaglom. The methodologists were represented by G. G. Maslova, the head of the mathematics education laboratory at the Scientific Research Institute on Educational Content and Methods under the aegis of the USSR Academy of Pedagogical Sciences, as well as this laboratory's members, Yu. N. Makarychev, K. I. Neshkov, A. D. Semushin, and A. A. Shershevsky (one of the best mathematics teachers in Moscow). A. I. Fetisov was a well-known methodologist and author of manuals and problem books in geometry. The prefatory note to the curriculum stated: "The final draft of the explanatory note was completed by A. N. Kolmogorov, A. I. Markushevich (introduction, arithmetic, algebra, and beginning calculus), and I. M. Yaglom (geometry)."

The 1968 curriculum provided for a radical reform of the existing course in mathematics.

The introduction of a series of major new topics significantly expanded the range of information covered; these included elements of calculus, geometrical transformations, vectors and coordinates. Students also were to be given a substantive introduction to the axiomatic method. All of these served the central aim of the curriculum, which was to enrich the course in mathematics with ideas that had become significant in an age of accelerating scientific-technological progress as elements of a common culture. Another important goal was to increase the logical purity of the exposition.

A substantial expansion of the range of subjects and ideas covered in school could be achieved only by allotting time to them in classes. This meant that certain traditional themes and topics had to be abandoned. In this connection, the following decisions were made:

1. The elementary school curriculum in mathematics was shortened from four to three years, while its overall substance was preserved intact.
2. "Arithmetic," as a separate subject, was eliminated. For grades 1–5, a single subject — "Mathematics" — was introduced. It contained

elements of arithmetic as well as preparatory materials for classes in algebra and geometry.

3. While the ideas were raised to a higher level, the level of technical skills that average students were expected to master was lowered, as was the level of difficulty of the problems that average students were given. Different requirements were introduced for different students through the creation of elective classes to be chosen by the students themselves in accordance with their interests, inclinations, and abilities.
4. The topic "complex numbers" was eliminated from the program. The study of elementary probability theory and mathematical statistics had to be abandoned due to the shortage of class time and the lack of sufficiently prepared teachers who had real experience with these subjects.
5. The list of traditionally-studied isolated facts and properties (trigonometric identities, the properties of chords and tangents, and so on) was reduced substantially.
6. The presentation of traditional topics was made more concise and simple through the effective use of new methods (for example, complicated derivations of the formulas for the volume of the pyramid and the sphere, and the area of the sphere, were to be simplified substantially by applying the concept of the integral).

The new curriculum exposed students to elementary set theory and mathematical logic early on. But on the whole, this innovation was moderate by comparison with the reforms that were taking place at the same time in France or Belgium. As the explanatory note that accompanied the curriculum emphasized: "The curriculum approaches the introduction of the concepts and terminology of set theory and mathematical logic with caution. The possibility of using them in schools on a broader scale is still under discussion."

One of the important general principles of the reform was the need to establish a more precise and complete system of notation and exposition for mathematical texts. Kolmogorov connected this directly with the explosive growth in information technology that was expected to take place in the future. Working with machines requires precision and familiarity with working with symbols.

The adoption of the 1968 curriculum opened the door for work on textbooks that could implement the reform's ideas. But existing textbooks were already being revised by the mid-1960s and some obvious shortcomings were being eliminated (Kolmogorov, 1966a, 1967a,b). At the same time, large-scale work was underway on elucidating the ideas of the reform and providing a preliminary presentation of the new topics. *Matematika v shkole* began to publish a series of articles by Kolmogorov and others, aimed at popularizing the new ideas. The publishing houses "Mir" and "Prosveschenie" published a number of books and pamphlets on the "new school mathematics" (Markushevich, Maslova, and Cherkasov, 1978) including translations of foreign texts and textbooks (Moise and Downs, 1968; Doneddu, 1979).

5 The Implementation of the Reforms

The reform involved a large amount of varied work on the territory of an enormous country whose population spoke many languages. In addition, the cultural map of the USSR was highly heterogeneous — there were obvious differences, for example, between the rural schools of Central Asia and the urban schools of the Baltic republics. In order to carry out the reforms, an effective system of management had to be created.

Political decisions were made at the top and passed down to lower governing bodies to be carried out in the school departments of specific party organizations: the hierarchy descended from the Central Committee of the CPSU to the central committees of the republics to the regional committees to the city committees to the district committees. At each of these levels, appropriate goals were set and appropriate decisions were made. It may be said that the role played by the organs of the party was a legislative one. The executive role was played by the educational organs of the Soviet government: the USSR Ministry of Education, the ministries of the republics, the regional school board, the city school board, and the district school board.

The creation of the USSR Ministry of Education in 1966 — prior to which point there had only been ministries in the separate republics — was largely motivated by the need to coordinate the implementation of the reforms. Minister of Education M. A. Prokofiev was a member of the existing establishment, but as a serious scientist (specialist in chemistry, member of the USSR Academy of Sciences) and a genuine activist in the field of education, he remained on mutually respectful terms with Kolmogorov, Kikoin, and other leaders of the reforms. He resigned in 1984, refusing to implement the newly formulated program of bringing informatics into the educational system, considering it unrealistic. After his resignation, he actively promoted the idea of making schools more differentiated, which was rejected by the leadership of the country at that time. He left a testament of sorts in his small book, *Postwar Schools in Russia* (Prokofiev, 1997). In a private conversation, M. A. Prokofiev told me that in the Politburo he had always been supported by Minister of Defense D. F. Ustinov, who understood the significance of schools for the modern army perfectly.

The key decisions (assessing the state of affairs, recommending textbooks, and so on) were made at regularly scheduled Ministry of Education board meetings. An important role was played by the inspectorate of the Ministry of Education, which regularly organized comprehensive inspections across the country.

The system for preparing teachers was also structured hierarchically: from central institutes in the republics to regional continuing education institutes for teachers to district offices to methodological associations in the schools. Regular courses for methodologists from the republics and RSFSR (Russian Federation) methodologists in mathematics were conducted for a number of years in Moscow, at the Central Continuing Education Institute for Teachers. The authors of new textbooks that were to go into use on the first day of school would give lectures; then, the same materials would be presented to teachers — somewhat less cogently, perhaps — during summer and winter courses in regional centers and major cities in the republics.

Responsibility for the scientific side of the reforms — analyzing students' knowledge, analyzing programs and textbooks, developing pedagogical and analytical materials, and so on — was given to the

USSR Academy of Pedagogical Sciences, which had been formed on the basis of the RSFSR Academy of Pedagogical Sciences, also in 1966. The Academy of Pedagogical Sciences communicated and collaborated with pedagogical institutes in all of the republics.

The Academy of Pedagogical Sciences' Scientific Research Institute on Educational Content and Methods oversaw the development of new trial textbooks. In mathematics, this work (making trips to districts where experimental textbooks were being used, analyzing the results, conducting tests, engaging in methodological work with teachers) was carried out by the mathematics education laboratory at the same research institute. The head of the laboratory was G. G. Maslova. Four districts were selected for testing out experimental textbooks: the Tosno district in the Leningrad region; the Beloyarsk district in the Sverdlovsk region; the Suzdal district in the Vladimir region; and the city of Sevastopol. All schools in these districts used two competing textbooks from the late 1960s until the mid-1970s, at which point a final selection of textbooks was made.

The Research Institute on Educational Content and Methods had a strong graduate school. During the 1970s, a large number of pedagogical doctoral dissertations defended at the graduate school dealt with problems connected with the reform of the school mathematics curriculum.

When work on the curricula was completed in 1970, the Central Committee on Content Development was dissolved; its work as a whole was approved at a joint meeting of the presidiums of the USSR Academy of Sciences and the USSR Academy of Pedagogical Sciences, chaired by Academy of Sciences President M. V. Keldysh. A new Scientific Methodological Council — made up of different subject committees — was established at the Ministry of Education in order to oversee the publication of the new textbooks and methodological manuals. Kolmogorov was appointed head of the mathematics committee in 1970. In 1980, he was replaced by the academician A. D. Aleksandrov. The Scientific Methodological Council remained in existence until 1991, i.e., until the collapse of the Soviet Union. Subsequently, it was reorganized into a council of experts, effectively remaining what it had always been, until finally being dissolved in early 2003.

