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Slow decay of correlations for multi-dimensional Intermittent maps

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Abstract

Polynomial decay of correlations typically happens for intermittent maps with respect to Gibbs measures associated to (piecewise) Holder continuous potentials with exponent greater than 1.

1 Introduction

In this paper, we shall consider the decay of correlations for piecewise C^1-maps admitting indifferent periodic points (*intermittent maps*) with respect to Gibbs measures associated to piecewise Holder continuous potentials. For hyperbolic systems there exists a unique equilibrium state μ for a Holder continuous function f which is a Gibbs measure. Then (auto)correlations of Holder continuous functions g with respect to μ,

$$C_{g,g}(n) = |\int_X (gT^n) g d\mu - (\int_X g d\mu)^2|$$

decay exponentially fast and the limitting behaviour of $\sum_{i=0}^{n-1} gT^i$ obeys the normal distribution ([1]). On the other hand, the classical Thermodynamic Formalism easily fails in nonhyperbolic situation, e.g., the papers [14-20] discussed different phenomena from statistical point of view for intermittent systems. We shall be interested in the following problem.

Question When does the dynamical system (T, μ) have a large class of functions in which we have polynomial decay of correlations ?

We will give a partial answer to this question in multi-dimensional situation. There are a few results describing polynomial decay of correlations for one-dimensional intermittent maps. Papers [6] and [7] contain results establishing polynomial bounds on correlations for a large class of functions with respect to an absolutely continuous invariant measure. Since the existence of indifferent fixed points causes the failure of bounded distortion, the measures are not Gibbs. On the other hand, the paper [8] discussed slow decay of correlations with respect to the Gibbs measure for subshift of finite type, where functions of summable variations were considered instead of Holder continuous functions. In section 2, we collect previous results and some observations for potentials of summable variations. In section 3, we show the convergence to Gibbs measures for piecewise invertible maps T defined on a subset of a compact metric space associated to piecewise Holder continuous functions and we establish bounds on correlations with respect to the Gibbs measure for piecewise Holder continuous functions. The bounds that we obtain are expressed by sizes of cylinders (see Theorem 3.1) which typically decays polynomially fast for maps admitting indifferent fixed points. In section 4, we apply our theorems to such intermittent maps so that we have a large class of functions in which we have polynomial decay of correlations with respect to Gibbs measures associated to Holder continuous functions with exponent greater than 1.

2 Preliminaries.

Let (X,d) be a bounded metric space, $\mathcal{F}$ be the σ-algebra of Borel subsets of X and $Q=\{X_a\}_{a\in I}$ be a disjoint partition of X with $X_a \in \mathcal{F}(\forall a \in I)$. We assume that there exists a compact metric space $\bar{X} \supseteq X$ such that $X_0 = \bigcup_{a\in I} intX_a$ is an open dense subset of $\bar{X}$. Let $T : X \to \bar{X}$ be a piecewise invertible map with finite range structure, i.e., Q is a generating partition, $T|_{X_a} : X_a \to TX_a$ is a homeomorphism $(\forall a \in I)$ and $\mathcal{U} = \{T^n(intX_{a_1\ldots a_n}) : \forall X_{a_1\ldots a_n}, \forall n > 0\}$ consists of finitely many open subsets of $\bar{X}$. We call the quadraple $(T, X, Q, \mathcal{U})$ a *picewise invertible system with finite range structure*([13-15]). We denote the local inverse $(T|_{X_a})^{-1}$ by ψ_a and $(T^n|_{X_{a_1\ldots a_n}})^{-1}$ by $\psi_{a_1\ldots a_n}$. For a function f, we define the Perron Frobenius operator $\mathcal{L}_f$ by

$$\mathcal{L}_f g(x) = \sum_{y\in T^{-1}x} \exp f(y) g(y).$$

We suppose that there is a positive number p and a Borel probability measure ν supported on X satisfying the equation, $\mathcal{L}_f^* \nu = p\nu$. Sufficient conditions for finite to one Markov maps to admit such ν and p were established in [4]. For infinite to one Bernoulli maps(i.e., $TX_a = X, \forall X_a \in Q$), the following conditions were obtained in [20](cf. [12],[9]).

