Notes on the Wigner Representation Theory of the Poicaré Group, Localization and Statistics

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Bert Schroer

Freie Universität Berlin

Institut für theoretische Physik

Arnimalle 14 14195 Berlin

e-mail schroer@physik.fu-berlin.de

June 1996

Abstract

It has been known that the Wigner representation theory for positive energy orbits permits a useful localization concept in terms of certain lattices of real subspaces of the complex Hilbert -space. This framework was recently used by Brunetti, Guido and Longo in order to construct interaction-free nets of local algebras without using non-unique "free field coordinates". Here it is shown that this structure preempts among other things properties of localization and braid-group statistics in low-dimensional QFT. It also sheds some light on string-like localization properties of Wigner's "continuous spin" representations. We formulate a constructive nonperturbative program to introduce interactions into such an approach based on the Tomita-Takesaki modular theory. The new aspect is the deep relation of the latter with the scattering operator.

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1 Introduction.

The main aim of this paper is the exploration and extension of Wigner's representation - theoretic approach to relativistic quantum theory [1] for the construction of particles and free fields in the context of d=2+1 abelian braid-group statistics (the particles are often referred to as "anyons "or abelian "plektons "). In this way one may hope to obtain a more direct understanding of the origin and the physical consequences of plektonic statistics than that by the somewhat vague (and often indirect and complicated) method via imposing Chern-Simons perturbation on standard fermionic matter, using the formalism of functional integrals (in a region where the necessary and sufficient conditions for Feynman-Kac representability of quantum physics is strictly speaking violated!).

The main issue in the adaptation of Wigner's theory to this question is the problem of "localization". The sharp and covariant concept used in these notes is not that of Newton and Wigner [2], but the more recent idea of localization via suitably defined real subspaces [3] of the complex Wigner representation space. This concept relates to the rather universal and deep mathematical Tomita-Takesaki modular theory for von Neumann algebras [4] which has been known to connect to such diverse looking physical issues as the existence of antiparticles (TCP), the stability of temperature states, the Unruh-Hawking effect [5] and many other structures of "Local Quantum Physics "[6]. The underlying "modular" wedge-localization has no counterpart in classical field theory or in nonrelativistic quantum theory (as the various quantization approaches and the interaction picture formalism). *Among all concepts in QFT it is the most intrinsic one,* and it achieves something which in e.g.in coordinate- free differential geometry was accomplished already a long time ago, namely the separation of intrinsic properties from mainly accidental "coordinatizations".

In section 2 entitled "ancient history " we review those aspects of Wigner's theory which are relevant for our purpose. This theory was the first successful attempt to formulate a framework of relativistic quantum theory which was *not based on quantization* parallelism to classical theories. Mainly through algebraic QFT, its spirit has been kept alive in present day field theory. To most physicists of the younger generation who are familiar with the more popular Lagrangian quantization approach, the Wigner theory remained unknown because modern texts often identify QFT with the Lagrangian approach.

A notable exception besides the (perhaps too brief on this issue) reference [6] is S.Weinberg's recent book [8]. There the Wigner approach is largely used in order to *support* the Lagrangian formulation of QFT and the path-integral approach (but the possibility of other non-Lagrangian approaches to interactions is not ruled out, as emphasized by the author), whereas here our aim is quite different, namely to understand areas which are not covered (and probably never will be) by the Lagrangian framework. It is worthwhile to mention that all the pertinent results on chiral conformal QFT as well as a large part of results on massive d=1+1 theories have been obtained by nonperturbative non-Lagrangian methods such as representation theory, S-matrix bootstrap, formfactor program etc. A Lagrangian name, where it appears, usually only served for "baptizing" the model in the traditional way. Sometimes, as in the case of the renaming of the current algebras associated to the generalized Thirring model (with its scale invariant fixed points in multicoupling space) of the 60's and 70's into the Wess-Zumino-Witten-Novikov models of the 80's , even this was unnessecary since the latter is a special case of the former at the scale-invariant subset in terms of "different field coordinates").

In fact the fields which appeared in the 1974 work on conformal field theory [7] were so far away from Lagrangian-and Euclidean- (and even from Wightman-)fields , that the problem of model construction was not pursued as a result of Zeitgeist prejudices. Nowadays it is very natural to consider charged fields which have nontrivial source and range projectors onto super-selection sectors but at that time this appeared as going against Wightman as well as Euclidean field theory.

In fact one would be very surprised if plektonic d=2+1 fields or d=3+1 "continuous spin" fields are not of this new *non-Lagrangian* kind. An educated guess is that all fields in d$\geq$2+1 with weaker than compact localization properties are of this new kind. This is in agreement with an old result of Yngvason about the obstructions posed by the d=3+1, m=0 Wigner "continuous spin" representation within a Wightman framework [12].

In section 3 entitled "recent history ", we briefly present those aspects of the Tomita Takesaki modular theory which are relevant in the present context. In particular we explain, how by introducing *real* subspaces of the Wigner representation space via a Tomita involution, one may implement a localization concept which is more useful for our purpose than the Newton-Wigner localization. These ideas, although known to some experts,

unfortunately had never been published in an accessible way. [3]. Section 3 also contains a brief sketch of a direct construction of local nets from the so called "wedge localization"[9] which, in the case of free bosonic theories, will be contained in forthcoming work of Brunetti, Guido and Longo [10]

The fourth section explains why the adaptation of Wigner's theory for d=2+1 anyonic spin is not compactly localizable, but still falls into the weaker spacelike cone (or semi-infinite string)-localizable category and presents the corresponding "anyonic" statistics in terms of a "twist "which is necessary in order to balance the dual of the wedge-localized real subspace (i.e. its symplectic complement) with the geometric (causal) dual in the sense of commutation relations of fields. A useful covariantization is that given by Gaberdiel and Fredenhagen. [11]

In section 5, we comment on a an interesting topological obstruction against Haag duality for non -simply connected regions which occurs in certain zero mass theories including the free Maxwell theory. These topological obstructions are absent in massive theories, but they are typical of local gauge theories i. e. those Lagrangian theories for which long range interaction can presently only be described by an indefinite metric vector potential and a formal return to quantum physics is not possible without invoking local gauge invariance. Algebraic QFT was not able (for good reasons in my opinion) to incorporate such classical ideas which have their natural formulation in fibre bundle theory. But through such duality obstructions as discussed in section 5, the algebraic approach at least perceives that there is a deep problem on the level of local quantum physics. In my view an adequate treatment of this problem can only be given in a framework of interaction which uses concepts which are characteristic of relativistic QFT as e.g. the modular properties used in this work.

In section 6 we show that Wigner's zero mass "continuous spin" representation falls into this weaker space-like cone localization category. In fact the natural covariant description [12] is in terms of semi-infinite light-like strings, very similar to the Gaberdiel-Fredenhagen covariantization of the d=2+1 Wigner anyons mentioned in section 4.