The members of the Scientific Methodological Council were famous mathematicians, methodologists, and teachers. When manuscripts were discussed, two or three principal reviewers would make presentations, summing up the numerous responses to the textbooks received from pedagogical institutes in different republics and regional continuing education institutes for teachers.

Meetings took place approximately every three or four weeks (depending on the number of manuscripts that had to be examined). The discussions were chaired by Kolmogorov, who always familiarized himself with the manuscripts beforehand. Kolmogorov possessed the rare talent of seeing the book in front of him as a whole: after looking through it rather quickly, he would locate what was most essential in it, whether this was an ineffective approach to a subject, obvious mistakes, or, on the contrary, some positive characteristic.

The textbooks and methodological manuals were edited at the mathematics division of the publishing house "Prosveschenie," at that time the largest publishing house in the world. The head of the publishing house, D. D. Zuev, took an active interest in the problematic aspects of school textbooks, created a special committee at the publishing house to work on them, and published 20 volumes of articles on "The Problematic Aspects of School Textbooks." Educational–methodological kits began to be published: these contained not only the textbook itself, but also a manual for teachers and educational materials (tests and quizzes). After work on the textbooks was finished, "Prosveschenie" began publishing a series entitled "The Mathematics Teacher's Library."

As a rule, final decisions about revising the textbooks would be made at the last moment, which made the editorial-and-publication process extremely difficult: new editions of four million copies of a textbook had to be made available by the beginning of the school year. Nonetheless, first editions contained relatively few major flaws (not counting misprints and mistakes in answers to problems).

Publishing houses in the different republics that specialized in education would translate the textbooks into the different languages spoken in the Soviet Union. They would also publish methodological literature by local authors. Relevant and up-to-date information would be published in the journal *Matematika v shkole* (for example, Kolmogorov and Semyonovich, 1970; Kolmogorov and Shvartsburd, 1975).

6 Elective Classes

As a mathematician, Kolmogorov was distinguished by astonishing scientific boldness. He took up problems that seemed unapproachable and managed to solve many of them. The problem that Kolmogorov set before himself in reforming mathematics education was also distinguished by the audacity of its conception. His premise was that the potential of the individual student and the potential of the education system were both high. Therefore, a rather high general level could realistically be attained if education was structured with intelligence and skill. Consequently, the level that the reforms aimed at was substantially higher than the level that was typical of virtually all other countries.

The first phase of the reform would be devoted to finding simple and succinct forms of presentation, a goal that was expressed in Kolmogorov's intention "to formulate the logical foundations of mathematics in a way that a teenager could understand."

But there was also another side to things. What kind of educational system could most effectively develop children's interests, inclinations, and abilities? This second problem had great significance for the government, since the government was particularly interested in finding a means to prepare large numbers of highly qualified experts.

The difficulty resided in ideological constraints: the misleading concept of the "uniformity of the school" (effectively, the idea that education meant the same thing to everyone) made it impossible to introduce differentiations into schools. A democratic solution to this problem was found: it consisted in offering students classes to choose for themselves, i.e., elective classes. Apparently, as the following documents show, this idea was first proposed by Kolmogorov:

> Letter from A. N. Kolmogorov to A. I. Markushevich
>
> (December 29, 1964)
>
> Dear Aleksey Ivanovich!
>
> Please forgive me for the way in which I expressed myself during our recent conversation.

In essence, however, creating possibilities for additional lessons in mathematics and physics in most of the schools in the country remains a very necessary goal if we wish to make further studies in these disciplines and in modern technology genuinely accessible to students. If we expand the programs in all schools by introducing integral calculus, etc., we will thereby also expand the program of college entrance exams. But in most schools, with mediocre teachers and six hours of classes in grades 9–10, students will assimilate the expanded program even worse than they absorb the current curriculum, and naturally, they will not be able to enter any college at all.

Placing all bets on mathematics circles and youth mathematical schools does not seem to me very promising.

But perhaps it is possible, without going against the "uniform school" dogma, to provide time for elective classes in the lesson plans for grades 9–10 (for example, three in ninth grade and six in tenth grade), with the school being obligated to organize them in accordance with the population's wishes. They may even be classes in drawing and radio technology, but they may also be classes in biology and the foundations of evolution, in foreign languages, or in mathematics and physics. What is important is that these will be hours allocated for classes during the entire year, and not just practice internships for some number of work days (now, I believe, 36 days in ninth grade and 12 days in tenth grade) for acquiring expertise and work qualifications.

I have just visited the neighboring Bolshevsky school no. 3. The youngsters take their qualifications as radio technicians quite seriously, but a large number of them would be enthusiastic about three or four hours per week of additional classes in mathematics. The parents, once they find out about such a possibility, would of course want their children to study mathematics, or technical drawing, or foreign languages, and would themselves find expert teachers.

I think that, in altered form, all of this also applies to good schools in state farms, although perhaps not to secondary schools in every backwater village.

There is another question concerning which I should like to know your opinion.

> I can understand the reluctance to expand the network of physics–mathematics schools such as our boarding school to a very large scale. But it is not clear to me whether the people making these decisions realize just how microscopic this whole initiative is, even if it is seen just as an experiment. Responsible government workers, ministers, and deputy ministers meet with university presidents for serious discussions, the television broadcasts my lectures, etc. Yet the idea of selecting 180 students from 40 regions is completely absurd if we believe that we will be able to identify and locate the talents hidden among "the people."
>
> Along what channels should one try to promote the idea that even experimental work must be done on a somewhat larger scale?
>
> Yours, A. Kolmogorov

A. I. Markushevich's response indicates that he too appreciated the absurdity of the "uniformity principle" in Soviet schools. As he wrote:

> In my view of physics–mathematics schools attached to universities as special points within the process as a whole, I apparently have no disagreement with you, Andrey Nikolayevich. But, by contrast with you, I attribute greater importance to schools that continue to prepare computer programmers. After all, it was supremely important to break the bleak bureaucratic monotony of our pre-reform secondary schools, which considered it a virtue to give all of our schoolchildren one and the same thing.

The idea of elective classes developed rapidly. This may be explained, on the one hand, by the fact that organizational problems met with an effective and timely solution. A resolution passed by the CPSU Central Committee in 1966 provided for allotting a certain amount of school time to elective classes and for paying teachers to conduct them. On the other hand, the experience of working with mathematics circles and schools specializing in mathematics that had been accumulated by that time, and most importantly, the involvement of highly qualified authors, made it possible to develop compact elective classes very quickly.

By 1970, the first textbooks for elective classes were completed. They were further developed in the following years (the laboratory for applied mathematics, headed by S. I. Shvartsburd, took charge of organizing the project as a whole, and a particularly prominent role was played by V. V. Firsov, who at that time was one of the laboratory's senior researchers).

For the 1968 curriculum, Kolmogorov had written a special note on elective classes. He proposed creating a course of "Additional Chapters," which would be conceptually connected with the general course. This idea did not take hold. Programs for 17-h and 34-h classes won more support, as did preparatory classes for competitive examinations.