(C-1) $\exists 0 < L_a < \infty$ such that $d(\psi_a x, d\psi_a y) \le L_a d(x,y)(\forall a \in I)$ and $\exists \theta \in \mathbf{R}$ such that $\sum_{a\in I} L_a^\theta < \infty$.

(C-2) $\exists 0 < L_f < \infty$ such that $|f(x) - f(y)| \le L_f d(x,y)^\theta, (\forall x, y \in X_a \in Q)$.

(C-3) $\exists 0 < K < \infty$ such that $\mathcal{L}_f 1(x) = \sum_{a \in I} \exp f(\psi_a x) \le K (\forall x \in X_0)$.

Definition For a function f and for $k \geq 1$ we define

$$var_k(f) = \sup_{X_{a_1 \ldots a_k}} \sup_{x,y \in X_{a_1 \ldots a_k}} \{|f(x) - f(y)|\}.$$

Definition A fixed point x_0 is said to be *indifferent* with respect to f if $f(x_0) = \log p$. A periodic point x_0 with period q is said to be indifferent with respect to f if $1/q \sum_{i=0}^{q-1} f(T^i x_0) = \log p$.

Remark A. When T is piecewise C^1-invertible, ν is the Lebesgue measure and $f = -\log|\det DT|$, the above definition coincides with the usual one of the indifferent periodic point(cf. [14]).

The next result gives a relation between summable variations and the existence of indifferent periodic points.

Proposition 2.1 *Suppose that f satisfies the summable variation i.e., $\sum_{k=1}^{\infty} var_k(f) < \infty$. Then there is no indifferent periodic points with respect to f. (Cf.[14,15]).*

Remark B. If there is an indifferent fixed point with respect to f, then $\sup_X f \geq \log p$. Since $d(\nu T|_{X_a})/d(\nu|_{X_a}) = \exp(\log p - f)$, the property $\log p > \sup_X f$ which gives $\sup_{x \in X} (\sum_{i=0}^{n-1} fT^i(x) - n \log p) < 0 (\forall n > 0)$ just implies the expanding property (in case when $f = -\log|\det DT|$ and ν is the Lebesgue measure.cf.[4,5]).

Proof of Proposition 2.1. Let x_0 be an indifferent fixed point with respect to f and let $X_{a_1 \ldots a_n}$ be a cylinder containing x_0. Then we see that

$$\sup_{x,y \in X_{a_1 \ldots a_n}} \exp(\sum_{i=0}^{n-1} fT^i(x) - \sum_{i=0}^{n-1} fT^i(y)) = \sup_{x,y \in X_{a_1 \ldots a_n}} \frac{\exp(\sum_{i=0}^{n-1} fT^i(x) - n \log p)}{\exp(\sum_{i=0}^{n-1} fT^i(y) - n \log p)}$$

$$\geq \frac{\exp(nf(x_0) - n \log p)}{\inf_{y \in X_{a_1 \ldots a_n}} \exp(\sum_{i=0}^{n-1} fT^i(y) - n \log p)}$$

$$\geq \frac{1}{\int_{T^n X_{a_1 \ldots a_n}} \exp(\sum_{i=0}^{n-1} fT^i - n \log p)(\psi_{a_1 \ldots a_n} y) d\nu(y)} = \frac{1}{\nu(X_{a_1 \ldots a_n})} \to \infty (n \to \infty).$$

On the other hand, if f is of summable variation, we have a finite bound of LHS. This is a contradiction. □.