The last section contains some speculative attempts of incorporating an intrinsic notion of interaction via an ansatz for an "interacting" Tomita involution J within the Fock-space defined by scattering theory. This ansatz generalizes the J_0 obtained from the Wigner theory and does not involve the interaction picture and time-ordering but uses only concepts of general quan-

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tum field theory. In this last section I also speculate on several presently insufficiently understood problems which have connections with modular ideas.

2   Ancient History.

In 1939 Wigner [1] classified the irreducible ray-representations of the Poincaré-group (or what amounts to the same in this case, the irreducible vector-representations of its covering). His main motivation was to understand in intrinsic physical terms the ever increasing "zoo" of linear relativistic (higher spin) field equations of those days, which were proposed in the aftermath of the Dirac equation. For this purpose he had to extend the Frobenius method of induced representations from finite groups to the non-compact Poincaré-group, a mathematical novelty which gave rise to new mathematical developments in group representations, [13]. He first determined all transitive momentum-space orbits under the Lorentz-group and then classified the (isomorphic for different momenta on the same orbit) the fixpoint-group of a conveniently chosen reference vector p_R on the orbit. The "induction" was done with the help of this "little group" and a suitably defined family of "boosts" served to identify the fixpoint-groups at different orbit points.

For the positive energy orbits $p^2 = m^2, p_0 > 0$ and $p^2 = 0, p_0 > 0$ (the only orbits of relevance for our purpose) in d=3+1, the (coverings of the) little groups are $SU(2)$ resp. $\widetilde{E}^{(2)}(2)$ (the two-fold covering of the two-dimensional Euclidean group).

The massive $[m, s]$-representations are most conveniently described in terms of 2s+1 component wave-function spaces:

$$H = \left\{ \underline{\psi}(p) \Big| \sum_{s_3} \int |\psi_{s_3}(p)|^2 \frac{d^3p}{2\omega} < \infty \right\} \tag{1}$$

on which the Lorentz transformation acts as:

$$(U(\Lambda)\underline{\psi})(p) = D^{(s)}(R_W(\Lambda, p))\underline{\psi}(\Lambda^{-1}p) \tag{2}$$

with R_W being the $\Lambda-$ and p-dependent (nonlocal) Wigner rotation.

In the $m^2 = 0$ case one has a greater wealth for the representation theory of the little group. In case the "translations" of $\widetilde{E}(2)$ are mapped to zero, one obtains the family of one-component semi-integers-helicity representations:

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$$(U(\Lambda)\psi)(p) = e^{is\Phi_W(\Lambda,p)}\psi(\Lambda^{-1}p) \qquad (3)$$

The conversion of the one-component Wigner wave functions into e.g. the standard local helicity description in terms of field strength $F_{\mu\nu}$ is well-known .

Explicit formulas for the Wigner phase Φ_W as well as the previous Wigner rotation R_W are to be found already in the original paper as well as in S. Weinberg's recent book.[8] Also the extensions to the full group including space and time reflections may be found in the literature. In the following we will need the formula for the TCP$=\theta$ acting on the (doubled, if particles are not self-conjugate) $[m, s]$-representation as:

$$\theta \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = D^{(s)}(i\sigma_2) \begin{pmatrix} \psi_-^* \\ \psi_+^* \end{pmatrix}, \quad \pm=(\text{anti})\text{particle doubling} \qquad (4)$$

Before we relate this TCP-transformation of the Wigner theory to a new localization concept, some more historical remarks are in order.

Wigner's work, although little noticed at the time (at least by the community of producers of new relativistic field equations), showed in one stroke that the problem of inventing more general looking field equations was of a somewhat academic nature; what really mattered was their irreducible p-space content and not their covariant appearance in x-space.

Wigner was aware that Poincaré-invariance was not the only physical requirement for relativistic particles, but there were also the important issues of causality and localization. In 1949 he wrote a paper together with R.Newton [2] in which they proposed, what became later known as the Newton-Wigner localization .This localization was not covariant and effectively violated Einstein causality at distances shorter than a Compton-wavelength, but it seemed to be the best one could do if one adapts the wave-packet localization of the Schroedinger theory to the relativistic domain .

As a result of these unsatisfactory aspects of this localization, Wigner became increasingly suspicious about the internal consistency of QFT (private remark obtained from R.Haag). However a short time later Wightman and other QFT theorist showed that there was no contradiction between the Heisenberg-Pauli canonical quantization approach and the Wigner theory[14]. In fact the latter can be used in order to obtain a more intrinsic access to the former [8].

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With one $[m,s]$-representation one connects a whole family of free fields Ψ , which all share the same canonical momentum space creation and annihilation operators affiliated (transforming) with the $[m,s]$ Wigner representation:

$$\Psi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(e^{-ipx} \sum_{s_3=-s}^{s} u(p,s_3)a(p,s_3) + e^{ipx} \sum_{s_3=-s}^{s} v(p,s_3)b^*(p,s_3) \right) \frac{d^3p}{2\omega}$$

$$(5)$$

Here u and v are explicitly known column-vectors in a space of $\geq 2s+1$ dimensions. They represent intertwiners between the Wigner representation and the covariant description of its content:

$$\sum_{s_3'} u(p,s_3')D_{s_3,s_{3'}}^{(s)}(R_W(\Lambda,p)) = D_{cov}^{[n,m]}(\Lambda)u(\Lambda^{-1}p,s_3) \qquad (6)$$

The indexing of the entries of u is given by a pair (n,m) of n un-dotted and m dotted symmetrised spinorial indices $(\alpha_1\alpha_2 ....\alpha_n, \beta_1\beta_2.....\beta_m)$. The only restrictions are that $\frac{n+m}{2}$ be semi-integer if s is semi-integer as well as the validity of the inequality $\left|\frac{n}{2} - \frac{m}{2}\right| \leq s \leq \frac{n}{2} + \frac{m}{2}$. Hence the matrix D_{cov} describes a finite-dimensional tensorial (and therefore non-unitary) representation of the Lorentz-group.A systematic determination of these intertwiners is not contained in the original work, but was carried out later by Joos[15] and Weinberg. In Weinberg's recent book [8] the reader finds an exhaustive treatment of this family of local fields. There one also finds a careful discussion of some peculiarities of the $[0,s]$ photon-neutrino class. In that case the covariantization of these nonfaithfull Wigner representation is much more restrictive than for massive theories. Whereas for the latter case one has the above inequality , the helicity for the former obeys the equality s=$\left|\frac{n}{2} - \frac{m}{2}\right|$. For the much more elusive infinite component "continuous spin" faithful zero mass representation, the covariantization was carried out in the 70's where also the lack of the standard localization property was noticed [12]

In all cases whether massive or zero mass, the local covariant fields live in the same Fock-space i.e they share the same momentum space creation and annihilation operators and in addition are local relative to each other in the sense of space-like (anti)commutation relations. Using the very appropriate concept of Borchers[16], one obtains a more concise description of this notion of relative locality. These fields are members of an equivalence class of relatively local fields. More specifically, they form a linear subset of the free field

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$[m, s]$ " Borchers class ", an object which has been explicitly computed in the 60's by H.Epstein and the present author[17]. Borchers showed in complete generality that fields, which are local with respect to a given local field, with the latter acting cyclically on a Hilbertspace, are automatically local (with respect to themselves) and he proved that this entails the following consequences:

(1) The cyclically acting members generate the same local von Neumann-algebras, i.e. if A(x) and B(x) are two such fields and $\mathcal{A}$ (A,O) denotes the local von Neumann-algebra generated by the field A(x) smeared with test-functions having support in O (a natural family of regions O are the so called double cones on which Poincare-transformations act stably) one has:

$$\mathcal{A}(A, O) = \mathcal{A}(B; O) \tag{7}$$

(2) The different members of the Borchers class do not only lead to the same local observables, but also entail the same S-matrix i.e. the S-matrix is a class invariant.