In the late 1960s and early 1970s, the following manual for elective classes was published: "Additional Chapters for the Course in Mathematics" for grades 7 and 8 (Sikorsky, 1969) and grades 9 and 10 (Additional chapters, 1970). In 1978 and 1980, "Selected questions of Mathematics" (Bokovnev and Shvartsburd, 1978; Firsov, 1980) were published. The courses found in these and certain other books are listed below:

V. G. Boltyansky and G. G. Levitas, "The Divisibility of Numbers and Prime Numbers"
R. S. Guter, "Number Systems and the Arithmetic Foundations of Computer Operations"
N. Ya. Vilenkin, "Elements of Set Theory"
I. M. Gelfand, Ye. G. Glagoleva, and A. A. Kirillov, "The Coordinate Method"
I. M. Gelfand, Ye. G. Glagoleva, and E. E. Shnoll, "Functions and Graphs"
K. P. Sikorsky, "Solutions to Problems for the General Course"
A. N. Zemlyakov, "Symmetry"
I. L. Nikolskaya, "Elements of Mathematical Logic"
A. N. Zemlyakov, "Sets on the Coordinate Plane"
N. Ya. Vilenkin, "Infinite Sets"
N. Ya. Vilenkin and A. G. Mordkovich, "Differential Equations"
A. A. Egorov and G. V. Dorofeyev, "Complex Numbers and Polynomials"
A. M. Abramov and A. N. Zemlyakov, "Elements of Spherical Geometry"

In practice, elective classes continued to be developed rather actively throughout the 1970s. But the lack of special measures for preparing teachers for them and the reduction in the number of hours allocated for mathematics held back their development. Although no exact statistics exist, there are reasons to believe that gradually the hours that had been originally intended for elective classes came to be used for preparing students for competitive exams. By the early 1990s, elective classes had dissolved within the school curriculum and ceased to exist.

A mathematics correspondence school was created in 1964 under the aegis of Moscow State University on the initiative of I. M. Gelfand, with the support of I. G. Petrovsky (the rector of Moscow State University). This was a major event — a fundamentally new form of schooling. A system of entrance exams was worked out, and even more importantly, an outstanding system of assignments for students was developed as well. The organization of the school was original and quite democratic; over the course of a two-year program, students were required to complete about 20 substantial assignments. Students' work was checked (and corrected) on a volunteer basis by undergraduates at the mathematics department of Moscow State University: every undergraduate oversaw 10 students, and the work of every 10 undergraduates was monitored by a supervisor — an upperclassman or a graduate student at the mathematics department. The mathematics correspondence school exists to this day (and now encompasses multiple subjects). About 200,000 schoolchildren from many cities and towns have graduated from it; many of them went on to enroll in various colleges.

7 Mathematics 1–5[1]

Before the reforms, students in grades 1–5 had a class called "Arithmetic," which included very minor sections in geometry that dealt mainly with formulas for areas and volumes, and units of

[1] This section and a few following will deal with specific textbooks. The literature review is provided in Shtokalo (1975). Readers can also opt to consult Kolyagin (2001).

measurements — the main motive here was a wish to diversify problems and to give them a practical meaning. The decision to name the new class "Mathematics" reflected those fundamental changes which the reforms had introduced into the education of children between the ages of 7 and 12.

The creators of the new curriculum and the authors of the new textbooks pursued two basic goals: (1) to present the traditional part of the course in a substantially more condensed fashion, including covering a number of topics earlier than before; (2) to include a number of new topics in preparation for classes in algebra and geometry in grades 6–8 — to this end, a number of topics were included in the curriculum for grades 1–5 that had been covered previously in grades 5 and 6.

The most fundamental change was the shortening of elementary school education from four to three years. The contents remained largely what it had been previously: the objective was to study natural numbers, to carry out operations using natural numbers, and to solve easy problems in arithmetic. The principal innovation was the appearance of letter notation and a basic idea of equations. Naturally, the loss of one year of schooling meant that standards for students' computational skills had to be lowered; word problems were made easier as well. The geometrical material was somewhat expanded — students studied the simplest figures and elementary straightedge-and-compass constructions.

After a review, a textbook by M. I. Moro *et al.* was selected for grades 1–3. Until the 1990s, it remained the only textbook in use. In the 1990s, the monopoly was abolished, but this textbook is still used to this day, along with others. A. G. Pchelko, the author of a previously used textbook, contributed to the first editions of this textbook (Moro, Bantova, and Beltiukova, 1968, 1969, 1970), thus helping to provide some continuity between the new curriculum and what had preceded it.

The class "Mathematics 4–5" (Vilenkin *et al.*, 1968, 1969) radically altered the traditional curriculum.

1. The concept of "set" and operations on sets ("intersection" and "union") were explicitly introduced. This terminology and notation was actively employed at subsequent stages of education.

2. In the sections on arithmetic, fractions and negative numbers were introduced earlier than they had been before. The level of difficulty of word problems was lowered. (Traditionally, the following scheme was employed in Soviet and Russian schools: the full solution to a word problem had to include a clearly written out sequence of questions posed by the student, and the calculations required to answer them. Traditional problem books contained extremely involved problems whose solution involved answering 6–10 different questions. Problems in the "reformed" textbook were usually shortened to 2–3 questions per problem.)
3. Explicit algebraization gave the new program a rather revolutionary character. Letter notation, formulas, simple (linear) algebraic equations, and corresponding problems were actively used. All of this was fundamentally new — previously, elements of algebra had first appeared only in grade 6.
4. The list of geometric topics was considerably expanded. These were distributed throughout the entire course. Students were taught coordinates on the line and in the plane. Elementary straightedge-and-compass and protractor problems were solved regularly. An important innovation was the concept of axial symmetry and point symmetry as well as of rotation. The concept of congruent figures was introduced (as a required part of the course). All of this created a foundation for the systematic course in geometry that would begin in sixth grade, in which geometrical transformations played a very important role.
5. A certain lowering in the problems' level of difficulty was compensated for by the inclusion of additional problems with higher levels of difficulty, aimed at developing students' inventiveness.

The textbook "Mathematics 4–5" occupies a special place among all the textbooks that were produced in the course of the reform of mathematics education: it had the calmest, or perhaps the happiest, fate. This conclusion is warranted not only by its longevity: 40 years later the textbook is still used in schools. By contrast with other textbooks, "Mathematics 4–5" was subjected to virtually no criticism either from above or from below. Probably the only shortcoming that teachers saw in it was that it contained too few arithmetical word problems.

I see two basic reasons for its success. First, the trial run of "Mathematics 4–5" lasted longer than the trial runs of other textbooks — four years. This made it possible to analyze its virtues and shortcomings calmly, and to go through several rounds of revisions.

Second (and most importantly), the group of authors who wrote the textbook was well-balanced. N. Ya. Vilenkin and A. I. Markushevich — the textbooks' editor — were the mathematicians among them, but in addition to being major mathematicians, they possessed the intuition of good methodologists and had literary talent. K. I. Neshkov was an exceptionally conscientious and highly talented teacher and methodologist. S. I. Shvartsburd was also a very experienced teacher, who had founded schools specializing in mathematics during the 1950s. He turned out to be a good mediator in discussions that took place among the authors, who were all very different people.

During the 1970s–1980s, the textbook went through rather minor changes. The most notable of them was the "eradication" of set theoretical terminology and notation following the events of 1978–1979 (see below) — although the discussion between Kolmogorov and Vilenkin, who was against introducing the term "congruence," dated back to 1972 (*Matematika v shkole*, 1972, no. 5).

In a competition in 1987–1988, the textbook by Vilenkin *et al.* retained its position, although a new manual by Nurk and Telgmaa (1988) also was introduced. A more significant and consequential event was the beginning of work on a new textbook in arithmetic by a working group led by the academician S. M. Nikolsky (Nikolsky *et al.*, 1988): the authors' main goal was to reestablish arithmetic as the core of the middle school mathematics curriculum.

8 Geometry 6–10

Traditionally, since the 1930s, a systematic course in geometry has been taught in Russia (USSR) from sixth to tenth grade. The reorganization of this course became the single most difficult problem that arose during the reforms. One of the reasons for this was the traditional difficulty of studying geometry at the elementary level, which has even earned a special designation: "the problem of the first lessons in geometry in grade 6." The problem stems from the fact that the deductive style of exposition — something fundamentally new for schoolchildren — requires overcoming both psychological and epistemic difficulties. Schoolchildren do not understand why one must prove things that are obvious. It is also not clear to them why obvious assertions must be proven by using other assertions that are equally obvious.

At the same time, the changes proposed by the new curriculum in this instance were of the most revolutionary nature. There was a great quantity of new materials. Both in terms of its substance and its methodology, this course possessed features that were fundamentally new.