Let $\mathcal{V}$ denote the finite disjoint partition generated by $\mathcal{U}$. Define for $x, x' \in V \in \mathcal{V}$,

$$C_f(x, x') = \sup_n \sup_{(a_1 \ldots a_n) \in \mathcal{A}_n} \sum_{i=0}^{n-1} (f(T^i \psi_{a_1 \ldots a_n} x) - f(T^i \psi_{a_1 \ldots a_n} x')).$$

Next result allows us to establish the existence of a Gibbs measure associated to potentials of summable variation for symbolic systems.

Proposition 2.2 *Let f be a potential of summable variation. Suppose that $\{X_{b_1\dots b_l}\}_{l>0} \to \{x\}$ as $l \to \infty$. Let $\{x'_n\}_{n>0}$ be a sequence of points in X such that $x'_n \in X_{b_1\dots b_n} \forall n > 0$. Then $C_f(x, x'_n) \to 0$ as $n \to 0$.*

Proof. It is easy to see that for $x, x' \in V \in \mathcal{V}, C_f(\psi_{a_1\dots a_k}x, \psi_{a_1\dots a_k}x') \leq \sum_{i=k}^{\infty} var_i(f)$. This completes the proof. □.

Let (Σ, σ) be the symbolic dynamics of T with respect to the generationg partition Q and $\rho : \Sigma \to X$ be the factor map. ¿From Proposition 2.2 a σ- invariant measure which is Gibbs for the function $f \circ \rho$ is obtained by applying P.Walter's method in ([12]) . For the existence of a T-invariant Gibbs measure (absolutely continuous with respect to ν), summable variation of f is not sufficient. In fact, even if f has summable variation, we do not know for $x, x' \in V \in \mathcal{V}$ belonging to different cylinders (i.e., $x \in X_a, x' \in X_b, a \neq b$) whether $C_f(x, x') \to 0$ as $d(x, x') \to 0$.

In the rest of this section, we remark that for the symbolic system (Σ, σ) Prospositons 3.3 and Theorem 3.2 in [16] which were obtained by Markov approximations method developed in [2-3] allow us to have polynomial bounds on correlations with respect to the Gibbs measure for potentials of summable variations immediately and the bounds are the same as those which were obtained by M.Pollicott in [8]. More precisely, let $h = d\mu/d\nu$. $i_j(k)$ denotes $i_j^1 \dots i_j^k$ and $X_{i_0(k)\dots i_l(k)}$ denotes the cylinder of rank $(k + l - 1), X_{i_0(k)} \cap T^{-1}X_{i_1(k)} \cap \dots \cap T^{-(l-1)}X_{i_l(k)}$. (See Remark C in [16]). In order to apply Proposition 3.3 and Theorem 3.2 in [16], we need to bound the following quantities.

(a) $\sup_{x,y \in X_{i_0(k)\dots i_l(k)}} \exp \sum_{i=0}^{l-1}(fT^i(x) - fT^i(y))$,

(b) $\sup_{x,y \in X_{i_0(k)\dots i_l(k)}} h(x)/h(y)$,

(c) $\sup_{x,y \in X_{i_0(k)\dots i_l(k)}} hT^l(x)/hT^l(y)$.

If f is of summable variation, then

$$(a) \leq \exp(\sum_{j=k+1}^{\infty} var_j(f)).$$

Since for $x, y \in X_{a_1\dots a_n}$

$$h(x)/h(y) \leq \exp(\sum_{j=n}^{\infty} var_j(f)),$$

both (b),(c) are bounded by $\exp(\sum_{j=k}^{\infty} var_j(f))$. Consequently the errors arising from Markovian approximations, $\Delta_3(k)$ which was given in Proposition

3.3 ([16]) can be bounded by $O(\sum_{j=k}^{\infty} var_j(f))$. Since the term related to the Doblin condition, $(1-s/2)^{[n/m]}$ in Theorem 3.2 ([16]) is stretched exponential, if $var_j(f) = O(j^{-r})(r\theta > 1)$ then for $n > k(n), C_{f,f}(n) \leq O(k(n)^{-r\theta+1})$. Taking the second parameter $k(n) = n^{1-\epsilon}(\epsilon > 0)$ gives

$$C_{f,f}(n) \leq O(n^{(1-\epsilon)(-r\theta+1)}) = O(n^{-r\theta+1+\epsilon(r\theta-1)}).$$