This suggests a viewpoint of QFT ("algebraic QFT ") which is quite different from the standard one most of the textbooks. By analogy with differential geometry, the pointlike covariant fields are like coordinates and the algebraic net, i.e.the assignment: $O \rightarrow A(O)$ contains all the intrinsic physical information [6].The terminology "field coordinates",which is used freely in the present work, is meant in this sense. The Borchers theory also gave a prominent role as a net invariant to the S-matrix. In the last section we will use the S-matrix as an invariant of the wedge- based modular theory.

Progress obtained from this net point of view has been slow, but steady and very solid indeed. Its mathematical pillars are the Tomita-Takesaki modular theory[4] and the V.Jones subfactor theory[18], both dealing with structural properties of von Neumann-algebras. It is not an accident, that both mathematical theories had their physical (less general) predecessors: the Haag-Hugenholtz-Winnink [6] description of KMS-states in the first case and the Doplicher-Haag-Roberts superselection theory [6] in the second (a fact which was not known at the time of the mathematical discoveries).

Here we want to show that the seeds for this intrinsic mode of physical thinking are already contained in the Wigner theory. More concretely ,the Wigner theory preempts some special aspects of both mathematical theories: localization properties are related to modular properties (explained in

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detail in the sequel) and duality obstructions related to properties of inclusions (only briefly mentioned in section 5). It is interesting to note that this progress occurs precisely at that structure which Wigner considered as questionable (namely localization)

Needless to add the remark that the $[m,s]$ fields are in general not "Lagrangian fields " i. e. the above local free fields are in generally not solutions of an Euler-Lagrange equation . To give an example, for s=$\frac{3}{2}$ the Rarita-Schwinger field is "Lagrangian ", but the e.g. minimal 4-component field in the same Borchers-class is not "Eulerian", i.e. its Lorentz transformation properties are not incorporated into the structure of field equations neither are those fields in the range of the canonical formalism which is an important property of the (classical) Lagrangian field theory and the Cauchy initial value problem. With the modular structures we will even move further away from classical structures and quantization as with the previous equivalence class structure.

In Weinbergs book one finds a formal argument that invariant (Wick-ordered)polynomial coupling terms lead to perturbations which are independent of the $[m,s]$ field- coordinates which one uses for the specification of the interaction density. With other words a given polynomial interaction may be rewritten in terms of any kind of field- coordinates one likes (and it stays polynomial in terms of the new fields).This suggests that one should try to avoid these "field-coordinates" altogether and aim for a description which restores Wigner's representation uniqueness on the level of the associated operator algebras, thus avoiding the confusing multitude of field coordinates(and hopefully eventually also arriving at a more intrinsic understanding of interactions).The next section takes a step towards this goal.

3 Recent History

For the sake of simplicity let us assume that we are using the Wigner formalism in order to describe a self-conjugate particle situation.Then,apart from a possible sign factor ,the previous action of $\theta = $ TCP on the momentum space wave-function simplifies as follows:

$$(\theta\psi)(p) = D^{(s)}(i\sigma_2)\psi^*(p) \tag{8}$$

where $D^{(s)}(i\sigma_2)$ represents the conjugation matrix in the $[m, s]$ Wigner space. Introduce now another conjugation j which differs from θ by a rotation with π around the x-axis :

$$j = R(e_x, \pi) \cdot \theta \tag{9}$$

An elementary calculation shows that this j commutes with the L-boost $\Lambda(01, \chi)$ in the 0-1 -plane :

$$U(\Lambda(01, \chi))j = jU(\Lambda(01, \chi)) \tag{10}$$

Whereas the boosts define a unitary subgroup, the continuation to imaginary χ yields an unbounded closable operator:

$$\delta := U(\Lambda(01, \chi = 2\pi i)) \tag{11}$$

In particular the unbounded operator:

$$s_W = j\delta^{\frac{1}{2}} \tag{12}$$

turns out to be a closed densely defined involution. This is as a result of the commutation relation :

$$j\delta^{\frac{1}{2}} = \delta^{-\frac{1}{2}}j \tag{13}$$

which follows from the one before.

In fact the above definition of s agrees with its polar decomposition. We now use this involution s in order to define a closed real subspace :

$$H_W^R = \{h \in H \mid s_W h = h\}$$

The properties of the positive operator δ entail the density of $H_W^R + iH_W^R$ in the Wigner representation space . This decomposition also allows to introduce a real inner product on H :

$$(\psi' + i\chi', \psi + i\chi)^R = (\psi', \psi) + (\chi', \chi) \tag{14}$$

where the primed wave-functions belong to the (-1) eigenspace $H_W^{R\prime}$ of the adjoint s^*. Both summands on the right hand side are real since:

$$(\psi', \psi) = -(\psi', s\psi) = -\overline{(s^*\psi', \psi)} = -\left(\psi, s^*\psi'\right) = (\psi, \psi') \tag{15}$$

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The real structure is the same as the one obtained by using the real part of the complex inner product and then restricting to the subspace $\mathcal{H}_W^R$. Conversely one can obtain H_W by introducing a complex structure on H_W^R. By applying Poincaré-transformations to those spaces one obtains a whole family of real subspaces which are eigenspaces of densely defined involutions s_{wedge} corresponding to the family of wedges obtained from the t-x wedge by Poincare-transformations. One then finds the following surprising theorem :

Theorem(Brunetti,Guido and Longo[10]): The family of real wedge subspaces form a covariant net of wedge-localized subspaces.

This means in particular that one has isotony i.e.$H_{W'}^R \subseteq H_W^R$ for $W' \subseteq W$ and it is interesting to note that this property is equivalent to the positivity of the energy *[10]*

It is well known that in case of integer Wigner spin there exists the so called Weyl functor [3] converts these localized real subspaces into local von Neumann algebras. The generators of these algebras in physicists notation are:

$$W(h) = e^{i(\Psi(h) + \Psi(h)^*)} , \quad h \in H_W^R \qquad (16)$$

In other words the algebras for the wedge regions can be directly defined in terms of the Wigner theory without reference to "local field coordinates". Algebras of e.g.double cones can be formed by intersections and their associated real subspaces lead to complexifications which are dense in the Wigner space (apart from the "continuous spin" representation). This can be seen by using localization properties of the u,v intertwiners of the previous section. A more elegant way, which presently has not been worked out, would be to isolate a property of the wedge subspaces which guarantees this density without using any intertwiners.