The competition of 1964 was won by a group of authors which included F. F. Nagibin, professor at the Kirov Pedagogical Institute; A. F. Semyonovich, associate professor (and subsequently full professor) at the Cherkassk Pedagogical Institute; and R. S. Cherkasov, professor and chairman of the mathematics teaching methodology department and the Moscow Municipal Pedagogical Institute, and for many years the editor of the journal *Matematika v shkole*. The group was headed by Kolmogorov, who became the co-author and editor of the textbook. His decision to become so involved was based on the extreme time constraints under which the authors had to work. In keeping with the government-mandated schedule for the transition from the old textbooks to the new ones, the first editions of the new textbooks had to meet certain deadlines and it was thus necessary to give the group of authors help in their work.

A first, experimental edition of a textbook in geometry for grade 6 (Kolmogorov *et al.*, 1970) appeared in 1970 (the textbooks "Geometry 7" and "Geometry 8" came out in 1971 and 1972, respectively (Kolmogorov *et al.*, 1971, 1972a,b)). The textbook was put to use on an experimental basis for a three-year trial, which immediately brought to light major problems stemming from the novelty of the theory and the practice for the students, an obviously overloaded curriculum, the teachers' lack of methodological experience, and the novelty of the material for the teachers.

The textbook for grade 6 was substantially reworked, and by 1972 it was introduced in schools across the Soviet Union. The new version of the course in geometry for grades 6–8, which consisted of three textbooks that appeared during the years 1972–1974 (Kolmogorov *et al.*, 1972a,b, 1973a,b, 1974a,b), was distinguished from the earlier, experimental edition by its more systematic approach. The authors sought to correct the numerous flaws that had come to light during the trial run.

Kolmogorov formulated the ideology of this course in the following way (Gusev *et al.*, 1972, p. 7):

> The new course in geometry for grades 6–8 is substantially different from the traditional one. The new textbook in geometry for the eight-year school includes the following changes:
>
> 1. The concept of geometrical figures as sets of points is consistently promoted.
> 2. It is made completely clear (already in sixth grade) that geometry inevitably makes use of certain fundamental concepts that have no obvious definitions, and that these concepts must be used to define precisely all other geometrical concepts.
> 3. The textbook systematically develops the concept of "geometrical transformations" as one-to-one mappings of the entire plane (and later, of all of space) onto itself. In sixth grade, this pertains to "motions" of the plane (in contemporary mathematical language, "isometries"). In seventh grade, students examine similarity transformations, in particular, dilations.
> 4. The textbook gradually prepares materials for understanding the possibility of various "geometries" that are non-Euclidean (such as that of Lobachevsky) or that contain Euclidean geometry as a particular case (such as notion of a "metric space"), which are prepared already in sixth grade through an examination of the basic properties of distances.
> 5. In grade 7, students are introduced to the notion of vector, which is then systematically used in the upper grades and in physics classes.

6. In grade 8, students study the trigonometric functions of angles from $-180°$ to $180°$.

Two more items should be added to this brief list of innovations:

(7) The new program introduced elementary space geometry — a special chapter in the eighth-grade textbook was devoted to this topic.

(8) Students were introduced to the coordinate method in geometry (the equations of the straight line and the circle; elementary problems).

While developing the new course, Kolmogorov paid particular attention to the rigor of the definitions. He believed that the ability to work with rigorous definitions was an absolutely indispensible part of the general skill set of every educated person. In view of the difficulty of some proofs, it is not possible to prove every proposition in a school-level textbook; however, it is important in such cases explicitly to indicate "unproven assertions," whose demonstrable meaning must be convincingly illustrated. But in the case of definitions — and in the formulation of propositions — precision and purity must be observed.

These principles were consistently promoted. The system of exposition that was adopted by the final version of "Geometry 6–8" (Kolmogorov *et al.*, 1979) may be considered flawless in terms of its logical underpinnings. Preference also was given to a precise system of notation and an explicit and comprehensive approach to writing out the solutions of problems.

While working on the textbook, in 1970, Kolmogorov proposed a new, original axiomatization of Euclidean geometry. It was impossible to adhere to the stated principles for structuring the textbook without a clearly articulated system of axioms and basic concepts. Kolmogorov's axiomatics took the following approach (in the 1974 edition of "Geometry 8").

The concepts "point," "straight line," and "distance" are posited as fundamental (undefined) notions. A plane is a set of points in which subsets ("straight lines") are distinguished, "distances" are defined, and the following axioms hold:

I. **Axioms of incidence**

I_1. Each straight line is a set of points.

I_2. For any two points, there exists one and only one straight line that contains them both.

I_3. There exists at least one straight line; every straight line contains at least one point.

II. **Axioms of distance**

II_1. For every two points A and B, there corresponds a nonnegative magnitude |AB|, which is called the distance from point A to point B.

$$|AB| = 0 \quad \text{if and only if } A = B.$$

II_2. The distance from point A to point B is equal to the distance from B to A:

$$|AB| = |BA|.$$

II_3. For any points A, B, and C

$$|AC| \leq |AB| + |BC| \text{ (triangle inequality).}$$

III. **Axioms of order**

III_1. Any point O on a straight line p divides the set of points on the straight line that are not O into two non-empty subsets of points in such a way that (a) for any two points A and B that belong to different subsets, point O lies between A and B; (b) for any two points A and B that belong to the same subset, one of them lies between the other point and O.

III_2. For any distance a on a given ray with its origin in O, there exists one and only one point whose distance from O is a: $|OA| = a$.

III_3. If a point C lies between points A and B, then the points A, B, and C belong to one straight line.

III_4. Any straight line p divides the set of points in the plane that do not belong to it into two non-empty sets in such a way that (a) any two points that belong to different sets are separated by the line p; (b) any two points that belong to the same set are not separated by the line p.

IV. **The axiom of congruence**

For any two pairs of points A and B and A′ and B′, such that $|AB| = |A'B'|$, there exists two "rigid motions" that take A into A′ and B into B′.

V. **The axiom of parallelism**

Through any point À in a plane, there passes not more than one straight line that is parallel to a given straight line.

Remark. All of the concepts used in the formulations of the axioms are subsequently defined in the text. Point X lies between two different points A and B if $|AB| + |XB| = |AB|$. A segment is defined as the figure consisting of two points and all points lying between them. "Rigid motion" is the mapping of a plane onto itself that preserves distance, etc. Kolmogorov laid out the axiomatics of scalar quantities in his article "Quantity" in the *Great Soviet Encyclopedia.*

Kolmogorov's axiomatic system made it possible to implement his conception of the course. The axioms of distance are the axioms of a metric space. The axiom of congruence makes it possible to begin talking about Felix Klein's ideas. The formulation of the axiom of parallels is the traditional approach to grasping the ideas of Lobachevsky.

It is important to emphasize that distance is not a number, but a magnitude. Such a view was accepted in Euclid's time. Kolmogorov attributed a great deal of meaning to the concept of magnitude. He developed an axiomatics of scalar magnitudes, and in a course for the Physics–Mathematics School he used this as a basis on which to construct a theory of real numbers, in which positive numbers are defined as monotonic additive operators on the set of scalar magnitudes (Kolmogorov, 1966b). None of this was mentioned in the new school curriculum, of course: the textbook merely called attention to the fact that numerical values of magnitudes depend on the choice of the unit of measurement. The discussion of magnitudes in the course on geometry paved the way for a discussion of dimensionality in the course on physics.

In 1970, when I became Kolmogorov's graduate student, he proposed that I take up the topic of "The Logical Foundations and Plan of Geometry." The aim was to work on the foundations of geometry, i.e., to construct a sequence of definitions and proofs for the main theorems of Euclidean plane geometry. He was interested above all in the theorem on angle measurement, which had remained without a proof. This work was completed in 1975, when I defended my thesis, which also contained many comments on the school curriculum. The material from this dissertation became the basis for specialized courses at pedagogical institutes.

