

# Advances and Perspectives in Ergodic Theory

Celebrating Dynamics on the occasion of Gerhard Keller's 60<sup>th</sup> Birthday

**Friday 30 May 2014, Lecture hall H12**

13:30 Opening address

14:00 **Lai-Sang Young** (Courant Institute, New York University)

Nonequilibrium steady states for certain particle systems

15:30 **Sébastien Gouëzel** (Université de Rennes)

Moment bounds for non-uniformly expanding maps

17:00 **Sebastian van Strien** (Imperial College London)

Stochastic stability of expanding circle maps with neutral fixed point

**Saturday 31 May 2014, Lecture hall H12**

9:30 **Viviane Baladi** (Ecole Normale Supérieure Paris)

Linear response for one-dimensional dynamical systems

11:00 **Daniel Lenz** (Friedrich-Schiller-Universität Jena)

Dynamical systems and the diffraction of quasicrystals

13:30 **Domokos Szász** (Budapest University of Technology and Economics)

The rare interaction limit in a fast-slow mechanical system

15:00 **Carlangelo Liverani** (Università di Roma Tor Vergata)

Beyond averaging in fast-slow partially hyperbolic systems

# Abstracts of the lectures

**Lai-Sang Young** (Courant Institute, New York University)  
Nonequilibrium steady states for certain particle systems

The topic of this talk lies on the interface between dynamical systems and nonequilibrium statistical mechanics. I will begin by describing a family of mechanical models in which local environments at lattice sites are represented by rotating disks, and energy exchange among these disks are mediated by moving particles. Each system is coupled to unequal heat baths and forced out of equilibrium. Relevant questions on nonequilibrium steady states are reviewed. Answers to these questions are, unfortunately, out of reach at the present time for the models described. I will instead consider stochastic models designed to capture the essence of their dynamics, and present recent results for these Markovian systems.

**Sebastian Gouezel** (Université de Rennes)  
Moment bounds for non-uniformly expanding maps

The growth rate of moments of Birkhoff sums is commonly used in physics to estimate specific parameters of dynamical systems. We give a theoretical justification to this process, in intermittent maps or more generally in Young towers: we get precise (matching) upper and lower bounds for this growth rate, in the whole range of parameters of interest, and deduce that the growth rate indeed encodes how non-uniform the expansion is. The methods involve precise dynamical estimates and martingale inequalities.

**Sebastian van Strien** (Imperial College London)  
Stochastic stability of expanding circle maps with neutral fixed point

One of the best known dynamical systems with intermittency behaviour is the well-known Pomeau-Manneville circle map. This map has a neutral fixed point at 0 which causes orbits to linger there for long periods. Nevertheless this map has always a physical measure: for  $\alpha \geq 1$  it is the Dirac measure at 0 while for  $\alpha \in (0, 1)$  it is absolutely continuous. It was also known for quite a while that this map is stochastically stable when  $\alpha \geq 1$ . In this talk I will discuss a result which implies that this map is also stochastic stable when  $\alpha \in (0, 1)$ . (Joint with Weixiao Shen)

**Viviane Baladi** (Ecole Normale Supérieure Paris)  
Linear response for one-dimensional dynamical systems

The transfer operators of piecewise expanding interval maps  $f$  have a spectral gap on  $BV$ , which implies in particular exponential mixing in the aperiodic case. For smooth one-parameter families of piecewise expanding maps  $f_t$ , Gerhard Keller showed in 1982 that the  $L^1$ -norm of the invariant density  $\rho_t$  has a  $|t \log |t||$  modulus of continuity. With Smiana, we proved in 2008 that this result is optimal. In particular, linear response fails. Smooth unimodal maps are nonuniformly expanding at best. In the good cases, their transfer operator also enjoys a spectral gap on a suitable Banach space (constructed e.g. via a Hofbauer-Keller or Young tower), which implies several nice properties, including an invariant density  $\rho \in L^1$  and exponential mixing. The same mechanism as for piecewise expanding maps leads to breakdown of linear response: We shall present recent work with Benedicks and Schnellmann on differentiable unimodal maps with nondegenerate critical points. Our upper bounds for the modulus of continuity of the a.c.i.m. (restricting to large sets of parameters  $t$  enjoying slow enough postcritical recurrence) are of the form  $|t^{1/2} \log |t|^M|$ . For the quadratic family, our result gives both upper bounds and (restricting to appropriate sequences) lower bounds of the form  $|t^{1/2}|$  for the modulus of continuity of the a.c.i.m.  $\rho_t dx$ .

**Daniel Lenz** (Friedrich-Schiller Universität Jena)  
Dynamical systems and the diffraction of quasicrystals

Since their discovery in 1982 quasicrystals have been considered from various points of view in several disciplines. In the mathematical treatment diffraction and the connection to dynamical systems has received ample attention. We provide a survey on this line of research.

**Domokos Szasz** (Budapest University of Technology)  
The Rare Interaction Limit in a Fast-Slow Mechanical System

Gaspard and Gilbert's 2008 two-step strategy to derive the 'macroscopic' heat equation from the 'microscopic' kinetic equation challenged many experts for its rigorous elaboration. The GG model consists of a chain of localized but interacting hard disks. For a 2-chain, our paradigm replaces one of the disks with a stick moving in an interval with reflecting barriers and interacting rarely with the localized disk. For this model - realizing the first, truly dynamical part of the GG-strategy - we obtain the 'mesoscopic' master equation describing a Markov jump process for the energies of the particles. A byproduct of our arguments: the calculation of the transition kernel provides at the same time rigorous foundation for *Boltzmann's microscopic chaos hypothesis*. The results are joint with Péter Bálint, Péter Nándori and Imre Péter Tóth.

**Carlangelo Liverani** (Università Roma Tor Vergata)  
Beyond averaging in fast-slow partially hyperbolic systems

Averaging theory provides a description for the motion on a moderately long time scale in a wide class of fast-slow systems. Yet, it is natural to wonder if something can be said for arbitrarily long times. At the moment almost nothing is known and it is not clear if there is any hope to develop a fairly general theory. To try to get some insight on the situation I will describe one of the simplest (non trivial) possible examples. I will illustrate what can be proven with particular emphasis on new limit theorems for the slow variable.

# Participants

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