Physicists familiar with another "miracle " from the quantum physics in curved space-time [19] namely the Unruh-Hawking effect for the Rindler wedge (i.e. the quantum physics of a uniformly accelerated observer) should take notice that this effect and the above theorem are two sides of the same coin.

Behind both miracles lies a very basic and universal theory [5] which, as already mentioned in the introduction, mathematicians refer to as the Tomita-Takesaki modular theory. We will here only limit ourselves to some salient features. This theory deals with von Neumann-algebras in "standard position " e. g. weakly closed operator algebras in a Hilbertspace possessing

a cyclic and separating vector Ω. In local quantum physics the vacuum is a vector which has this property with respect to all local subalgebras with a nontrivial causal complement of their localization region [6] . In order to construct the basic objects of this theory,one starts from the star-structure of the von Neumann-algebra A and defines an unbounded but closable involutive operator S :

$$SA\Omega = A^+\Omega , A \in \mathcal{A} \tag{17}$$

Its polar decomposition:

$$S = J\Delta^{\frac{1}{2}} \tag{18}$$

defines the Tomita conjugation J and the modular operator Δ . The latter gives rise to the modular automorphism σ_t. The nontrivial part of the T.T. theorem is the behaviour of these operators with respect to the von Neumann-algebra A:

$$J(\mathcal{A}) := J\mathcal{A}J = \mathcal{A}', \quad \mathcal{A}' = commutant \ of \ \mathcal{A}. \tag{19}$$

$$\sigma_t(\mathcal{A}) = \Delta^{it}\mathcal{A}\Delta^{-it} = \mathcal{A}, modular \ automorphism \ of \ \mathcal{A} \tag{20}$$

For a physicist, the K in $\Delta^{it} = e^{itK}$ is like a generalized Hamiltonian and J is like a generalized TCP-operator of the pair $\mathcal{A}, \Omega$. The only miracle as far as the application of this theory to the local algebras of QFT is concerned , is that these modular quantities for the pair $A(wedge), \Omega \ v$vacuum-vector becomes geometric:

$$\Delta^{it} = U(\Lambda(01, \chi = 2\pi t) \tag{21}$$

$$J = \begin{cases} R(e_x, \pi)\Theta, \ \text{s= integer} \\ KR(e_x, \pi)\Theta, \ \text{for} \ \ \text{s= semi-integer} \end{cases} \tag{22}$$

Here K is the well-known Klein-twist (not to be confused with the closely related Jordan-Wigner transformation) for fermions: K=$\frac{1+iV}{1+i}$ with V= $\exp i\pi N_{fermi}$. In the integer spin case the Wigner theory preempts this modular structure through the existence of the previously introduced family of real subspaces H$^R_{wedge}$ which are converted into bosonic net algebras via the Weyl functor. In the semi-integer spin case we expect the CAR-functor to play the analogous role.In the Wigner theory the Fermi-statistics manifests itself through:

$$jH^R_{wedge} \neq H^R_{oppositewedge} \quad \text{for s=semi-integer} \tag{23}$$

This mismatch is repaired on the level of the algebras by the above Klein-twist K:

$$\mathcal{K}\mathcal{F}(jH^R_{wedge}) = \mathcal{F}(H^R_{opp.wedge}) \tag{24}$$

Here F is the CAR functor. K restricted to the Wigner-space of fermions is just a numerical factor i, which is precisely the obstruction factor between j and the $\pi-$rotation. So the Klein factor just permits to express the Tomita J in terms of geometrical objects. The rest consists in applying the CCR resp. CAR functor which maps the net of Hilbertspaces into the net of von Neumann algebras.

Note that all recent contributions of modular theory to the understanding and construction of Borchers classes (including the present one) could have been given two decades ago ever after the prominent role of wedge algebras was discovered by Bisognano and Wichmann. [9]

4   Fractional Wigner-Spin and Statistics of Anyons.

In d=2+1, the little group of a point on the forward mass shell is the abelian $U(1)$ and therefore the Wigner theory allows (at least a priori) for *any* value of s, i.e.one expects "anyons " (the more restrictive non-abelian plektons will only be mentioned at the end of this section). Using the methods of the previous section, and checking the prerequisites for the existence of a TCP operation on the direct sum of particle-antiparticle Wigner spaces, one again establishes the properties of a family of real subspaces which can be associated with localization and statistics properties of field theoretic two point functions. However the difference between the modular complement $jH^R_W = H^{R\prime}_W$ and the geometric complement $H^R_{W\prime} = Rot(\pi)H^R_W$ is bigger than in the previous fermionic theory i.e.the Klein transformation which accounts for this difference is more complicated.

In order to keep the Klein-twist simple, let us imagine that we are dealing with a Z_N-spin i.e. we assume that $s = \frac{1}{N}$ .Then the Klein factor which corrects the mismatch between localization via commutativity and the geometric localization turns out to be a suitable "square root"[20] of the action

of the $2\pi-$rotation in Fock space

$$K = \sum_n e^{-i\pi s n^2} P_n \qquad (25)$$

The physically relevant question is: what is the a priory best possible localization of the anyonic algebras. It turns out that a compact localization as in the previous section is not possible, i. e. in H_W^R there are no compactly localized wave functions. If such a wave function would exist, one could perform a $2\pi-$rotation such that the support remains inside one wedge for all angles, however the nontrivial phase created by such a rotation contradicts its affiliation to the real subspace H_W^R.

From the general structure of algebraic QFT we expect that the spectral gap leads to a (arbitrarily thin) space like cone localization. In d=2+1 only genuine braid group statistics is able to exhaust this possibility, whereas permutation group statistics resulting from semi-integer spin leads back to the compact localization. Since the core line of a semi-infinite spacelike cone is characterized by an initial point x and a spacelike unit direction e, we expect a string like localized wave function depending on x and e.