An optional chapter entitled "The Logical Construction of Geometry" appeared in an edition of the eighth-grade textbook (1974–1977). Here, Kolmogorov offered a brief and accessibly written overview of the axiomatic method, including a discussion of the consistency, completeness, and independence of axioms, illustrated with examples from finite geometry. At the Physics–Mathematics School in the early 1970s, he taught a wonderful course in which geometry was constructed on the basis of axioms of incidence. The course began with an examination of finite affine and projective planes and spaces. An outline of Kolmogorov's lectures has survived, but has not been published.

The edition of 1972–1974 did not solve the problems that arose when the textbook was used in schools. Teachers absorbed the course slowly and with difficulty. During these years, there appeared a new genre of published literature that could offer practical help to teachers: lesson-by-lesson methodological analyses which contained very detailed recommendations on how to organize every lesson.

Work on a new version of the textbook "Geometry 6–8," which began in 1973, lasted much longer than expected. The unification of three textbooks in one book precluded the possibility of transferring problems from one year to the subsequent year. Several transitional drafts were prepared. The authors systematically searched for a more compact structure, and simpler methods and proofs. The system of problems went through substantial revisions.

Finally, a unified textbook came out in 1979 (Kolmogorov *et al.*, 1979). But this textbook did not last long — its last edition came out in 1982, at which point a political decision was made to replace it with a textbook by A. V. Pogorelov (see the section on the "counter-reform" below).

I actively participated in the preparation of the 1979 edition as a co-author. The decision to replace this textbook probably was premature. Over 10 years of work, teachers had accumulated considerable experience; the situation had begun to improve. In the early 1980s, Kolmogorov and I actively discussed ways to improve the textbook; a new prospectus was prepared. But these plans were never realized. A. F. Semyonovich and R. S. Cherkasov, who continued working on the textbook, submitted their new version to a competition in 1987, but their project did not meet with success.

I will now turn to the space geometry textbooks. Two groups of authors, which had emerged from a competition held in 1964, competed with one another during the developmental stages. The first group included teachers from the Kursk Pedagogical Institute, V. M. Klopsky and M. I. Yagodovsky. The author of the second textbook was K. S. Barybin, a methodologist from Moscow. At the beginning of the reform, the first group was enlarged to include Z. A. Skopets, a professor at the Yaroslavl Pedagogical Institute. Skopets, who was the author of famous problem books in geometry, became the textbooks' co-author and scientific editor (Klopsky, Skopets, and Yagodovsky, 1969, 1971). The second textbook was edited by A. B. Sossinsky, an associate professor at the Moscow State University mathematics department (Barybin, 1970, 1971).

In the relatively unanimous opinion of numerous reviewers, Barybin's textbook, which contained a multitude of mathematical inaccuracies, failed to reflect the ideas of the reform. Experiments with putting this textbook into use pointed to the same conclusion. Consequently, it was rejected.

In the mid-1970s, I worked at the publishing house "Prosveschenie" and edited the first, large-scale editions of the textbook by Z. A. Skopets *et al.* (Klopsky, Skopets, and Yagodovsky, 1975, 1976). I must say that this group of authors was made up of very experienced, highly qualified, and conscientious people who produced a sound textbook.

In terms of its contents, the textbook adhered to the principles described above — the principles employed in the course in plane geometry. The foundation for the course in space geometry consisted of Kolmogorov's axiomatics, supplemented by spatial axioms of incidence. Definitions were kept rigorous and formulations precise, although adhering to this approach when dealing with complicated notions such as "vector," "polyhedron," and "volume" made things very difficult for the authors, and subsequently for the students and teachers as well. Students were systematically taught to conceive geometrical figures as sets of points.

New methods, which had been prefigured in classes 6–8, were actively developed both in theory and in problem solving. Considerable attention was devoted to vector methods for solving problems (these methods were explicitly emphasized); the notion of scalar product appeared as well. But the idea of constructing the course in geometry on a purely vectorial foundation — which was fashionable at the time — was not even discussed.

The isometry classification theorem for three-dimensional space was not formulated, but certain types of isometries of space were discussed in theory and applied in solving problems. Problems "on visualizing symmetry," which were included in the course, were employed specifically in order to develop spatial imagination. Nonetheless, problems such as "How many axes (planes) of symmetry does a cube have?" invariably created difficulties for students and teachers alike.

By comparison with the traditional approach, the new textbook greatly simplified the derivation of formulas of volume by using the notion of integral. The area of the sphere was determined using Minkowski's method: as the derivative with respect to the radius of the volume of the ball that it bounds.

Sets of problems on constructing the cross-sections of polyhedra and problems on "imaginary constructions" helped to develop spatial imagination. For the first time, rules for representing spatial figures on a plane appeared in ordinary schools: the properties of parallel projection were formulated and proved.

The beginning of the course followed a more or less traditional approach: theorems about the mutual positions of straight lines and planes in space were proved on the basis of axioms of incidence; and related problems were discussed, with the aim of developing students' deductive skills and spatial imagination.

On the whole, it must be noted that, despite the introduction of many new topics, the course in space geometry drew less criticism from teachers than the course in plane geometry. This is explained, first, by the fact that many of the ideas developed in the upper grades had already been explained in grades 6 and 7 in a preliminary fashion. Second, with respect to methodology, the textbook was quite good, and a system of problems had been worked out that contained a "spectrum of assignments" from simple to difficult ones. Methods for solving problems were formulated clearly and precisely, and they were accompanied by examples. In addition, a substantial body of problems that were familiar to teachers from A. P. Kiselev's old textbook were retained. Nonetheless, difficulties did arise. They were connected with the conceptual intensity of the course and the shortage of class time.

Despite its strengths, the textbook edited by Z. A. Skopets was quickly replaced during the "counter-reforms" of the 1980s. As in the case of Kolmogorov's textbook, I believe that rejecting it completely was a mistake. Despite initial difficulties, teachers were beginning to get used to the new curriculum. The last editions of the textbooks, substantially revised, were free of many of the defects present in earlier versions.

9 Algebra and Elementary Calculus

In keeping with the program of 1968, the modernization of the course in algebra initially was moderate in character. Conspicuously greater attention was paid to functions and graphs. Set theoretical concepts and symbols were used. The main innovation consisted in shifting the topic "Exponential Functions and Logarithms" and the advanced sections to the eighth grade. Thus, the overall contents of the course "Algebra 6–8" was well known to teachers. This meant that the new textbooks were accepted and put into use with little difficulty (Makarychev, Mindiuk, and Muravin, 1972, 1973, 1974).

One should also note that the group of authors who wrote these textbooks — by contrast with the groups of authors who worked on the other textbooks — did not include any professional mathematicians and consisted entirely of methodologists (N. G. Mindyuk, Yu. N. Makarychev, and S. B. Suvorova). The textbooks' editor, A. I. Markushevich, was, of course, a professional mathematician, but he had a very moderate notion of modernization. In addition, the authors were realistic about how difficult the problems in the textbook could be. Due to all of these circumstances, the new textbooks in algebra were favorably received by teachers.

Probably the only fundamental difficulty arose in connection with the fact that the authors — who were very diligent and conscientious methodologists — wished to explicate all concepts fully; as a result, they somewhat overloaded their textbooks with problems aimed at testing students' understanding of the concepts of "mapping" and "correspondence." Consequently, in the late 1970s, the algebra textbooks that had been edited by A. I. Markushevich were subjected to significant criticism. At the beginning of the 1980s, when S. A. Telyakovsky (a researcher at the Mathematics Institute of the USSR Academy of Sciences) became the new editor of the series, the shortcomings were eliminated in new editions of the textbooks (Makarychev, Mindiuk, and Suvorova, 1981, 1982, 1983). Following a competition in 1987, these textbooks were recommended for use in schools across the country. Beginning in the following year, they started to be used in schools along with the textbooks of Alimov, Kolyagin *et al.* (1988, 1989a,b, 1990), which had also received honorable mentions in the competition.

The earliest experience with teaching calculus in Russian schools dates back to the beginning of the 20th century. Elements of differential calculus were part of the curriculum in "real-schools"; A. P. Kiselev (1908) wrote a calculus textbook. Strangely, this historical episode exerted a substantial influence on decisions made in the 1980s. In discussions of the mathematics curriculum, many people favored omitting calculus from the program. The case was decided by the opinion of the academician I. M. Vinogradov, who himself had graduated from a "real-school": "We already studied calculus before the Revolution. So we need calculus in the schools."