In particular, for $\theta = 1$,

$$C_{f,f}(n) \leq O(n^{-(r-1-\epsilon(r-1))})(\forall \epsilon > 0).$$

3 Main results

For the original intermittent map T, we shall establish the convergence of $\{\mathcal{L}_{f-\log p}^{n} 1\}$ to a density of the Gibbs measure with respect to ν, which allows us to have a nice property of the limit point.

Theorem 3.1 (Main Theorem.) *Let $(T, X, Q, \mathcal{U})$ be a piecewise invertible systems satisfying Bernoulli condition(i.e., $TX_a = X$ for $\forall a \in I$). Suppose that (C-1,2,3) are satisfied. Assume further that (C-4) $\sum_{i=1}^{\infty} \sigma(i)^{\theta} < \infty$. Then f is of summable variation and $\sum_{i=n}^{\infty} var_i(f) \leq L_f \sum_{i=0}^{\infty} \sigma(i)^{\theta}$. For a bounded function g (with respect to ν) satisfying (C-2), we have that for $m \geq 1, k \geq 1$*

$$||\mathcal{L}_{f-\log p}^{m+k} g - \mathcal{L}_{f-\log p}^{m} g||_{\infty} \leq O(\sum_{i=m}^{\infty} (\sigma(i))^{\theta})$$

and we have a bounded function $h > 0$ satisfying

$$||\mathcal{L}_{f-\log p}^{n} g - (\int_X g d\nu) h|| \leq O(\sum_{i=n}^{\infty} \sigma(i)^{\theta})$$

and

$$|h(x) - h(y)| \leq \sum_{i=m}^{\infty} (\sigma(i))^{\theta} (\forall x, y \in X_{a_1 \ldots a_m}, \forall X_{a_1 \ldots a_m}.)$$

Corollary 3.1 *Let $\mu = h\nu$. Then μ is a T-invariant Gibbs measure satisfying*

$$C_{g,g}(n) = |\int_X (gT^n) g d\mu - (\int_X g d\mu)^2| \leq \sum_{i=n}^{\infty} (\sigma(i))^{\theta}.$$

Lemma 3.1 *Suppose that (C-2,4) are satisfied. Then f is a potential of summable variation and $\{\mathcal{L}_{f-\log p}^{n} 1\}$ is uniformly bounded. Further we have for a bounded function g satisfying (C-2)*

$$|\mathcal{L}_{f-\log p}^{m} g(\psi_{b_1 \ldots b_k} x) - \mathcal{L}_{f-\log p}^{m} g(\psi_{b_1 \ldots b_k} y)| \leq \exp(\sum_{i=0}^{\infty} var_i(f)) O(L_f \sum_{i=k}^{\infty} \sigma(i)^{\theta})).$$

Proof of Lemma 3.1 The first assertion is immediate from the definition of $var_i(f)$. Note that

$$(1) \quad \sup_{x,y \in T^n X_{a_1 \ldots a_n}} \frac{\exp(\sum_{i=0}^{m-1} f(\psi_{a_i \ldots a_n} x))}{\exp(\sum_{i=0}^{m-1} f(\psi_{a_i \ldots a_n} y))} \le \exp(\sum_{i=0}^{\infty} var_i(f)).$$

(The property is just the Renyi condition when $f = -\log|\det DT|$). Then the second assertion easily follows from the conformality of ν. Similarly the last assertion is obtained by (1) and the following inequalities.