Starting from the Wigner wave function which transforms according to (ψ is one-component) :

$$(U(g)\psi)(p) = e^{is\Phi_W(g,p)}\psi(\Lambda^{-1}(g)p), \qquad g \in \widetilde{SO(2,1)} \qquad (26)$$

with Φ_W being the (nonlocal) Wigner phase. One looks for a factorization into covariant factors in analogy with the u-v intertwiners of the previous section. This is achieved by [11] defining:

$$\psi_{cov}(p,g) = F(L^{-1}(p)g)\psi(p) \qquad (27)$$

where any function F on $\widetilde{SO(2,1)}$ is acceptable as long as it fulfills the equivariance law:

$$F(\omega \cdot g) = e^{is\omega}F(g) \qquad \omega \in R \subset \widetilde{SO(2,1)} \quad g \in \widetilde{SO(2,1)} \qquad (28)$$

As a consequence we find the covariance law:

$$(U(g')\psi_{cov})(p,g) = \psi_{cov}(\Lambda^{-1}(g')p, gg') \qquad (29)$$

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With this covariant wave function we now affiliate a Dirac state vector which, as usual, is created from the vacuum:

$$|p, g\rangle = a^*(p, g)\Omega \tag{30}$$

It obeys the contragradient transformation law:

$$U(g)a^*(p, g')U(g)^{-1} = a^*(\Lambda(g), g'g^{-1}) \tag{31}$$

Transforming to x-space fields (the subscript cov will be omitted in the sequel):

$$\psi(x, g) = \int \frac{d^2p}{2\omega}(e^{ipx}a(p, g) + e^{-ipx}a^*(p, g)) \tag{32}$$

one obtains the desired transformation law:

$$U(g')\psi(x, g)U(g')^{-1} = \psi(\Lambda(g')x, g'g) \tag{33}$$

The two-point function is a quadratic expression in the function F:

$$(\Omega, \psi(x_1, g_1)\psi(x_2, g_2)\Omega) = \int \frac{d^2p}{2\omega}e^{ip(x_1 - x_2)}\overline{F(L^{-1}(p)g_1)} \cdot F(L^{-1}(p)g_2) \tag{34}$$

Choosing x_2 and g_2 "opposite" to x_1 and g_2 i.e. such that:

$$x_2 = Rot(\pi)x_1 \qquad g_2 = Rot(\pi)g_1 \tag{35}$$

the covariant transformation law gives:

$$(\Omega, \psi(x_1, g_1)\psi(x_2, g_2)\Omega) = e^{2\pi is}(\Omega, \psi(x_2, g_2)\psi(x_1, g_1)\Omega) \tag{36}$$

which is in agreement with the expected anyonic statistics of the non-compact localization .

   In order to obtain fields which are localized on semi-infinite strings, one has to chose a model for F. The choice :

$$F(g) = e^{isg(0)} \tag{37}$$

with g $=(\gamma, \omega)$ acting fractionally on the line u$\in R = \widetilde{S^1} \subset \widetilde{SO(2, 1)}$ as:

$$(\gamma, \omega)(u) = \omega + \arg \frac{e^{iu} + \gamma}{1 + e^{iu}\overline{\gamma}} \tag{38}$$

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The two-point function specializes to :

$$\langle \psi(x_1, u_1)\psi(x_2, u_2)\rangle = \int \frac{d^2p}{2\omega} e^{ip(x_1-x_2)} e^{-is(L(p)^{-1}(u_1)-L^{-1}(p)(u_2))} \tag{39}$$

and the difference between the left and right hand side in (37) replaces the bosonic commutator function for our anyonic case:

$$\Delta(\xi, u_1, u_2) = \tag{40}$$

$$\int \frac{d^2p}{2\omega} \left\{ e^{ip\xi} e^{-is(L(p)^{-1}(u_1)-L(p)^{-1}(u_2))} - e^{-ip\xi} e^{-is(L(p)^{-1}(u_2)-L(p)^{-1}(u_1))} e^{4\pi is\left(\left[\frac{u_1-u_2}{2\pi}\right]+\frac{1}{2}\right)} \right\}$$

where ξ is the difference of the x's and the square bracket indicates the nearest larger integer.

Δ has the property of L-covariance:

$$\Delta(g\xi, gu_1, gu_2) = \Delta(\xi, u_1, u_2) \tag{41}$$

The vectors u_1 and u_2 on the unit circle correspond to a wedge W and its spacelike complement W'. The simultaneous stability group of u_i leaves this wedges invariant. For each $\xi \in W \cup W'$ there exists a transformation $K \in \widetilde{SO(2,1)}$ which is in the conjugacy class of the $\pi-$rotation which reflects ξ and flips the u's :

$$K\xi = -\xi \qquad Ku_1 = u_2 \qquad K^2 = 2\pi - rotation \tag{42}$$

Under the action of this "square root" of the 2π-rotation the Δ behaves as:

$$\Delta(K\xi, Ku_1, Ku_2) = \Delta(-\xi, u_2, u_1 + 2\pi) = e^{-2\pi is}\Delta(\xi, u_1, u_2) \tag{43}$$

Consistency between the two transformation formulas yields:

$$\Delta(\xi, u_1, u_2) = 0, \qquad for \ \ \xi \in W \cup W' \tag{44}$$

i.e. as long as u_1 is different from u_2 any separation ξ of the string starting points x and y, such that a string crossing is avoided, will lead to a vanishing Δ function. A representation in terms of known functions is presumably easier than for the analog problem of the d=3+1 continuous spin representation (presented below).

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This situation corresponds to light-like strings and the covariantization just presented is due to Gaberdiel and Fredenhagen[11]. The G-F description does not reveal in a manifest way that these anyons permit a spacelike cone localization. Only the formation of *wave packets* in the *light-like string direction e* or a covariantization based directly on space-like strings (using a de Sitter space representation of the $\widetilde{SO(2,1)}$) could reveal sharper localizations inside wedges. We presented these rather explicit calculations because the covariantization of the d=3+1 m=0 continuous spin representations in section 6 is completely analogous albeit analytically more complicated.

Our argument in favour of space-like cone localization for anyons is based on the observation of non-triviality of intersections[23] of wedge spaces H_W^R. Let W_1 and W_2 be two wedges and form the dense set of states obtained by averaging with smooth functions of compactly localized Fourier transform:

$$\int ds dt\, f(t,s) \delta_{W_1}^{it} \delta_{W_2}^{is} \Phi, \quad \Phi \in \mathcal{H} \tag{45}$$

This set of vectors is certainly in the domain of $\delta_{W_1}^{\frac{1}{2}}$. In order to see that also $\delta_{W_2}^{\frac{1}{2}}$ can be applied, we need a commutation relation of $\delta_{W_2}^{\frac{1}{2}}$ with $\delta_{W_1}^{it}$. For orthogonal wedges such a commutation relation is well-known from SO(2,1) group theory:

$$\delta_{W_2}^{\frac{1}{2}} \delta_{W_1}^{it} \delta_{W_2}^{-\frac{1}{2}} = \delta_{W_1}^{-it} \tag{46}$$

and the case of W's in a more general position may be reduced to this orthogonal situation and in this way one proves the existence of a simultaneous dense domain for the $\delta's$ associated to different L-boosts

At this point one may be tempted to think that our one-particle analysis, which relates to the field theoretic two-point function, may be generalized to the standard commutation relation between creation and annihilation operators:

$$a(p,u)a^*(q,v) = e^{4\pi i s\left(\left[\frac{u-v}{2\theta}\right]+\frac{1}{2}\right)} a^*(q,v)a(p,u) + 2\omega\delta(p-q)e^{-is(L^{-1}(p)(u)-L^{-1}(p)(v))} \tag{47}$$

This is however inconsistent (except for bosonic and fermionic phases) because it can be shown to lead to a contradiction with the associativity of multiplication for three space-like cone localized anyon operators [22]. The correct multiparticle space from scattering theory has a different structure.