Elementary calculus was introduced into schools across the country in 1965, when the Kochetkovs' textbook went into use (Kochetkova and Kochetkov, 1965). This textbook, which was written by famous methodologists, was rather sloppy from the mathematical point of view; therefore, Kolmogorov agreed to edit a text written by B. Ye. Veits and I. T. Demidov (1969, 1970), teachers at the Murmansk Pedagogical Institute. In terms of mathematics, this book was better than the other one. After a comparative study of the two textbooks, Veits and Demidov's text was recommended for use in schools.

At the same time, it was recognized that this text was rather difficult for ordinary schools, particularly the sections concerned directly with calculus. The authors effectively adhered to the method of exposition used in colleges (limits of series — limits of functions — continuity, etc.). This textbook was taken as a foundation, and the group of authors was substantially enlarged to include Kolmogorov (as author and editor), S. I. Shvartsburd, O. S. Ivashev-Musatov, and B. M. Ivlev. The authors produced a new version of the textbook for use in ordinary schools (Kolmogorov *et al.*, 1973a,b, 1974a,b). Although the text was simplified, it still gave rise to considerable difficulties in schools; therefore, work on it continued in 1978–1979, with the final edition appearing in 1980 (Kolmogorov *et al.*, 1980).

The 1980 edition contained the following key changes. (1) Due to a reduction of class hours, sections on combinatorics and the principle of mathematical induction had to be omitted. (2) The exposition of key concepts (limit, derivative, and integral) was greatly abridged and simplified. (3) Due to severe criticism of the textbooks by the USSR Academy of Sciences (see below), the use of set theoretical concepts was reduced.

The next revision (Kolmogorov *et al.*, 1988) was completed in 1987, when a textbook competition was organized. At this time, the following fundamental changes were made:

1. The basic concepts of calculus (continuity, derivative, and integral) were formulated on the basis of illustrative geometrical and physical notions; attempts to provide a precise definition of limits were abandoned.
2. The system of problems and the methodological apparatus were substantially developed: (a) questions for review and model problems were added to each chapter, thus specifying what exactly students were expected to know; (b) the sections containing "Historical Facts" were systematized; (c) while the new edition as a whole placed lower demands on students' knowledge, it was necessary to preserve texts that were aimed at students who were interested in mathematics — to this end, a set of "Advanced Problems" was compiled and published initially as a supplement to the textbook, and subsequently as part of the text.

This textbook, edited by Kolmogorov and still in use, has not been revised in any significant way since 1987.

Intensive work on developing alternative textbooks began in the early 1980s. One of them was prepared as part of a large project headed by the academician A. N. Tikhonov. An experimental textbook by a new group of authors began to be published in the late 1970s (see, for example, Alimov *et al.*, 1984). Its distinctive features were a minimization of facts about calculus and a search for the simplest methods of exposition.

Another textbook was produced by M. I. Bashmakov, the famous Leningrad mathematician who worked on the problems of school-level education. This textbook was characterized by its conciseness and attention to practical application (Bashmakov, 1989).

Following the competition of 1987, all three textbooks were recommended by the Ministry of Education. The Kolmogorov textbook and the Bashmakov textbook won second place, while the textbook by Alimov *et al.* won third place (first place was not awarded). The results of the competition were published in the journal *Matematika v shkole* (1987, no. 1; 1988, no. 2).

10 The Counter-Reform

The events of the late 1970s and 1980s may be characterized as a counter-reform, since their logic was determined by sharp criticism of the still-incomplete reform of mathematics education and a wish to revise the transformations that had already taken place.

A decisive role during this period was played by the mathematics division of the USSR Academy of Sciences; the counter-reform was spearheaded by academicians I. M. Vinogradov (director of the Steklov Mathematics Institute), L. S. Pontryagin, A. N. Tikhonov (director of the Institute of Applied Mathematics). Teachers and methodologists who had invested a great deal of labor in the development of the "new school mathematics" were beginning to get used to the new textbooks and were not opposed to the Kolmogorov reform, but several groups that could potentially support the counter-reform movement were roused into action in the course of a discussion that had been initiated by mathematicians.

I believe that it is not an accident that the counter-reform began specifically in 1978. One year earlier, high-school students who had been educated in schools that implemented the reforms started taking college entrance exams for the first time. The colleges were confronted by a serious and acute problem: how to organize the exams?

There were two alternatives. The first was to preserve the already well-developed style of the exams, which often consisted of artificial problems that required students to use extremely intricate technical methods for solving them. It should be noted that this approach led many colleges to develop their own, idiosyncratic traditions, which greatly increased demand for the services of tutors and college instructors. This phenomenon became so prominent that a new name was invented for the "science" of writing problems for college entrance exams and tutoring methodology — "college-ology" (*"vuzomatika"*).

The second option was to undertake the difficult and serious work of substantially modernizing the system of college entrance exams in light of the radical changes that had taken place in the schools, and of making the transition to a system of problems that tested students' knowledge of mathematics and inventiveness, their readiness for college-level studies, rather than their ability to pass a specific college's entrance exam. But this option, which was supported by Kolmogorov, was much more difficult than the first, which determined many college workers' attitudes toward the reform.

The most influential group that came to support the counter-reform movement was the RSFSR's Ministry of Education. When the USSR Ministry of Education was established, tensions quickly arose between the education ministries of the USSR and the RSFSR, as often happens when two bureaucratic organizations fight over a sphere of influence. (There was even a joke in educators' circles: it is a good thing that there is only a USSR Ministry of Defense and no Ministry of Defense for the RSFSR. The joke turned out to be prophetic: in 1990, the RSFSR Ministry of Defense was created, and the collapse of the Soviet Union soon followed.)

The discussion about mathematics in the schools allowed RSFSR education minister A. I. Danilov (later replaced by N. V. Aleksandrov and G. P. Veselov) to assume an independent and aggressive position.

As shown above, there were plenty of grounds for constructive criticism. But the harshness with which the Academy of Sciences' mathematics division came out against the reforms was hardly justified. In 1967, the mathematics division had approved the plan for the new curriculum, and the basic goals of the school reform had been supported at a joint meeting of the presidiums of the Academy of Sciences and the Academy of Pedagogical Sciences (1970), chaired by M. V. Keldysh, the president of the Academy of Sciences. During the 1970s, academic circles took no part in the reform and were not interested in it.

Undoubtedly, subjective factors exerted a strong influence on the way in which events unfolded. By this time, for a number of reasons, Kolmogorov's personal relations with I. M. Vinogradov, L. S. Pontryagin, and A. N. Tikhonov had become quite complicated. There were also certain differences in their views of mathematics. Kolmogorov's diary contains the following entry, dating back to 8 January 1944: "Graduate student committee with Pontryagin and Plesner. Total chaos. Pontryagin keeps picking on Fomin and

Millionshchikov (not without justification, but with a special antipathy toward set-theorism)."

The counter-reform was launched in May 1978, during a discussion of school-related problems at a special meeting of the board of the Academy of Sciences' mathematics division, chaired by M. V. Keldysh. (This was practically the only time that Keldysh made a speech criticizing the reforms. He died in June of that year.) Kolmogorov was invited to attend the meeting. A critical resolution was passed and the decision was made to hold a special meeting of the mathematics division wholly devoted to school-related issues. This meeting took place on 5 December 1978. I was present at it, accompanying Kolmogorov, and have described the event in detail in a pamphlet "On the Situation of Mathematics Education in Secondary Schools in the USSR" (Abramov, 2003).

In preparing for the meeting, Kolmogorov made no attempt to draw his colleagues' votes over to his side. He spent a long time preparing his presentation and he prepared for it thoroughly — uncharacteristically for him, he prepared not just the key points, but a complete text (see Abramov, 2003). In his speech, he analyzed the situation quite critically and gave his assessment of the textbooks (criticizing in particular the textbook on space geometry). He indicated weak points and outlined a program of action.