$$|\mathcal{L}^m_{f-\log p} g(\psi_{b_1 \ldots b_k} x) - \mathcal{L}^m_{f-\log p} g(\psi_{b_1 \ldots b_k} y)|$$

$$\le \sum_{a_1 \ldots a_m} p^{-m} \exp(\sum_{i=0}^{m-1} f(\psi_{a_i \ldots a_m b_1 \ldots b_k} x) |1 - \exp(\sum_{i=0}^{m-1} (f(\psi_{a_i \ldots a_m b_1 \ldots b_k} x) - f(\psi_{a_i \ldots a_m b_1 \ldots b_k} y))|$$

$$+ \sum_{a_1 \ldots a_m} p^{-m} \exp(\sum_{i=0}^{m-1} f(\psi_{a_i \ldots a_m b_1 \ldots b_k} y)) L_g d(\psi_{a_1 \ldots a_m b_1 \ldots b_k} x, \psi_{a_1 \ldots a_m b_1 \ldots b_k} y). \square.$$

Proof of Theorem 3.1. We can prove the theorem along the line of the Proof in [11]. It follows from Lemma 3.1 that $\exists 0 < K_1 < \infty$ satisfying $K_1^{-1} < \mathcal{L}^m_{f-\log p} g < K_1 (\forall m \ge 0)$. Then for $\forall k \ge 1, \forall m \ge 0$, we have that

$$K_1^{-2} \mathcal{L}^m_{f-\log p} g(x) < \mathcal{L}^{m+k}_{f-\log p} g(x) < K_1^2 \mathcal{L}^m_{f-\log p} g(x).$$

We put $K_1^{-2} = r_0, K_1^2 = R_0$, and $C = \exp(\sum_{i=0}^{\infty} var_i(f))$. Since (2):

$$\mathcal{L}^{k+m}_{f-\log p} g(x) - r_0 \mathcal{L}^m_{f-\log p} g(x) - C^{-1} \sum_{a_1 \ldots a_m} \int_{X_{a_1 \ldots a_m}} (\mathcal{L}^k_{f-\log p} g(y) - r_0 g(y)) d\nu(y)$$

$$= \sum_{a_1 \ldots a_m} \mathcal{L}^k_{f-\log p} g(\psi_{a_1 \ldots a_m} x) \exp(\sum_{i=0}^{m-1} fT^i(\psi_{a_1 \ldots a_m} x)) p^{-m}$$

$$-r_0 \sum_{a_1 \ldots a_m} \exp(\sum_{i=0}^{m-1} fT^i(\psi_{a_1 \ldots a_m} x)) p^{-m} g(\psi_{a_1 \ldots a_m} x)$$

$$-C^{-1}(\sum_{a_1 \ldots a_m} \int_{X_{a_1 \ldots a_m}} (\mathcal{L}^k_{f-\log p} g(y) - r_0 g(y)) d\nu(y)),$$

the conformality of ν allows us to have a lower bound of (2):

$$C^{-1} \sum_{a_1 \ldots a_m} \int_{X_{a_1 \ldots a_m}} (\mathcal{L}^k_{f-\log p} g(\psi_{a_1 \ldots a_m} x) - \mathcal{L}^k_{f-\log p} g(y)) d\nu(y)$$

$$-C^{-1} r_0 \sum_{a_1 \ldots a_m} \int_{X_{a_1 \ldots a_m}} (g(\psi_{a_1 \ldots a_m} x) - g(y)) d\nu(y).$$

Then it follows from Lemma 3.1 that

$$\mathcal{L}^{k+m}_{f-\log p} g(x) - r_0 \mathcal{L}^m_{f-\log p} g(x) - C^{-1} \sum_{a_1 \dots a_m} \int_{X_{a_1 \dots a_m}} (\mathcal{L}^k_{f-\log p} g(y) - r_0 g(y)) d\nu(y)$$

$$\geq C^{-1}(-O(\sum_{i=m}^{\infty} \sigma(i)^\theta) - L_g \sigma(m)^\theta).$$