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The anyonic momentum space creation and annihilation operators associated with scattering states have source and range projections which have to match the superselection charges of the state vectors. In this case no "anyonic functor" which transforms localized subspaces into von Neumann subalgebras is known. We intend to return to this interesting problem in a separate publication.

As a consequence of the appearance of the directional degrees of freedom for elementary strings (i.e. strings which cannot be represented in terms of line integrals of other fields as in Mandelstam type exponential line integrals) the Wigner anyons are more analogous to infinite component L-covariant wave functions. Their relation to "Chern-Simons anyons" is presently not clear. Note that the geometric pictures about anyons from the days of Leinaas-Myrheim to the Chern-Simons inspired path integral proposals suffer from one serious defect: they did not lead to a concrete operator description of "free anyons". QFT with its intrinsic concepts is expected to be a much more suitable place for their understanding than quantum mechanics.[24] Even if further studies of our construction reveal that the fourier-transformed anyonic two-point function have additional spectrum above the one particle hyperboloid (similar to particle number conserving integrable models in d=1+1), our modular methods should explain the necessity for such an extension of the Wigner spectrum.

Before closing, a brief comment about the d=1+1 situation is in order. In this case the localization properties of free fields depend on the "Lorentz-spin" i.e. on the value of s in the one-dimensional L-representation factor $\exp s\chi$ with χ being the rapidity. All s> 0 representations may be obtained in the s=$\frac{1}{2}$ Fockspace of fermions by using appropriate intertwiners u and v. But only for (half)integer s does one obtain pointlike localized covariant fields. At generic values one does not get beyond wedge localization. The bad localization property does not improve in the zero mass limit. The localizable fields of chiral conformal field theory have a different origin which is further removed from the Wigner representation theory. They owe their existence to the peculiar structure of current operators which lead to Weyl algebras with a nontrivial center. It seems that also the structurally rich plektonic theories (nonabelian braid group statistics) can be traced back to this property [24].

5   Haag Duality and E.M.Duality.

Massive free fields obey Haag duality not only for double cones, but also for topologically more complicated localization regions e. g.toroidal regions. Algebras associated to massless fields for helicity s≥1 however cause a topological obstruction resulting in a breakdown of toroidal Haag duality [3]. Let T be a space-like torus i.e.a spatial circle fattened by double cones which is the space-like complement of a region T' which consists of a double cone of diameter r and a "double cone at infinity": $|\vec{x}| \geq R + |t|$ separated by a distance R-r ≥0 with T being the toroidal corona [24] region in between . Then one obtains the following proper inclusion:

$$H^{\mathcal{R}}(\mathcal{T}) \subset H^{\mathcal{R}}(\mathcal{T}')' \tag{48}$$

where $H^R(..)'$ denotes the previously defined symplectic complement. The inclusion is genuine and[3] the defect is finite dimensional. This obstruction can be traced back to the appearance of δ' the E-H commutation relation.It entails the violation of Haag duality for the corresponding algebras. The introduction of vector potentials removes this obstruction formally (principles of locality, Einstein causality etc. loose their physical meaning in the presence of indefinite metric) at the prize of indefinite metric without effecting its continued presence in the physical factor space von Neumann algebra generated by the field strength. The vector potential description is avoidable in the structural analysis of Maxwell-like interacting theories but it is essential in the present day perturbative approach in QED. From an intrinsic QFT point of view the "local gauge principle" is not a principle at all ,but a (conceptionally questionable) compromise between the covariantized Wigner theory and the necessity to formulate the correct longrange interaction within the standard interaction picture which is the starting point of the perturbative approach . Only if one believes in the quantization philosophy , one may speak of a" gauge principle" as the quantum substitute of classical fibre bundles. The Wigner space can be described in terms of a semidefinite metric vector-valued wave functions. The trouble of indefinite metric fields only appears on the level of second quantization. Therefore it should be very interesting to compute the real subspaces directly by intersecting a family of wedge algebras instead of using the canonical structure of suitably chosen covariant fields (as the afore mentioned Leyland, Roberts Testard method). In that

case one can avoid indefinite metric and the Gupta-Bleuler method in favour of a positive semidefinite real Fock-subspace: $\mathrm{H}_W^R(semidef.) \subset \mathcal{H}(semidef.)$.

Since the $F_{\mu\nu}$ intertwiners are obtained from the semi-definite $A_\mu$intertwiners by forming derivatives, one expects that this defect is related to one which occurs in chiral conformal field theory.[25] where the defect turns out to be finite dimensional in terms of Hilbertspaces.

This phenomenon of a defect of topological nontrivial Haag duality is well- known from the charge-neutral observable algebras associated with massive charged free fields. In that case this deficiency has a natural explanation in terms of the existence of nontrivial charged sectors [6].It is also well-understood for maximally extended current algebras in chiral conformal QFT.

The imperfection of photon like m=0 representations with respect to duality for topologically nontrivial regions however is more serious. It is tempting to interpret this obstruction as indicating the necessity of an interaction i.e.the presence of non-vanishing electric or magnetic (or both) currents.

$$\partial^\mu F_{\mu\nu}(x) = j_\nu(x), \ \ \partial^\mu \tilde{F}_{\mu\nu}(x) = \tilde{j}_\nu(x) \tag{49}$$

i.e. the interaction is necessary to maintain perfect duality which is violated in the free theory. Such a point of view would attribute a very distinguished role to electromagnetic duality i.e. those superselection rules which originate from the quantum version of the Maxwell structure. This issue of problematizing the notion of "magnetic field" on the same level of depth as the notion of "charge" in the DHR superselection theory is presently ill-understood in QFT. In low dimensional QFT the analogous issue of order-disorder duality and the connection with Haag duality is in a much better shape. There, even in free theories, it is not possible to have *no* charge sectors with both order and disorder the realization of both charges being related in d=1+1 to the zero mass limit. This analogy is another reason to believe that the free Maxwell situation is peculiar. The remaining three non-peculiar cases namely the appearance of objects with e.-, m.- or e.m.-charges have a severe infrared structure whose implications for localization properties are outside the present scope of understanding. A better understanding of the connection between these properties and the modular theory (in the vein of the remarks about interactions in the last section) seems to be essential.

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6 The Localization Properties of the Positive Energy Continuous Spin Representations.

Already in the late 60's the question of how to covariantize and incorporate Wigner's zero mass continuous spin representation into existing frameworks of QFT arose some interest notably with physicists who were familiar with the spirit of algebraic QFT [12]. Whereas in standard QFT based on quantization and Lagrangians these representations were usually dismissed as uninteresting because "nature apparently does not make use of them", the spirit in which algebraic field theorists approached this problem was more "Wignerian". They asked whether these representations fulfill the localization properties which are inexorable attributes (in addition to their indecomposability expressed in the irreducibility requirement)of particles. It was found that they do not permit a compact localization as the standard Wightman fields do.