The other speeches and presentations were quite critical. The harshest remarks were made by L. S. Pontryagin and A. N. Tikhonov. Only L. V. Kantorovich and S. L. Sobolev came out in favor of the ideas of the reform, calling for moderation. The final resolution, which was passed virtually unanimously, was quite severe: "The existing state of curricula and textbooks is acknowledged to be unsatisfactory." A committee on mathematics education — to be chaired by I. M. Vinogradov — was formed, and support was voiced for the RSFSR Ministry of Education's idea to develop a new curriculum and new textbooks, and to begin testing them out in practice (Abramov, 2003).

Subsequent decisions (1982) were made by Central Committee of the CPSU: geometry textbooks edited by Kolmogorov and Z. A. Skopets were removed from schools; A. V. Pogorelov's textbook was introduced in an accelerated fashion. These decisions were undoubtedly influenced by I. M. Vinogradov and A. N. Tikhonov, who were in direct contact with very powerful people in the Central Committee.

A considerable role both in shaping public opinion and in acquiring support for the "counter-reformers" in top government circles was played by L. S. Pontryagin. In 1979, he published an article entitled "Ethics and Arithmetic" (*Sotsialisticheskaya industriya*, 26 May 1979), in which, without actually referring to Kolmogorov by name, he accused him of irresponsibility and immorality. The article provoked a significant response, but no organizational steps were taken as a result of it. What tipped the scales in favor of the critics of the reform was an article that L. S. Pontryagin published in the main ideological journal of the Central Committee of the CPSU, *Kommunist* (1980, *14*). Through this article, the issue was elevated to the realm of ideology (although Kolmogorov's name was, again, not mentioned) and consequently now required a decision by the top leadership of the country. The political significance of the problem of school mathematics was emphasized by the academician A. A. Logunov, President (rector) of Moscow State University, in a speech at a session of the Supreme Soviet of the USSR.

The mood which determined the actions of the "counter-reformers" is well illustrated by an episode at which I was present. Somewhat unexpectedly, in May 1980, I found myself together with V. V. Firsov in the office of I. M. Vinogradov, who had by then already turned 90. As soon as the meeting began, Vinogradov summoned practically all of the people present in the institute — very famous members of the Academy. Vinogradov sat at the head of the table; V. V. Firsov and I, as guests, sat across from L. S. Pontryagin.

Vinogradov posed a question with which he was preoccupied: "Could we not replace these anti-government textbooks by September 1?" (i.e., within three months). Pontryagin said that unfortunately this was impossible — three months was not enough time to publish many millions of new textbooks, which would also have to be written first.

The difficulty of the "counter-reformers"' task resided in the lack of an alternative. M. A. Prokofiev, the USSR education minister, understood perfectly well that sudden changes were inadmissible in the inevitably conservative, gigantic system of secondary education (in the USSR, there were over 40 million schoolchildren), and diplomatically, but quite concertedly, opposed attempts at radical and rapid changes.

The opposition understood the need for an alternative as well. In 1979, two promptly prepared drafts for a mathematics curriculum were published. One of them was created by I. M Vinogradov's committee (*Matematika v shkole*, 1979, *2*), the other by a committee at the RSFSR Ministry of Education, headed by A. N. Tikhonov (*Matematika v shkole*, 1979, *3*).

The only educational texts that could in principle be put to use in schools were two books on elementary geometry ("Plane Geometry" and "Stereometry") by A. V. Pogorelov (1969, 1970), which had been published during the 1970s by Fizmatgiz, the State Physics–Mathematics Publishing House. Correspondence between Kolmogorov and the academician A. V. Pogorelov, in which Kolmogorov reviewed these texts, has survived. Kolmogorov had a favorable attitude toward the books and had recommended them for publication.

The problem was that Pogorelov had written his books as textbooks for pedagogical institutes. This meant that the text would have to be adapted urgently for schools and that a system of exercises would have to be developed. This work took up about a year and in September 1980, the new textbooks (Pogorelov, 1980) were introduced on a trial basis in the cities of Sevastopol and Kharkov (where Pogorelov was working at the time), under the aegis of the education ministry of the Ukraine.

The "anti-Kolmogorov" coalition broke up relatively quickly. A. N. Tikhonov became the head of authors' groups created at the RSFSR Ministry of Education, thus acquiring great administrative power. L. S. Atanasyan (a professor at the Lenin Pedagogical Institute in Moscow and the author of geometry textbooks for pedagogical institutes) and three physics professors from Moscow State University — E. G. Poznyak, V. I. Butuzov, B. V. Kadomtsev — began writing new geometry textbooks in 1979 (see, for example, Atanasyan *et al.*, 1979). Textbooks in algebra for grades 6–8 and algebra and elementary calculus for grades 9–10 were written by Professor Sh. A. Alimov (A. N. Tikhonov's student); the well-known methodologist

Yu. M. Kolyagin, who would soon become a member of the Academy of Pedagogical Sciences; Moscow Institute of Physics and Technology professor M. I. Shabunin; and V. A. Ilyin, a physics professor at Moscow State University (currently a member of the Russian Academy of Sciences).

Vinogradov's committee (after his death in 1982, L. S. Pontryagin took over as chairman) supported other authors' groups. The decision was made to retain but substantially revise the textbook in algebra for grades 6–8. S. A. Telyakovsky, who at the time was the secretary of the "Vinogradov committee," became the science editor of the project. The academician S. M. Nikolsky became the head of an authors' group that produced a textbook in arithmetic for grades 5 and 6, and the textbooks "Algebra 6–8" and "Algebra and Elementary Calculus 9–10" (see, for example, Nikolsky *et al.*, 1984). M. K. Potapov, a mathematics professor at Moscow State University, and N. N. Reshetnikov, a researcher at the Academy of Pedagogical Sciences, joined this collective.

The academician A. D. Aleksandrov, who replaced Kolmogorov in 1980 as head of the Scientific Methodological Council, became the chairman of an authors' group that wrote textbooks in geometry. Professor A. A. Werner of the Leningrad Pedagogical Institute and V. I. Ryzhik, one of Leningrad's best teachers, became his co-authors (see, for example, Alexandrov, Werner, and Ryzhik, 1984).

Also in Leningrad, an authors' group chaired by D. K. Faddeev — an associate (corresponding) member of the Academy of Sciences — began working on an algebra textbook (see, for example, Faddeev, 1983). Preliminary materials were prepared, but the work was soon interrupted by Faddeev's illness.

11 The 1980s

The extraordinary activity of the scientists from the USSR Academy of Scientists created a situation that was fundamentally new for Soviet schools. The existing programs (Kolmogorov's, Vinogradov's, and Tikhonov's) were all different from one another. Even more importantly, the USSR had a tradition of using the same textbooks for the whole country. A new problem arose: How could textbooks be diversified? This question was answered in 1981 when a new mathematics curriculum was created (*Matematika v shkole*, 1982, *2*).

In 1980, V. V. Firsov was appointed director of the mathematics education laboratory at the USSR Academy of Pedagogical Sciences. He went on to exert a great influence on mathematics education in Soviet schools during the 1980s. In effect, the waning of Kolmogorov's influence and the creation of competing authors' groups headed by famous mathematicians resulted in a certain ideological vacuum. Under these circumstances, V. V. Firsov unexpectedly became a leading figure and in large measure determined the subsequent course of events. This was facilitated by his good mathematical education (Moscow State University mathematics department), his deep interest in school-related problems and his experience with working in Moscow State University mathematics circles, his intellectual freedom, and his communication skills. He was convinced that the only way out of the existing situation was through constructive action. To this end, it was first and foremost necessary to create good working conditions for many different authors' groups.