Consequently, we have the lower bound $C^{-1}O(\sum_{i=m}^{\infty} \sigma(i)^\theta)$ and that

$$\mathcal{L}^{m+k} g(x) = (\mathcal{L}^{m+k} g(x) - r_0 \mathcal{L}^k g(x)) + r_0 \mathcal{L}^m g(x)$$

$$\geq -C^{-1} O(\sum_{i=m}^{\infty} \sigma(i)^\theta) + C^{-1} \sum_{a_1 \dots a_m} \int_{X_{a_1 \dots a_m}} (\mathcal{L}^k g(x) - r_0 g(x)) d\nu(x) + r_0 \mathcal{L}^m g(x)$$

$$= \mathcal{L}^m g(x)(-C^{-1} K_1^{-1} O(\sum_{i=m}^{\infty} \sigma(i)^\theta) + C^{-1} K_1^{-1} \sum_{a_1 \dots a_m} \int_{X_{a_1 \dots a_m}} \mathcal{L}^k g(y) d\nu(y)$$

$$+ r_0(1 - C^{-1} K_1^{-1} \sum_{a_1 \dots a_m} \int_{X_{a_1 \dots a_m}} g(x) d\nu(y))).$$

Then we see that $\exists \alpha(m) < 1$ and $\beta(m,k) > 0$ (for sufficiently large m)

$$\mathcal{L}^{m+k}_{f-\log p} g(x) \geq \mathcal{L}^m_{f-\log p} g(x)(\alpha(m) r_0 + \beta(m,k)).$$

Replacing $\mathcal{L}^{m+k} g(x) - r_0 \mathcal{L}^m(x)$ by $R_0 \mathcal{L}^m g(x) - \mathcal{L}^{m+k} g(x)$ a similar argument allows us to have $\delta(m,k) > 0$ such that

$$\mathcal{L}^{m+k}_{f-\log p} g(x) < \mathcal{L}^m_{f-\log p} g(x)(\alpha(m) R_0 + \delta(m,k)).$$

Put $r_1 = \alpha(m) r_0 + \beta(m,k)$, $R_1 = \alpha(m) R_0 + \delta(m,k)$. Then we have

$$r_1 \mathcal{L}^m_{f-\log p} g(x) < \mathcal{L}^{m+k}_{f-\log p} g(x) < R_1 \mathcal{L}^m_{f-\log p} g(x).$$

Inductively we have two sequences:

$$r_n = \alpha(m) r_{n-1} + \beta(m,k), R_n = \alpha(m) R_{n-1} + \delta(m,k)$$

and we can show that

$$\lim_{n \to \infty} r_n = \frac{\beta(m,k)}{1 - \alpha(m)} = \gamma(m,k) + O(\sum_{i=m}^{\infty} \sigma(i)^\theta),$$

where

$$\gamma(m,k) = \frac{\sum_{a_1 \dots a_m} \int_{X_{a_1 \dots a_m}} \mathcal{L}^k_{f-\log p} g(y) d\nu(y)}{\sum_{a_1 \dots a_m} \int_{X_{a_1 \dots a_m}} g(y) d\nu(y)},$$

$$\lim_{n\to\infty} R_n = \frac{\beta(m,k)}{1-\delta(m,k)} = \gamma(m,k) + O(\sum_{i=m}^{\infty} \sigma(i)^{\theta})$$

and

$$(\lim_{n\to\infty} r_n)\mathcal{L}^m_{f-\log p}g(x) < \mathcal{L}^{m+k}_{f-\log p}g(x) < (\lim_{n\to\infty} R_n)\mathcal{L}^m_{f-\log p}g(x).$$