Looking again at the old computations with the hindsight of the non-compact space-like cone localization of section 4, one easily realizes that their natural covariantization leads to the same wedge-localized light-like strings with an infinite component unitary representation on light-like directions and complex variables ξ_1, ξ_2 taking over the role of the u in the case of the anyonic representation of section 4. The identification of complex 2-vectors with light-like directions is done with the help of the Pauli matrices:

$$l_\mu = \xi^+ \sigma_\mu \xi \tag{50}$$

the intertwiners for this light-like covariantization are:

$$u(p,\xi) = f_\lambda^{\rho\chi}(\xi B_p), \quad f_\lambda^{\rho\chi}(\xi) = |\xi_2|^{2c-2} e^{-i\lambda\Phi_1} J_{l_0-\lambda}(\frac{\rho}{\kappa}|z|) e^{il_0\Phi_2} \tag{51}$$

$$z = \frac{\xi_1}{\xi_2}, \quad \Phi_{1,2} = \pm\pi + \arg\xi_1 \mp \arg\xi_2$$

The $f_\lambda^{\rho\chi}(\xi)$ corresponds precisely to the equivariant function F in section 4.

The step from wedge-localization to space-like cone localization is also analogous to section 4. Instead of two orthogonal wedges one now considers three. The smoothening with testfunctions of compact Fourier-transform:

$$\int dtdsdu f(t,s,u)\Delta_1^{it}\Delta_2^{is}\Delta_3^{iu}\Phi, \quad \Phi \in \mathcal{H} \tag{52}$$

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together with commutation relations between the three boosts analogous to the ones used in section 4 will give a simultaneous dense domain for $\Delta_i^{\frac{1}{2}}$ i=1,2,3 . However it is not clear if the case of three wedges in general position can be reduced to the orthogonal case.

It is interesting to compare this space-like cone localization with that established by Buchholz and Fredenhagen on the basis of the spectral gap assumption[6]. In their massive case it is not possible to realize this in a theory with a on mass-shell supported two-point function.

7   Intrinsic Understanding of Interactions? Programmatic Remarks.

Since the wedge regions have a preferential status with respect to the construction of interaction-free algebras, it is tempting to think that this may be helpful in obtaining some intrinsic insight into interactions. Let us take a helping hand from scattering theory. There it is shown that out- and ingoing-fields share the same Poincaré transformations with the interacting fields i.e. they both possess the same modular transformations Δ^{it} for the wedge region. Only the TCP conjugation is sensitive with respect to interactions. In order to see this we will derive the following representation for S which is valid in an asymptotically complete theory :

$$S = \theta \cdot \theta_0 = J \cdot J_0 \tag{53}$$

J= modular conjugation for interacting wedge algebra

J_0 = modular conjugation for the interaction- free incoming wedge algebras.

The formula follows from the Θ =TCP transformation of Heisenberg fields.Taking the LSZ limit on this transformation formula and noticing that both sides approach different (in and out) limits (and remembering that the spatial rotation factors between Θ and J are independent of interactions) one obtains the above representation.

The interacting and incoming wedge algebras are of the same type, in fact they are expected to be type III_1factors .Since both are living in the same Hilbertspace, their isomorphism amounts to a unitary equivalence. Unfortunately this kind of argument does not lead to a "natural" unitary operator

which we expect to be some kind of natural "square root" of S i.e. some kind of "algebraic" Möller operator. On a very formal level the method of Bogoliubov leads to such unitaries, but this would bring us back to the interaction picture and the formal time ordered operator expressions.

Let us therefore be more modest and just ask for a modular net of real Hilbert-subspaces of the incoming Fock-space. If we pose this problem in two space-time dimensions, we could take a j operator which is different from j_0 by one of those rather simple rapidity dependent factorizing S-matrices of the "bootstrap construction" which are the long-distant limits of the class of theories with the same superselection rules [24]. Here our modular proposal is expected to give a more field theoretic understanding of the so-called formfactor bootstrap program and the Bethe-ansatz approach. Both the formfactor program and the present "modular program" point into the same direction: the construction of local fields resp. of local nets from a given S-matrix. Whereas the formfactor program has only been formulated for factorizable S-matrices, the modular idea in principle does not suffer from such a restriction. Presently I do not know if the latter leads to a unique wedge algebra; the above argument only yields unique real local subspaces of the Fock space of scattering states. For the uniqueness of the algebras one would need a finer split into so called positivity cones.

Note that in higher dimensions such a starting point with a model S is not available since these long distance limits in d=3+1 would be trivial i.e a free field realization of the superselection rules affiliated with S=1 (it may however lead to a solid proof of the "folklore" statement that S=1 leads necessarily to the free field Borchers class.).

In the d=3+1 $S \neq 1$ case one could start with a unitary Poincaré-invariant operator S_{aux} which only fulfills the weaker requirement which a modular reflection J has to obey in order to yield a net of real subspaces of the Fockspace: $H_{W,F}^R \subset \mathcal{H}_F$. The requirement that these wedge spaces contain space-like cone localized Fock-space vectors, may already lead to a nontrivial restriction on the auxiliary S_{aux}. One would hope that such restrictions resulting from localizations which are sharper than the original wedge localization may give rise to an inductive procedure (a kind of algebraic perturbation theory in which the unitarity relations are fulfilled in every order). Hence the starting auxiliary S_{aux} could be a unitary operator as in Heisenberg's ill-fated S-matrix theory [26] since the wedge localization of states requires much less then the locality properties of local interpolating fields leading to severe an-

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alyticity restrictions for the scattering amplitudes. So the issue of the true S-operator would then appear in a later step: how to go (presumably by a series of iterative steps) from a net of wedge-localized state vectors to a net of space-like(or double-) cone localized von Neumann algebras. Therefore at the end ,unlike in Heisenberg's proposal, the physical S-matrix would carry all those subtle properties resulting from localization of fields.

The wedge localization theory may also provide some new concepts and techniques for external field problems.In the past the causality "diseases" in the presence of external electromagnetic fields of certain higher spin field equations were used in order to point out some problems with the "elementarity" (versus compositeness) of higher spin equations (supersymmetry is of no help here!)[27][28]. The physical argument (in favour of taking the causality structure of external field problems that serious) is a consequence of Weinberg's interpretation [8] of the external field approximation as resulting from infinitely heavy dynamical objects. The causality properties of light fields are not expected to depend on the size of the masses of heavy fields.

A closely related problem is the question of what state should play the role of the vacuum reference state for such external interactions, in particular if the interaction is stationary. Traditionally one has used adiabatic arguments, either overtly or, as in Schwinger's $\mu-$pair production calculation, buried inside an elegant looking formalism. For QFT in curved space- time the deviation of causality from its standard Minkowski-space form has a natural physical interpretation, but the problem of a vacuum-like reference state becomes even more dramatic, since such global concepts as "vacuum" and Wigner "particles" (and as a consequence normalization conditions for curved spacetime anomalous magnetic moment and Lamb-shift)are meaningless.