Firsov pinned his hopes on the Academy of Sciences' mathematics division and the USSR Ministry of Education. He spearheaded and actively participated in the preparation of A. V. Pogorelov's school-level textbook, repeatedly meeting with Pogorelov and convincing him that it was necessary to make substantial revisions (something that was not easy to do, since Pogorelov was difficult to convince). Firsov also spearheaded the writing of methodological supplements for Pogorelov's textbook. He had it tested out in practice. Firsov had no illusions about Pogorelov's text, but he believed that putting it into use was the lesser of all evils, considering the instability of the situation and the inferior quality of the other available textbooks. Firsov also supported an authors' group working on the textbook "Algebra 6–8" (edited by S. A. Telyakovsky). The USSR Ministry of Education instructed Firsov's laboratory to conduct a comparison between the knowledge levels of students who had been educated using different textbooks.

The development of the 1981 curriculum was probably the decisive event that stabilized the situation. This curriculum, which was developed by Firsov, N. N. Reshetnikov, and myself, was founded on the following ideas:

1. A comparison of the three existing plans revealed that, despite differences between the orders in which topics were arranged, disagreements about the role of set theory in a school-level course, and different notions about how much time to spend on each topic, they had a great deal in common. The body of knowledge that all three plans proposed to cover was largely the same, overlapping by roughly 90%. This revealed the possibility of a compromise and at the same time showed that — despite all the loud rhetoric — the academic community effectively had supported the basic principles of the 1968 curriculum.

2. In order for schools to be able to use different textbooks — given the fact that these textbooks presented topics in different sequences and devoted different amounts of time to the same topics — the following solution was proposed.

(a) The freedom of action allowed to authors' groups was restricted in the following way: For each stage of education (grades 1–3, 4 and 5, 6–8, 9 and 10), a universal, mandatory level of knowledge was established and the general requirements that students had to fulfill in order to pass from one stage to the next were explicitly formulated. All of this was specified in a section of the curriculum entitled "The Contents of Education."

(b) A sensible structure for the exposition of the material had to be found and headings had to be established for the sections into which the fundamentally new curriculum was to be subdivided. The decision was made to structure the curriculum along the principal substantive-methodological lines of the course in mathematics. Sections such as "Geometric Figures," "Elements of Calculus," etc., appeared, and the key concepts were distributed among them.

(c) Every set of textbooks was accompanied by a special (variable) section of the curriculum entitled "Subject Planning." This section constituted a curriculum in the familiar sense of the word, i.e., it described the methodology of exposition recommended for the various different textbooks, apportioned the material among different classes, and precisely scheduled the presentation of the subjects in each class throughout the school year.

In this way, the curriculum of 1981, for the first time in the history of Soviet schools since the 1930s, made it possible to teach using different textbooks. The section on "The Contents of Education" ensured the uniformity of education; the section on "Subject Planning" provided for its variety.

12 Epilogue

During the 1970s–1980s, the reform of mathematics education gave rise to many heated discussions; echoes of these arguments can be heard to this day. Without claiming to know the truth, I would like to express my view of the "pluses" and "minuses" of the reform.

1. Over the course of the 20th century, beginning with the work of the International Committee chaired by Felix Klein (1908), the mathematics community actively discussed various ideas of modernizing the contents of the school course in mathematics — introducing the elements of calculus, analytic geometry, and vector algebra. These ideas were first fully implemented in Russia during the "Kolmogorov reform."

2. During the reform of the 1960s–1980s, the literature for students and teachers became substantially richer and more diverse. The reform greatly stimulated the development of mathematics education methodology: many new ideas and names appeared. Much of what was done during those years remains relevant both for Russia and for other countries.

3. For students interested in mathematics, the reform was a positive phenomenon; however, its aim — to construct a conceptually rich and at the same time universally accessible school course in mathematics — was not achieved. The knowledge and skill levels of a considerable part of the students were lower than expected.

4. The relative failure of the reform had several causes.

First, mistakes of a substantial nature were made. One of the biggest among them was the sharp reduction of problems in arithmetic, which play a considerable role in the mathematical formation of schoolchildren and in their preparation for the study of algebra and geometry. Second, the very radical and rapid changes in the course in geometry were also a mistake. The set of problems and style of exposition were substantially altered — and many teachers turned out to be unprepared for these changes.

There is a widespread view that the reform was harmed by the elements of set theory that were introduced into the curriculum. I do not believe that this view is correct. There were, indeed, certain excesses, but they did not take up too much time and did not play a decisive role. The complete "eradication" of sets from the curriculum was an excessive over-reaction.

5. Many difficulties and negative results stemmed from the unrealistic nature of the objectives and time constraints imposed on the reformers by top government officials. Worldwide experience confirms that education reform inevitably requires extended periods of time for developing new content and preparing teachers. The two or three years from the creation of the new textbooks to their large-scale implementation were clearly not sufficient.

6. The situation was severely exacerbated by the decision to make universal education mandatory (1973). The decision to make the presentation of mathematical materials more scientific (1966) and the sudden, rapid growth in the scale of education were in conflict with one another.

7. A negative role was also played by the — mildly speaking — reserved attitude of the professional community toward the reform. Institutions of higher learning failed to restructure their systems of entrance exams, as envisioned at the beginning of the reform: the conceptual intensity of the new curriculum presupposed an easing in the requirements for technical skill. Even more significant was the absence of any restructuring in the contents of education at pedagogical institutes in accordance with the aims and goals of the reform.

8. The crisis of the late 1970s and early 1980s ultimately played a positive role: premises were created for putting an end to identical schools, curricula, and textbooks. New authors' groups were formed.

9. Finally, it should be noted that for all the drama of the history of the reform, today, 40 years later, schools mainly use curricula and

many of the textbooks that were created during the 1960s–1980s. On the other hand, it is also true that today this fact serves to hold back development. By the beginning of the 21st century, very serious events had taken place in the USSR and Russia that now call for very serious changes in school-level education.

I will conclude with a quote from the academician A. P. Ershov. Starting in the 1960s, Ershov worked a great deal on the problems of teaching informatics in schools, and in 1988, when a decision was made to bring informatics into the educational system, he became the leader of the new reform. Naturally, therefore, he was vitally interested in the reform of mathematics education.

In 1988, Ershov moved from Novosibirsk to Moscow and, in May, he asked me to visit him. He was preparing a talk for the International Congress on Mathematics Education, which was to take place in Budapest in August. It subsequently emerged that Ershov was at this time terminally ill (he died in December of that year). Our conversation, during which he asked me at length about the details of the "Kolmogorov reform," lasted several hours. In the end, he asked me to leave him a selection of documents.

Speaking about the reform of the 1960s–1980s in Budapest, Ershov said:

> The general situation, of course, is not a return to what we had 20 years ago. A new generation of successful mathematicians has been brought up on Kolmogorov's reforms. These individuals play a dominant role in the finest expressions of our mathematical thought and practice. In addition, the teachers, for all the difficulties that they have gone through, have been introduced to a great number of fresh and innovative ideas and have thus risen to a new level of self-awareness. A. N. Kolmogorov's activities stirred the creative energy of his academician colleagues, as a result of which the mathematical literature on school-level mathematics has become much richer. The journal *Kvant* came into being, along with its wonderful collection of supplementary volumes.
>
> I believe that we cannot assess the meaning, role, and fate of the Kolmogorov reform while confining ourselves to its scientific–methodological content. Its fate cannot be separated from the fate

of education as a whole — of the country as a whole — during that decade which our media delicately refers to as "the period of stagnation."

I would put it this way: if the Kolmogorov reform as a movement turned out to be a failure, then its failure represents nothing more than the projection onto mathematics of a more global failure of another grand movement, which consisted in the transition to mandatory secondary education with the retention of all of the former rigidity, homogeneity, and authoritarianism in the content and methodology of school-level education...

Thinking about the dramatic fate of the Kolmogorov reform and its conceptual leader, I cannot avoid drawing a parallel to the fate of another brilliant contemporary of Andrey Nikolayevich Kolmogorov. I have in mind the writer and poet Boris Leonidovich Pasternak and his main work, *Doctor Zhivago.* The same degree of talent, high professionalism, and capacity for ordinary work. The same incompatibility with many aspects of quotidian reality. The same inseparable connection with culture and with nature. The same extreme jealousy and prejudice on the part of his colleagues. The same exalted sense of his uncompromising predestination for some universal human mission... (Tikhomirov, 1999).

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