Integrating the inequality

$$|\mathcal{L}^{m+k}g(x) - \gamma(m,k)\mathcal{L}^m g(x)| \leq O(\sum_{i=m}^{\infty} \sigma(i)^{\theta})$$

gives $|\gamma(m,k) - 1| \leq O(\sum_{i=m}^{\infty} \sigma(i)^{\theta})$. Finally we have

$$|\mathcal{L}^{m+k}_{f-\log p}g(x) - \mathcal{L}^m_{f-\log p}g(x)|$$

$$\leq |\mathcal{L}^{m+k}g(x) - \gamma(m,k)\mathcal{L}^m g(x)| + |\gamma(m,k) - 1||\mathcal{L}^m g(x)| \leq O(\sum_{i=m}^{\infty} \sigma(i)^{\theta})\square.$$

Proof of Corollary 3.1. Since we have $C \equiv \exp(\sum_{i=0}^{\infty} var_i(f)) \geq 1$ such that

$$\frac{\frac{d(\nu T^n|_{X_{a_1\ldots a_n}})}{d(\nu|_{X_{a_1\ldots a_n}})}(x)}{\frac{d(\nu T^n|_{X_{a_1\ldots a_n}})}{d(\nu|_{X_{a_1\ldots a_n}})}(y)} < C,$$

we can easily see the Gibbs property of $\mu\square$.

Theorem 3.2 *Suppose that all conditions in Theorem 3.1 are satisfied. Assume further that*

$$(C-5)\sum_{k=1}^{\infty} k(\sum_{i=k}^{\infty} \sigma(i)^{\theta}) < \infty.$$

Then the central limit theorem holds for a bounded function g satisfying (C-2).

Proof. We can apply Proposition 5.2 in [19]. $\square$.

4 Examples– Maps admitting indifferent periodic points

Example 1 (A one-parameter family of maps on the interval [0,1]) For $0 < \beta < 1$, define $T_\beta(x) = \frac{x}{(1-x^\beta)^{1/\beta}}$ on $[0,(1/2)^{1/\beta})$ and $T_\beta(x) = \frac{x}{(1/2)^{1/\beta}-1} + \frac{1}{1-(1/2)^{1/\beta}}$ on $[(1/2)^{1/\beta}, 1]$. T_β admits an indifferent fixed point 0. Since $\sigma(i) = i^{-1/\beta}$, for a potential f satisfying (C-2) with $\theta > \beta$, we can apply Theorem 3.1 and Corollary 3.1. If $\theta > 3\beta$ CLT holds. (Cf.[13-20].)

The next two examples satisfy $\sigma(i) = i^{-1}$. For a potential f satisfying (C-2) with $\theta > 1$, we can apply Theorem 3.1 and Corollary 3.1. If $\theta > 3$, then CLT holds.

Example 2 (Brun's map) Let $X = \{(x_1, x_2) \in \mathbf{R}^2 : 0 \leq x_2 \leq x_1 \leq 1\}$ and for $i = 0, 1, 2, X_i = \{(x_1, x_2) \in X : x_i + x_1 \geq 1 \geq x_{i+1} + x_1\}$, where we put $x_0 = 1$ and $x_3 = 0$. T is defined by $T(x_1, x_2) = (\frac{x_1}{1-x_1}, \frac{x_2}{1-x_1})$ on $X_0, T(x_1, x_2) = (\frac{1}{x_1} - 1, \frac{x_2}{x_1})$ on $X_1, T(x_1, x_2) = (\frac{x_2}{x_1}, \frac{1}{x_1} - 1)$ on X_2. T admits an indifferent fixed point $(0, 0)$. (Cf. [10,14,20].)

Example 3 (A skew product map which is related to Diophantine approximation in inhomogeneous linear class) Let X be $\{(x_1, x_2) \in \mathbf{R}^2 : 0 \leq x_2 \leq 1, -x_2 \leq x_1 \leq -x_2 + 1\}$. T is defined by $T(x, y) = (1/x_1 - [(1 - x_2)/x_1] + [-(x_2/x_1)], -[-(x_2/x_1)] - (x_2/x_1))$. T admits indifferent periodic points $(1, 0)$ and $(-1, 1)$ with period 2.(Cf.[13-20]).

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