Very recently some marvelous progress on this problem was obtained through the discovery of the "microlocal spectrum condition" which also permits to take into account interactions between the matter fields[29]. Although the perturbation theory is not based on quantization (it rather uses the Bogoliubov Weinberg dispersion theoretic framework refined by the Epstein-Glaser theory), it is not as intrinsic as a modular-based approach proposed (but unfortunately not carried out) here. The time-ordering used in that formalism originates from Dirac's formalism for time dependent Hamiltonian perturbations. Although being a natural part of quantum mechanics and hence independent of (canonical, functional)quantization , such concepts using the 4^{th}component of a vector and distinguishing hyperplanes are not a

good starting point for an *intrinsic* approach to interactions. The modular wedge theory however is a necessary consequence of covariant nets. It is not only characteristic for the latter, but also explains and underlines the big distance which QFT maintains to classical field theory as well as to quantum mechanics .

The modular approach to interactions advocated here may also be useful for a more intrinsic understanding of renormalization. The usual presentation of renormalization is intimately related to quantization and actions. It only repairs a very formal and slightly illegitimate starting point[21]. Looking only at the renormalized correlation functions, it is not so easy to isolate an intrinsic aspect of renormalization [21]. Even in Wilson's renormalization group approach the intrinsic characterization of fix points outside of Gaussians remains unclear.

In the problem of d=2 critical indices an intrinsic understanding has finally be achieved in terms of very subtle properties of their attached noncommutative real time chiral theories[21]. It turned out that the critical indices are classified by certain numerical values of superselected charges (related to the so called "statistical dimensions" of algebraic QFT) which are in turn related to such (at first sight) remote looking issues as the classification of physically admissible braid-group statistics and modular theory. An intuitive understanding in terms of values of charges appears first in Kadanoff"s work[30].

Recently a framework was proposed which allows to understand an intrinsic association of a scale invariant theory to a massive theory (with the possibility of new superselection rules emerging in the short distance limit i.e. short-distance quark deconfinement)[31]. I believe that this framework together with ideas from modular theory may also cast some light on a possible distinction between "renormalizable and unrenormalizable nets". In addition there is the interesting issue of "semirenormalizable" theories i.e. the question of how to deal with theories like massive (non Higgs) vectormesons coupled to charged matter fields via conserved currents. In that case the neutral fields stay renormalizable. To have (observable) renormalizable subsets of fields could very well (as the causality issue) be a general phenomenon of higher spin interactions.

Finally we want to emphasize that our modular proposal based on the S-matrix does not cover zero mass theories. Whereas for those massless theories which can be viewed as scaling limits of massive theories (e.g. chiral confor-

mal QFT) this poses no serious problem (since the conceptual complication is compensated for by an analytic simplification), the physically interesting cases in which the dual e.and m. charges are of Maxwellian origin remain presently outside the modular approach. In this case neither the conceptual nor the analytic aspects are simple.

If in these notes I created the impression that QFT, despite its more than 60 years of existence, is a very young and fresh branch of intellectual endeavour (looking at the many basic but insufficiently understood problems), then this was not without intentions.

A large part of this work (the main exception being the section on "continuous spin") was carried out during a visit of the UFES Brazil. I am deeply indebted to some of my colleagues at the UFES, in particular to proof. Julio Cesar Fabris for their kind hospitality.

8   References

References

[1] E.P. Wigner, Ann. Math.**40**, 149 (1939)

[2] R. Newton and E.P. Wigner, Rev. Mod. Phys.**21**,400 (1949)

[3] P.Leyland, J. Roberts and D. Testard, "Duality for Quantum Free Fields" CNRS,Marseille preprint July 1978, unpublished

[4] M. Takesaki "Tomita's theory of modular Hilbert algebras and its application." Lecture Notes in Mathematics, Berlin:Springer 1970

[5] G.L. Sewell, Phys. Rev. Lett.**79A**,23 (1980)

[6] R.Haag, "Local Quantum Physics, Fields, Particles, Algebras", Springer Verlag 1992

[7] B. Schroer and J.A. Swieca, Phys. Rev. **D10**, 480 (1974), B.Schroer, J.A. Swieca and A.H. Völkel Phys.Rev. **D11**, 1509 (1975)

[8] S.Weinberg, "The Quantum Theory of Fields" Cambridge University Press 1995

[9] J.J. Bisognano and E.H. Wichmann, J. Math. Phys. **16**, 985 (1975),
J. Math. Phys. **17** ,303 (1976)

[10] R. Brunetti, D. Guido and R. Longo, to be published

[11] M. Gaberdiel and K. Fredenhagen, unpublished notes on covariant representations for massive d=2+1 anyonic spin

[12] G.J. Iverson and G. Mack, Annals of physics **64**,211 (1970)
J.Yngvason, Commun. Math. Phys. **18**, 195 (1970)

[13] V. Bargmann, Ann. Math.**59**, 1(1954),
C.C.Moore, Trans.Amer. Math.Soc. **221**, 35 (1976)

[14] A.S.W ightman, see bibliography in ref.17

[15] H. Joos, Fortschr. Physik, **10**,65 (1962)

[16] H.J. Borchers, Nuovo Cimento **15**,784 (1960)

[17] R.F. Streater and A.S. Wightman "PCT, Spin and Statistics and all that"New York: Benjamin 1964

[18] V.F.R. Jones, Inventiones Math.**72**, 1,(1983)

[19] R. Brout, S. Massar, R. Parentani and Ph. Spindel, "A Primer for Black Hole Quantum Physics", Physics Reports **260**,329 (1995). The reader will find many beautiful and miraculous calculations (but no relation to modular theory) in this review.

[20] B. Schroer, "Modular theory and symmetry in QFT" Proc.of the workshop on "Mathematical Physics towards the 21st Century" ed.R.N.Sen and A. Gersten, Ben Gurion University of the Negev Press 1994

[21] B. Schroer, "Motivations and Physical Aims of Algebraic QFT", FU-Berlin preprint, June 1996 hep-th/9608083

[22] J. Mund, private communication

[23] On this point I acknowledge a helpful discussion with R. Brunetti and H.W. Wiesbrock

[24] B. Schroer, Rev. Math. Phys.**7**, (1995),645

[25] J. Yngvason, LMP **31,** 127,(1994)

[26] W. Heisenberg, Z.Phys.**120**, 513 ,673 (1943)

[27] G. Velo and D. Zwanziger, Phys.Rev. **186**, 1337 (1969)

[28] B. Schroer, R. Seiler and J.A. Swieca, Phys.Rev.**D 2**,2927 (1970)

[29] R. Brunetti and K. Fredenhagen and M. Köhler,"The Microlocal Spectrum Condition And Wick Polynomials of Free Fields on Curved Spacetimes" DESY 95-196, to appear in CMP, and R.Brunetti and K.Fredenhagen, work on interacting QFT in CST, to appear.

[30] L.P. Kadanoff, J. Phys. **A11**, 1399 (1978)

L.P. Kadanoff and A.C. Brown, Ann.Phys.(N.Y.)**121** (1979) 318, and references therein.

[31] D. Buchholz, Nucl. Phys. **B469,** 333 (1996).